Separation axioms in generalized fuzzy soft topological spaces with sense of Ganguly and Saha

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Abstract - *The topological structure of generalized fuzzy soft sets introduced by Chakraborty and Mukherjee [14]. Khedr et al [3] defined separation axioms in generalized fuzzy soft topological spaces by using generalized fuzzy soft quasi$-$coincident relation and generalized fuzzy soft $Q-$neighborhood system. In this paper, we introduce the separation axioms using generalized fuzzy soft quasi$-$coincident with sense of Ganguly and Saha [15]. Also, we give some basic theorems of separation axioms in generalized fuzzy soft topological spaces.*

Keywords: Fuzzy soft set, generalized fuzzy soft set, generalized fuzzy soft topology, generalized fuzzy soft separation axioms.

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I. INTRODUCTION

Most of our real live problems in engineering, social and medical science, environment, economics, etc. have uncertainties. Several set theories have been given in order to mathematically model these uncertainties. Soft sets, fuzzy soft sets and generalized fuzzy soft sets are leading two of these theories.

In recent times, the process of fuzzification of soft set theory is rapidly progressed. In 2010, Majumdar and Samanta [12] introduced the generalized fuzzy soft set. In 2015, Chakraborty and Mukherjee [14] defined the topological structure of generalized fuzzy soft sets. To improve this concept many researchers studied on this field.

The concept of separation axioms is one of the most important concepts in topological spaces. In fuzzy soft topological space has been studied by Mahanta and Das [8], and Khedr et al. [6]. Some others ([7], [9]) studied some separation axioms of intuitionistic fuzzy soft separation axioms and established several equivalent forms of fuzzy soft spaces. Khedr et al. [2] mentioned separation axioms in generalized fuzzy soft topological spaces. They [3] introduced separation axioms in generalized fuzzy soft topological spaces by using generalized fuzzy soft quasi-coincident relation and generalized fuzzy soft $Q-$neighborhood system.

In our present article, we introduced new separation axioms, which are more general than separation axioms of Khedr et al. [2,3], by the sense of Ganguly and Saha [15] in generalized fuzzy soft topological spaces. By using this notions, we also give some basic theorems which are important for separation axioms and taking place in classical topological spaces.

II. PRELIMINARIES

In this section, we will give some fundamental definitions and theorems about generalized fuzzy soft sets which will be needed in the sequel.

Definition 2.1. [10] Let X be a non-empty set. A fuzzy set A in X is defined by a membership function $\mu_A : X \to [0,1]$ whose value $\mu_A(x)$ represents the "grade of membership" of x in A for $x \in X$. The set of all fuzzy sets in a set X is denoted by I^X , where I is the closed unit interval $[0.1]$.

Definition 2.2. [1] Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denotes the power set of X and $A \subseteq E$. A pair (f, A) is called a soft set over X if f is a mapping from A into $P(X)$ i.e.,

$f : A \to P(X)$ In other words, a soft set is a parameterized family of subsets of the set X. For $e \in A$, $f(e)$ may be considered as the set of $e-$approximate elements of the soft set (f, A).

Definition 2.3. [11, 16] Let X be an initial universe set and E be a set of parameters. Let $A \subseteq E$. A fuzzy soft set f_A over X is a mapping from E to I^X, i.e., $f_A : E \to I^X$, where $f_A(e) \neq 0_X$ if $e \in A \subseteq E$, and $f_A(e) = 0_X$ if $e \notin A$, where 0_X denoted empty fuzzy set in X.

Definition 2.4. [12] Let X be a universal set of elements and E be a universal set of parameters for X. Let $F : E \to I^X$ and μ be a fuzzy subset of E, i.e., $\mu : E \to I$. Let F_μ be the mapping $F_\mu : E \to I^X \times I$ defined as follows:
$F_\mu(e) = (F(e), \mu(e))$, where $F(e) \in I^X$ and $\mu(e) \in I$. Then F_μ is called a generalized fuzzy soft set ($GFSS$ in short) over (X, E). The family of all these generalized fuzzy soft sets over (X, E) denoted by $GFSS(X, E)$.

Definition 2.5. [12] Let F_μ and G_δ be two $GFSSs$ over (X, E). F_μ is said to be a GFS subset of G_δ denoted by $F_\mu \tilde{\subseteq} G_\delta$ if

(i) μ is a fuzzy subset of δ (ii) $F(e)$ is also a fuzzy subset of $G(e)$ $\forall e \in E$.

Definition 2.6. [12] Let F_μ be a $GFSS$ over (X, E). The complement of F_μ, denoted by F_μ^c, is defined by $F_\mu^c = G_\delta$, where $\delta(e) = \mu^c(e)$ and $G(e) = F^c(e)$, $\forall e \in E$. Obviously $(F_\mu^c)^c = F_\mu$.

Definition 2.7. [14] Let F_μ and G_δ be two $GFSSs$ over (X, E). The union of F_μ and G_δ, denoted by $F_\mu \tilde{\cup} G_\nu$, is The $GFSS$ H_ν, defined as $H_\nu : E \to I^X \times I$ such that $H_\nu(e) = (H(e), \nu(e))$, where $H(e) = F(e) \vee G(e)$ and $\nu(e) = \mu(e) \vee \delta(e)$ $\forall e \in E$.

Definition 2.8. [14] Let F_μ and G_δ be two $GFSSs$ over (X, E). The Intersection of F_μ and G_δ, denoted by $F_\mu \tilde{\cap} G_\nu$, is the $GFSS$ M_σ, defined as $M_\sigma : E \to I^X \times I$ such that $M_\sigma(e) = (M(e), \nu(e))$, where $M(e) = F(e) \wedge G(e)$ and $\sigma(e) = \mu(e) \wedge \delta(e)$ $\forall e \in E$.

Definition 2.9. [12] A $GFSS$ is said to be a generalized null fuzzy soft set, denoted by $\tilde{0}_\theta$, if $\tilde{0}_\theta : E \to I^X \times I$ such that $\tilde{0}_\theta(e) = (\tilde{0}(e), \theta(e))$ where $\tilde{0}(e) = \bar{0}$ and $\theta(e) = 0$ $\forall e \in E$ (Where $\bar{0}(x) = 0$ $\forall x \in X$).

Definition 2.10. [12] A $GFSS$ is said to be a generalized absolute fuzzy soft set, denoted by $\tilde{1}_\Delta$, if $\tilde{1}_\Delta : E \to I^X \times I$, where $\tilde{1}_\Delta(e) = (\tilde{1}(e), \Delta(e))$ is defined by $\tilde{1}(e) = \bar{1}$ and $\Delta(e) = 1$ $\forall e \in E$ (Where $\bar{1}(x) = 1$ $\forall x \in X$).

Definition 2.11. [5] Let $GFSS(X,E_1)$ and $GFSS(Y,E_2)$ be the families of all generalized fuzzy soft sets over (X,E_1) and (Y,E_2), respectively. Let $u:X\to Y$ and $p:E_1\to E_2$ be two functions. Then a mapping $f_{up}:GFSS(X,E_1)\to GFSS(Y,E_2)$ is defined as follows: for a generalized fuzzy soft set

$F_\mu\in GFSS(X,E_1)$, $\forall e'\in p(E)\subseteq E_2$ and $y\in Y$. Then

$$f_{up}(F_\mu(e')(y)=\left(\vee_{x\in u^{-1}(y)}\vee_{e\in p^{-1}(e')}F(e)(x),\vee_{e\in p^{-1}(e')}\mu(e)\right)\text{ if } u^{-1}(y)\neq\phi, p^{-1}(e')\neq\phi,$$

$$=(0,0)\quad\text{otherwise.}$$

f_{up} is called a generalized fuzzy soft mapping [GFS mapping for short] and $f_{up}(F_\mu)$ is called a GFS image of a $GFSS$ F_μ.

Definition 2.12. [5] Let $u:X\to Y$ and $p:E_1\to E_2$ be mappings. Let $f_{up}:GFSS(X,E_1)\to GFSS(Y,E_2)$ be a GFS mapping and $G_\delta\in GFSS(Y,E_2)$.
Then, $f_{up}^{-1}(G_\delta)\in GFSS(X,E_1)$ defined as follows:

$$f_{up}^{-1}(G_\delta)(e)(x)=\left(G(p(e))(u(x),\delta(p(e))\right)\quad\text{for } e\in E_1, x\in X.$$

$f_{up}^{-1}(G_\delta)$ is called a GFS inverse image of G_δ.
If u and p are injective then the generalized fuzzy soft mapping f_{up} is said to be generalized fuzzy soft injective (GFS injective for short). If u and p are surjective then the generalized fuzzy soft mapping f_{up} is said to be generalized fuzzy soft surjective (GFS surjective for short). The generalized fuzzy soft mapping f_{up} is called generalized fuzzy soft constant (GFS constant for short), if u and p are constant.

Proposition 2.13. [5] Let $F_\mu,H_\nu\in GFSS(X,E_1)$ and $G_\delta,M_\sigma\in GFSS(Y,E_2)$ For the generalized fuzzy soft mapping $f_{up}:GFSS(X,E_1)\to GFSS(Y,E_2)$, the following statements hold:
(1) If $F_\mu\,\tilde{\subseteq}\,H_\nu$, then $f_{up}(F_\mu)\,\tilde{\subseteq}\,f_{up}(H)\forall F_\mu,H_\nu\in GFSS(X,E_1)$,
(2) If $G_\delta\,\tilde{\subseteq}\,M_\sigma$, then $f_{up}^{-1}(G_\delta)\,\tilde{\subseteq}\,f_{up}^{-1}(M_\sigma)\forall G_\delta,M_\sigma\in GFSS(Y,E_2)$.

Definition 2.14. [4] The generalized fuzzy soft set $F_\mu\in GFSS(X,E)$ is called a generalized fuzzy soft point (GFS point in short) if there exist $e\in E$ and $x\in X$ such that
(i) $F(e)(x)=\alpha(0\prec\alpha\leq 1)$ and $F(e)(y)=0$ for all $y\in X-\{x\}$,
(ii) $\mu(e)=\lambda(0\prec\lambda\leq 1)$ and $\mu(e')=0$ for all $e'\in E-\{e\}$. We denote this generalized fuzzy soft point $F_\mu=(x_\alpha,e_\lambda)$.
(x,e) and (α,λ) are called respectively, the support and the value of x_α,e_λ. The class of all GFS points in (X,E), denoted by $GFSP(X,E)$. Two GFS points (x_α,e_λ) and (y_β,e'_γ) are said to be distinct if $e\neq e'$.

Definition 2.15. [4] Let F_μ be a $GFSS$ over (X,E). We say that $(x_\alpha,e_\lambda)\,\tilde{\in}\,F_\mu$ read as (x_α,e_λ) belongs to the $GFSS$ F_μ if for the element $e\in E,\alpha\leq F(e)(x)$ and $\lambda\leq\mu(e)$.

Evidently, every $GFSS$ F_μ can be expressed as the union of all the GFS points which belong to F_μ.

Definition 2.16. [13] For any two $GFSSs$ F_μ and G_δ over (X,E). F_μ is said to be a generalized fuzzy soft quasi−coincident [GFS quasi−coincident in short] with G_δ, denoted by $F_\mu q G_\delta$, if there exist $e \in E$ and $x \in X$ such that $F(e)(x)+G(e)(x) \succ 1$ and $\mu(x)+\delta(e) \succ 1$.

If F_μ is not [GFS quasi−coincident with G_δ, then we write $F_\mu \bar{q} G_\delta$, i.e., for every $e \in E$ and $x \in X$, $F(e)(x)+G(e)(x) \le 1$ or for every $e \in E$ and $x \in X$, $\mu(x)+\delta(e) \le 1$.

Definition 2.17. [13] Let (x_α, e_λ) be a GFS point and F_μ be a $GFSS$ over (X,E). (x_α, e_λ) is said to be [GFS quasi−coincident with F_μ, denoted by $(x_\alpha, e_\lambda) q F_\mu$, if and only if there exists an element $e \in E$ such that $\alpha + F(e)(x) \succ 1$ and $\lambda + \mu(e) \succ 1$.

Theorem 2.18. [13] Let $F_\mu, G_\delta \in GFSS(X,E)$ and $(x_\alpha, e_\lambda) \in GFSP(X,E)$. Then:

(1) $F_\mu \bar{q} G_\delta \Rightarrow F_\mu \tilde{\subseteq} G_\delta^c$,

(2) $F_\mu q G_\delta \Rightarrow F_\mu \tilde{\cap} G_\delta \ne \tilde{0}_\theta$,

(4) $F_\mu \bar{q} F_\mu^c$,

(3) $(x_\alpha, e_\lambda) \bar{q} F_\mu \Leftrightarrow (x_\alpha, e_\lambda) \tilde{\notin} F_\mu^c$.

Proposition 2.19. [3] Let $F_\mu, G_\delta, H_\nu \in GFSS(X,E)$ and $(x_\alpha, e_\lambda), (y_\beta, e'_\gamma) \in GFSP(X,E)$. Then:

(1) $F_\mu \bar{q} G_\delta \Leftrightarrow G_\delta \bar{q} F_\mu$,

(2) $F_\mu \tilde{\cap} G_\delta = \tilde{0}_\theta \Rightarrow F_\mu \bar{q} G_\delta$,

(3) $F_\mu \bar{q} G_\delta, H_\nu \tilde{\subseteq} G_\delta \Rightarrow F_\mu \bar{q} H_\nu$,

(4) $F_\mu q G_\delta \Leftrightarrow$ there exists an $(x_\alpha, e_\lambda) \tilde{\in} F_\mu$ such that $(x_\alpha, e_\lambda) q G_\delta$,

(5) $F_\mu \tilde{\subseteq} G_\delta \Leftrightarrow [(x_\alpha, e_\lambda) q F_\mu \Rightarrow (x_\alpha, e_\lambda) q G_\delta]$ or $[(x_\alpha, e_\lambda) \bar{q} G_\delta \Rightarrow (x_\alpha, e_\lambda) \bar{q} F_\mu$,

(8) $(x_\alpha, e_\lambda) \bar{q} (y_\beta, e')_\gamma \Leftrightarrow (x \ne y, e \ne e')$ or $(x = y, e = e'$ but $\alpha + \beta \le 1$ or $\lambda + \gamma \le 1)$ or $(x = y, e \ne e', \alpha + \beta \succ 1)$ or $(x \ne y, e = e', \lambda + \gamma \succ 1)$,

(9) $x \ne y$ or $e \ne e' \Rightarrow (x_\alpha, e_\lambda) \bar{q} (y_\beta, e')_\gamma \, \forall \alpha, \beta, \lambda, \gamma \in I$ and $\forall e, e' \in E$.

Definition 2.20. [14] Let T be a collection of generalized fuzzy soft sets over (X,E). Then T is said to be a generalized fuzzy soft topology ($GFST$ in short) over (X,E) if the following conditions are satisfied:

(i) $\tilde{0}_\theta$ and $\tilde{1}_\Delta$ are in T,

(ii) Arbitrary GFS unions of members of T belong to T,

(iii) Finite GFS intersections of members of T belong to T.

The triple (X,T,E) is called a generalized fuzzy soft topological space ($GFST-$space, in short) over (X,E). The members of T are called generalized fuzzy soft open sets [GFS open in short] in (X,T,E).

Definition 2.21. [5] Let (X,T,E) be a $GFST-$ space. A $GFSS$ F_μ in $GFSS(X,E)$ is called generalized fuzzy soft $Q-$neighborhood (briefly, $GFSQ-$nbd) of H_ν [resp. (x_α,e_λ)] if there exists $G_\delta \in T$ such that $H_\nu q G_\delta$ and $G_\delta \tilde{\subseteq} F_\mu$ [resp. $(x_\alpha,e_\lambda) q G_\delta$ and $G_\delta \tilde{\subseteq} F_\mu$].

The family of all $GFSQ-$nbds of H_ν [resp. (x_α,e_λ)], denoted by $N_q(H_\nu)$ [resp. $N_q(x_\alpha,e_\lambda)$].

Definition 2.22. [14,4] Let (X,T,E) be a $GFST-$space. A $GFSS$ F_μ in $GFSS(X,E)$ is called generalized fuzzy soft neighborhood (briefly, $GFS-$nbd) of H_ν [resp. (x_α,e_λ)] if there exists $G_\delta \in T$ such that $H_\nu \tilde{\subseteq} G_\delta \tilde{\subseteq} F_\mu$ [resp. $(x_\alpha,e_\lambda) \tilde{\subseteq} G_\delta \tilde{\subseteq} F_\mu$].

The family of all $GFS-$nbds of H_ν [resp. (x_α,e_λ)], denoted by $N(H_\nu)$ [resp. $N(x_\alpha,e_\lambda)$].

Theorem 2.23. [5] Let $F_\mu \in GFSS(X,E)$ and $(x_\alpha,e_\lambda) \in GFSP(X,E)$. Then $(x_\alpha,e_\lambda) \tilde{\in} cl(F_\mu)$ if and only if each open $GFSQ-$nbd of (x_α,e_λ) is GFS quasi$-$coincident with F_μ.

Definition 2.24. [5] Let (X,T_1,E_1) and (Y,T_2,E_2) be two $GFST-$ space, and $f_{up}:(X,T_1,E_1)\to(Y,T_2,E_2)$ be a GFS mapping. Then f_{up} is called generalized fuzzy soft continuous [$GFS-$continuous for short] if $f_{up}^{-1}(G_\delta)\in T_1$ for all $G_\delta \in T_2$.

Theorem 2.25. [5] Let (X,T_1,E_1) and (Y,T_2,E_2) be two $GFST-$ spaces. For a GFS mapping $f_{up}:(X,T_1,E_1)\to(Y,T_2,E_2)$, the following statements are equivalent:

(1) f_{up} is $GFS-$continuous,

(2) for $GFSS$ F_μ in $GFSS(X,E)$, the inverse image of every $GFS-$nbd of $f_{up}(F_\mu)$ is a $GFS-$nbd of F_μ,

(3) for each $GFSS$ F_μ in $GFSS(X,E)$ and each $GFS-$ nbd M_σ of $f_{up}(F_\mu)$, there is a $GFS-$ nbd H_ν of F_μ such that $f_{up}(H_\nu)\tilde{\subseteq} M_\sigma$.

III. GENERALIZED FUZZY SOFT QUASI SEPARATION AXIOMS

Definition 3.1 [2] **.** A $GFST-$ space (X,T,E) is said to be:

(1) generalized fuzzy soft T_0-space ($GFST_0-$space for short) if for every pair of distinct GFS points $(x_\alpha,e_\lambda),(y_\beta,e'_\gamma)$ there exists a GFS open set containing one of the points but not the other,

(2) generalized fuzzy soft T_1-space ($GFST_1-$space for short) if for every pair of distinct GFS points $(x_\alpha,e_\lambda),(y_\beta,e'_\gamma)$ there exists a GFS open sets F_μ and G_δ such that $(x_\alpha,e_\lambda)\tilde{\in} F_\mu$, $(y_\beta,e'_\gamma)\tilde{\notin} F_\mu$ and $(y_\beta,e'_\gamma)\tilde{\in} G_\delta$, $(x_\alpha,e_\lambda)\tilde{\notin} G_\delta$,

(3) generalized fuzzy soft T_2-space ($GFST_2-$space for short) if for every pair of distinct GFS points $(x_\alpha,e_\lambda),(y_\beta,e'_\gamma)$ there exists disjoint GFS open sets F_μ and G_δ such that $(x_\alpha,e_\lambda)\tilde{\in} F_\mu$ and $(y_\beta,e'_\gamma)\tilde{\in} G_\delta$,

(4) generalized fuzzy soft regular space GFS regular space for short) if for every GFS closed set H_ν and and every GFS point (x_α,e_λ) such that $(x_\alpha,e_\lambda)\tilde{\cap} H_\nu$ there exist disjoint GFS open sets F_μ and G_δ

such that $(x_\alpha, e_\lambda) \tilde{\in} F_\mu$ and $H_\nu \tilde{\subseteq} G_\delta$. (X,T,E) is called a generalized fuzzy soft $GFST_3-$ space ($GFST_3-$space for short) if it is GFS regular and $GFST_1-$space,

(5) generalized fuzzy soft normal space (GFS normal space for short) if for every disjoint GFS closed sets H_ν, K_γ there exist disjoint GFS open sets M_ψ and N_η such that $H_\nu \tilde{\subseteq} M_\psi$, $K_\gamma \tilde{\subseteq} N_\eta$. (X,T,E) is called a generalized fuzzy soft $GFST_4-$space ($GFST_4-$space for short) if it is GFS normal and $GFST_1-$space.

Definition 3.2 [3] **.** A $GFST-$space (X,T,E) is said to be:

(1) generalized fuzzy soft quasi T_0- space ($GFSQ-T_0-$ space for short) if for every $(x_\alpha, e_\lambda),(y_\beta, e'_\gamma) \in GFSP(X,E)$ with $(x_\alpha, e_\lambda)\bar{q}(y_\beta, e'_\gamma)$ implies there exist $O_{(x_\alpha, e_\lambda)} \tilde{\in} N_q(x_\alpha, e_\lambda)$ such that $O_{(x_\alpha, e_\lambda)}\bar{q}(y_\beta, e'_\gamma)$ or there exist $O_{(y_\beta, e'_\gamma)} \tilde{\in} N_q(y_\beta, e'_\gamma)$ such that $O_{(y_\beta, e'_\gamma)}\bar{q}(x_\alpha, e_\lambda)$,

(2) generalized fuzzy soft quasi T_1- space ($GFSQ-T_1-$ space for short) if for every $(x_\alpha, e_\lambda),(y_\beta, e'_\gamma) \in GFSP(X,E)$ with $(x_\alpha, e_\lambda)\bar{q}(y_\beta, e'_\gamma)$ implies there exist $O_{(x_\alpha, e_\lambda)} \tilde{\in} N_q(x_\alpha, e_\lambda)$ such that $O_{(x_\alpha, e_\lambda)}\bar{q}(y_\beta, e'_\gamma)$ and there exist $O_{(y_\beta, e'_\gamma)} \tilde{\in} N_q(y_\beta, e'_\gamma)$ such that $O_{(y_\beta, e'_\gamma)}\bar{q}(x_\alpha, e_\lambda)$,

(3) generalized fuzzy soft quasi T_2- space ($GFSQ-T_2-$ space for short) if for every $(x_\alpha, e_\lambda),(y_\beta, e'_\gamma) \in GFSP(X,E)$ with $(x_\alpha, e_\lambda)\bar{q}(y_\beta, e'_\gamma)$ implies there exist $O_{(x_\alpha, e_\lambda)} \tilde{\in} N_q(x_\alpha, e_\lambda)$ and $O_{(y_\beta, e'_\gamma)} \tilde{\in} N_q(y_\beta, e'_\gamma)$ such that $O_{(x_\alpha, e_\lambda)}\bar{q}O_{(y_\beta, e'_\gamma)}$,

(4) generalized fuzzy soft quasi T_3- space ($GFSQ-T_3-$ space for short) if $GFSQ$ regular and $GFSQ-T_1-$space,

(5) generalized fuzzy soft quasi T_4- space ($GFSQ-T_4-$ space for short) if $GFSQ$ normal and $GFSQ-T_1-$space.

Definition 3.3. Let (X,T,E) be a $GFST-$space and $(x_\alpha, e_\lambda),(y_\beta, e'_\gamma) \in GFSP(X,E)$. If there exist GFS open sets F_μ and G_δ such that

(a) When $e \neq e'$ or $x \neq y$, $F_\mu \in N(x_\alpha, e_\lambda)$, $F_\mu\bar{q}(y_\beta, e'_\gamma)$ or $G_\delta \in N(y_\beta, e'_\gamma)$, $G_\delta\bar{q}(x_\alpha, e_\lambda)$.

(b) When $e = e'$, $x = y$ and $\alpha \prec \beta, \lambda \prec \gamma$ (say), $F_\mu \in N_q(y_\beta, e'_\gamma)$ such that $(x_\alpha, e_\lambda)\bar{q}F_\mu$.

Then (X,T,E) is called a generalized fuzzy soft $q-T_0-$space ($GFS\ q-T_0-$ space for short).

Theorem 3.4. Let (X,T,E) be a $GFST-$ space and (X,T,E) $GFS\ q-T_0$. Then (X,T,E) is $GFST_0$.

Proof. Let (X,T,E) be a $GFST-$space and (X,T,E) $GFS\ q-T_0$. Suppose that (X,T,E) is not $GFST_0$. Then there exist distinct GFS points $(x_\alpha, e_\lambda),(y_\beta, e'_\gamma)$ such that for every GFS open set G_δ

which is containing $(x_\alpha,e_\lambda),(y_\beta,e'_\gamma)$ is GFS subset G_δ. Since (x_α,e_λ) and (y_β,e'_γ) disjoint GFS sets and $(x_\alpha,e_\lambda)\tilde{\in}(x_\alpha,e_\lambda)\tilde{\subseteq}G_\delta$, $(y_\beta,e'_\gamma)\tilde{\in}(y_\beta,e'_\gamma)\tilde{\subseteq}G_\delta$, $G_\delta\in N(x_\alpha,e_\lambda)$ and $\beta\le G(e')(y),\gamma\le\delta(e')$. Now,

Case I. When $\beta\succ 0.5,\gamma\succ 0.5$, then $G(e')(y)+\beta\succ 1$, $\delta(e')+\gamma\succ 1$. Therefore, we have $G_\delta q(y_\beta,e'_\gamma)$. This is contradiction.

Case II. When $\beta\le 0.5,\gamma\le 0.5$, if we choose $\theta\succ 1-\beta,\vartheta\succ 1-\gamma$, then $G_\delta\tilde{\cup}(y_\theta,e'_\vartheta)\in N(x_\alpha,e_\lambda)$ and $G(e')(y)\vee y_\theta(y)+\beta\succ 1$, $\delta(e')\vee e'_\vartheta(y)+\gamma\succ 1$. Therefore, we have $[G_\delta\tilde{\cup}(y_\theta,e'_\vartheta)]q(y_\beta,e'_\gamma)$. This is contradiction.

Theorem 3.5. (X,T,E) $GFST-$space is $GFS\ q-T_0$ if and only if for every pair of distinct GFS points (x_α,e_λ) and (y_β,e'_γ), $(x_\alpha,e_\lambda)\tilde{\notin}cl(y_\beta,e'_\gamma)$ or $(y_\beta,e'_\gamma)\tilde{\notin}cl(x_\alpha,e_\lambda)$.

Proof. Let (X,T,E) be GFS, (x_α,e_λ) and (y_β,e'_γ) be two distinct GFS points in $GFSP(X,E)$.

Case I. When $e\ne e'$ or $x\ne y$, (x_α,e_λ) has a $GFS-$nbd F_μ such that $F_\mu\bar{q}(y_\beta,e'_\gamma)$ or (y_β,e'_γ) has a $GFS-$nbd G_δ such that $G_\delta\bar{q}(x_\alpha,e_\lambda)$. Suppose (x_α,e_λ) has a $GFS-$nbd F_μ such that $F_\mu\bar{q}(y_\beta,e'_\gamma)$. Then F_μ is a $GFS-$nbd of (x_α,e_λ) and $F_\mu\bar{q}(y_\beta,e'_\gamma)$. Hence $(x_\alpha,e_\lambda)\tilde{\notin}cl(y_\beta,e'_\gamma)$.

Case II. When $e=e'$ and $x=y$ and $\alpha\prec\beta,\lambda\prec\gamma$ (say), then (y_β,e'_γ) has a $GFSQ-$nbd which is not GFS quasi$-$coincident with (x_α,e_λ) and so in this case also $(y_\beta,e'_\gamma)\tilde{\notin}cl(x_\alpha,e_\lambda)$.

Conversely, let (x_α,e_λ) and (y_β,e'_γ) be two distinct GFS points in $GFSP(X,E)$. We suppose without loss of generality, that $(x_\alpha,e_\lambda)\tilde{\notin}cl(y_\beta,e'_\gamma)$. When $e\ne e'$ or $x\ne y$, since $(x_\alpha,e_\lambda)\tilde{\notin}cl(y_\beta,e'_\gamma)$ for all $0\prec\alpha,\lambda\le 1$, $(y_\beta,e'_\gamma)=(0,0)$ and hence $(cl(y_\beta,e'_\gamma))^c(e)(x)=(1,1)$. Then $(cl(y_\beta,e'_\gamma))^c$ is a $GFS-$nbd of (x_α,e_λ) such that $(cl(y_\beta,e'_\gamma))^c\bar{q}(y_\beta,e'_\gamma)$. Also, in case $e=e'$ and $x=y$ we must have $\alpha\succ\beta$, $\lambda\succ\gamma$ and then (x_α,e_λ) has a $GFSQ-$nbd which is not GFS quasi$-$coincident with (y_β,e'_γ).

Definition 3.6. Let (X,T,E) be a $GFST-$space and $(x_\alpha,e_\lambda),(y_\beta,e'_\gamma)\in GFSP(X,E)$. If there exist GFS open sets F_μ and G_δ such that

(a) When $e\ne e'$ or $x\ne y$, $F_\mu\in N(x_\alpha,e_\lambda)$, $F_\mu\bar{q}(y_\beta,e'_\gamma)$ and $G_\delta\in N(y_\beta,e'_\gamma)$, $G_\delta\bar{q}(x_\alpha,e_\lambda)$.

(b) When $e=e'$, $x=y$ and $\alpha\prec\beta,\lambda\prec\gamma$ (say), $F_\mu\in N_q(y_\beta,e'_\gamma)$ such that $(x_\alpha,e_\lambda)\bar{q}F_\mu$.

Then (X,T,E) is called a generalized fuzzy soft $q-T_1-$space ($GFS\ q-T_1-$ space for short).

Theorem 3.7. Let (X,T,E) be a $GFST-$ space and (X,T,E) $GFS\ q-T_1$. Then (X,T,E) is $GFST_1$.

Proof. The proof is similar with the proof of Theorem 3.4.

Theorem 3.8. (X,T,E) is $GFS\ q-T_1$. if and only if each $(x_\alpha,e_\lambda)\in GFSP(X,E)$ is a GFS closed set.

Proof. Suppose that for each $(x_\alpha,e_\lambda)\in GFSP(X,E)$ is a GFS closed set. Then $(x_\alpha,e_\lambda)^c$ is a GFS open set. Let $(x_\alpha,e_\lambda),(y_\beta,e'_\gamma)\in GFSP(X,E)$.

Case I. When $e\neq e'$ or $x\neq y$, for $(x_\alpha,e_\lambda)\in GFSP(X,E)$, $(x_\alpha,e_\lambda)^c$ is a GFS open set such that $(x_\alpha,e_\lambda)^c\,\tilde{\in}\,N(y_\beta,e'_\gamma)$ and $(x_\alpha,e_\lambda)\bar{q}(x_\alpha,e_\lambda)^c$.

Similarly $(y_\beta,e'_\gamma)^c$ is a GFS open set such that $(y_\beta,e'_\gamma)^c\,\tilde{\in}\,N(x_\alpha,e_\lambda)$ and $(y_\beta,e'_\gamma)\bar{q}(y_\beta,e'_\gamma)^c$.

Case II. When $e=e'$, $x=y$ and $\alpha\prec\beta,\lambda\prec\gamma$ (say), then (y_β,e'_γ) has a $GFSQ-$nbd $(x_\alpha,e_\lambda)^c$ which is not GFS quasi$-$coincident with (x_α,e_λ). Thus (X,T,E) is a $GFS\ q-T_1$.space.

Conversely, let (X,T,E) be a $GFS\ q-T_1$. space. Suppose that each GFS point (x_α,e_λ) is not GFS closed set in T. Then $(x_\alpha,e_\lambda)\neq cl(x_\alpha,e_\lambda)$ and there exist $(y_\beta,e'_\gamma)\,\tilde{\in}\,cl(x_\alpha,e_\lambda)$ such that $(x_\alpha,e_\lambda)\neq(y_\beta,e'_\gamma)$.

Case I. When $e\neq e'$ or $x\neq y$. Suppose that $\beta,\gamma\leq 0.5$. Since $(y_\beta,e'_\gamma)\,\tilde{\in}\,cl(x_\alpha,e_\lambda)$, by theorem 2.23 for each $F_\mu\in N_q(y_\beta,e'_\gamma)$, $F_\mu q(x_\alpha,e_\lambda)$ Then there exist GFS open set H_ν such that $(y_\beta,e'_\gamma)qH_\nu$, $H_\nu\,\tilde{\subseteq}\,F_\mu$. Hence $H(e')(y)+\beta\succ 1$, $\nu(e')+\gamma\succ 1$ and $H(e')(y)\succ 1-\beta$, $\nu(e')\succ 1-\gamma$.

Since $(y_\beta,e'_\gamma)\,\tilde{\subseteq}\,(y_{1-\beta},e'_{1-\gamma})\,\tilde{\subseteq}\,H_\nu\,\tilde{\subseteq}\,F_\mu$, we have $GFSQ-$ nbd F_μ of (y_β,e'_γ) such that $F_\mu q(x_\alpha,e_\lambda)$. This is contradiction. If $\beta,\gamma\succ 0.5$, we choose $1-\beta$, $1-\gamma$ the proof can be done as above.

Case II. When $e=e'$, $x=y$ and $\alpha\prec\beta,\lambda\prec\gamma$ (say), Since $(y_\beta,e'_\gamma)\,\tilde{\in}\,cl(x_\alpha,e_\lambda)$, by theorem 2.23 for each $F_\mu\in N_q(y_\beta,e'_\gamma)$, $F_\mu q(x_\alpha,e_\lambda)$. This is contradiction.

Definition 3.9. Let (X,T,E) be a $GFST-$space and $(x_\alpha,e_\lambda),(y_\beta,e'_\gamma)\in GFSP(X,E)$. If there exist GFS open sets F_μ and G_δ such that

(a) When $e\neq e'$ or $x\neq y$, $F_\mu\in N(x_\alpha,e_\lambda)$, $G_\delta\in N(y_\beta,e'_\gamma)$ such that $F_\mu\bar{q}G_\delta$.

(b) When $e=e'$, $x=y$ and $\alpha\prec\beta,\lambda\prec\gamma$ (say), $F_\mu\in N(y_\beta,e'_\gamma),G_\delta\in N_q(y_\beta,e'_\gamma)$ such that $F_\mu\bar{q}G_\delta$.

Then (X,T,E) is called a generalized fuzzy soft $q-T_2-$space ($GFS\ q-T_2-$space for short).

Theorem 3.10. Let (X,T,E) be a $GFST-$space and (X,T,E) $GFS\ q-T_2$. Then (X,T,E) is $GFST_2$.

Proof. The proof is similar with the proof of Theorem 3.4.

Remark 3.11. From definitions one deduce the following implications hold:

$$GFS\ q-T_2\ \Rightarrow\ GFS\ q-T_1\Rightarrow\ GFS\ q-T_0$$

Theorem 3.12. (X,T,E) be a $q-T_2$ if and only if for every $(x_\alpha,e_\lambda)=\tilde{\cap}\{cl(F_\mu):F_\mu\in N(x_\alpha,e_\lambda)\}$

Proof. Let (X,T,E) be a GFS $q-T_2$ −space. (x_α,e_λ) and (y_β,e'_γ) are GFS points in $GFSP(X,E)$ such that $(x_\alpha,e_\lambda)\neq(y_\beta,e'_\gamma)$. If $e\neq e'$ or $x\neq y$, then there are GFS open sets F_μ and G_δ containing (y_β,e'_γ) and (x_α,e_λ) respectively such that $F_\mu\bar{q}G_\delta$. Then G_δ is a GFS open −nbd of (x_α,e_λ) and F_μ is a $GFSQ$ open −nbd of (y_β,e'_γ) such that $F_\mu\bar{q}G_\delta$ i.e., $F_\mu\bar{q}(y_\beta,e'_\gamma)$. Hence $(y_\beta,e'_\gamma)\tilde{\notin}cl(G_\delta)$.

If $e=e'$, $x=y$, then $\alpha\prec\beta,\lambda\prec\gamma$ and hence there are a $GFSQ$ −nbd F_μ of (y_β,e'_γ) and GFS −nbd of (x_α,e_λ) such that $F_\mu\bar{q}G_\delta$. Hence $(y_\beta,e'_\gamma)\tilde{\notin}cl(G_\delta)$.

Conversely, let (x_α,e_λ) and (y_β,e'_γ) be two distinct GFS points in $GFSP(X,E)$.

Case I. When $e\neq e'$ or $x\neq y$. We first suppose that $0\prec\alpha,\lambda\prec1$ or $0\prec\beta,\gamma\prec1$, say $0\prec\alpha,\lambda\prec1$. Then there exist a positive real numbers r,s with $0\prec\alpha+s\prec1$ and $0\prec\lambda+r\prec1$.

By hypothesis, there exists a GFS open −nbd F_μ of (y_β,e'_γ) such that $(x_s,e_r)\tilde{\notin}cl(F_\mu)$. Then (x_s,e_r) has a $GFSQ$ open − nbd G_δ such that $G_\delta\bar{q}F_\mu$. Now, $s+G(e)(x)\succ1$ and $r+\delta(e)\succ1$ so that $G(e)(x)\succ1-s\succ\alpha$ and $\delta(e)\succ1-r\succ\lambda$ and hence G_δ is a GFS − nbd of (x_α,e_λ) such that $F_\mu\bar{q}G_\delta$, where F_μ is a GFS −nbd of (y_β,e'_γ).

In Case $\alpha=\beta=\lambda=\gamma=1$, by hypothesis, there exist a GFS open −nbd F_μ of (x_α,e_λ) such that $cl(F_\mu)(e')(y)=(0.0)$ i.e., $cl(F)(e')(y)=0$, $cl(\mu)(e')=0$. Then $G_\delta=[cl(F_\mu)]^c$ is a GFS −nbd of (y_β,e'_γ) such that $F_\mu\bar{q}G_\delta$.

Case II. Let $e=e'$, $x=y$ and $\alpha\prec\beta,\lambda\prec\gamma$ (say), then there exists a GFS −nbd F_μ of (x_α,e_λ) such that $(y_\beta,e'_\gamma)\tilde{\notin}cl(F_\mu)$. Consequently, there exist a $GFSQ$ −nbd G_δ of (y_β,e'_γ) such that $F_\mu\bar{q}G_\delta$. Then (X,T,E) is GFS $q-T_2$.

Theorem 3.13. Let $F_\mu,G_\delta\in GFSS(Y,E_2)$, $f_{up}:(X,T_1,E_1)\to(Y,T_2,E_2)$ be GFS mapping and F_μ is not GFS quasi−coincident with G_δ. Then $f_{up}^{-1}(F_\mu)$ is not GFS quasi−coincident with $f_{up}^{-1}(G_\delta)$.

Proof. Let $F_\mu\bar{q}G_\delta$ $\Rightarrow$ For all $k\in E_2$ and $y\in Y$: $F(k)(y)+G(k)(y)\le1$, $\mu(k)+\delta(k)\le1$

$\Rightarrow$ For all $e\in E_1$ and $x\in X$: $F(p(e))(u(x))+G(p(e))(u(x))\le1$, $\mu(p(e))+\delta(p(e))\le1$

$\Rightarrow$ For all $e\in E_1$ and $x\in X$: $f_{up}^{-1}(F)(e)(x)+f_{up}^{-1}(G)(e)(x)\le1$, $f_{up}^{-1}(\mu)(e)+f_{up}^{-1}(\delta)(e)\le1$

$\Rightarrow$ $f_{up}^{-1}(F_\mu)\bar{q}f_{up}^{-1}(G_\delta)$.

Theorem 3.14. Let $f_{up}:(X,T_1,E_1)\to(Y,T_2,E_2)$ be GFS− continuous. Then if corresponding $GFSQ$ open − nbd G_δ of (y_β,e'_γ) in (Y,E_2) there exist a $GFSQ$ open − nbd F_μ of (x_α,e_λ) in (X,E_1) such that $f_{up}(F_\mu)\tilde{\subseteq}G_\delta$, where $f_{up}(x_\alpha,e_\lambda)=(y_\alpha,e'_\lambda)$.

Proof. Let f_{up} be GFS −continuous and let G_δ be a $GFSQ$ open −nbd of (y_β,e'_γ) in (Y,E_2). Then $\alpha+G(e')(y)\succ1$, $\lambda+\delta(e')\succ1$ and hence there exist two positive real number β,γ such that

$G(e')(y) \succ \beta \succ 1-\alpha$, $\delta(e') \succ \gamma \succ 1-\lambda$ so that G_δ is a GFS open $-$nbd of (y_β, e'_γ). Since f_{up} is $GFS-$continuous, there exists a GFS open$-$nbd F_μ of (x_β, e_γ) such that $f_{up}(F_\mu) \tilde{\sqsubseteq} G_\delta$.

Now, $\beta \le F(e)(x)$, $\gamma \le \mu(e)$ implies $1-\alpha \prec F(e)(x)$, $1-\gamma \prec \mu(e)$ and so F_μ a $GFSQ$ open$-$nbd of (y_β, e'_γ).

Theorem 3.15. Let (X, T_1, E_1) be a $GFST-$ space, (Y, T_2, E_2) be a GFS $q-T_2-$ space and $f_{up} : (X, T_1, E_1) \to (Y, T_2, E_2)$ be GFS injective, $GFS-$ continuous mapping. Then (X, T_1, E_1) is a GFS $q-T_2-$space.

Proof. Let (Y, T_2, E_2) be a GFS $q-T_2-$space and $f_{up} : (X, T_1, E_1) \to (Y, T_2, E_2)$ be GFS injective, $GFS-$continuous mapping. $(x_\alpha, e_\lambda), (y_\beta, e'_\gamma) \in GFSP(X, E)$.

Case I. When $e \ne e'$ or $x \ne y$, then $f_{up}(x_\alpha, e_\lambda) = f_{up}(y_\beta, e'_\gamma)$. Then (Y, T_2, E_2) be a GFS $q-T_2-$space, $f_{up}(x_\alpha, e_\lambda)$, $f_{up}(y_\alpha, e'_\lambda)$ have GFS open $-$nbds F_μ, G_δ such that $F_\mu \bar{q} G_\delta$. Then by Theorem 2.25, 3.14 $f_{up}^{-1}(F_\mu)$ and $f_{up}^{-1}(G_\delta)$ are GFS open $-$ nbds of (x_α, e_λ) and (y_β, e'_γ) respectively such that $f_{up}^{-1}(F_\mu) \bar{q} f_{up}^{-1}(G_\delta)$.

Case II. When $e = e'$, $x = y$ and $\alpha \prec \beta, \lambda \prec \gamma$, then $f_{up}(x_\alpha, e_\lambda) = f_{up}(y_\alpha, e'_\lambda)$. Then (Y, T_2, E_2) be a GFS $q-T_2-$ space, $F_\mu \in N(f_{up}(x_\alpha, e_\lambda))$, $G_\delta \in N_q(f_{up}(y_\beta, e'_\gamma))$, such that $F_\mu \bar{q} G_\delta$. Then by Theorem 2.25, 3.14 $f_{up}^{-1}(F_\mu) \in N(x_\alpha, e_\lambda)$ and $f_{up}^{-1}(G_\delta) \in N_q(y_\beta, e'_\gamma)$ such that $f_{up}^{-1}(F_\mu) \bar{q} f_{up}^{-1}(G_\delta)$. Then (X, T_1, E_1) is a GFS $q-T_2-$space.

Definition 3.16. A $GFST$ $-$space (X, T, E) is a generalized fuzzy soft $q-$regular (GFS $q-$regular for short) if and only if for any GFS closed set H_ν in $GFSS(X, E)$ and any GFS point (x_α, e_λ) in $GFSP(X, E)$ such that $(x_\alpha, e_\lambda) \tilde{\not\in} H_\nu$.

(a) When $H(e)(x) = 0$, $\nu(e) = 0$, there are GFS open sets F_μ and G_δ such that $(x_\alpha, e_\lambda) \in F_\mu, H_\nu \tilde{\sqsubseteq} G_\delta$, and $F_\mu \bar{q} G_\delta$.

(b) When $H(e)(x) \ne 0$, $\nu(e) \ne 0$, there are GFS open sets F_μ and G_δ such that $(x_\alpha, e_\lambda) q F_\mu$, $H_\nu \tilde{\sqsubseteq} G_\delta$ and $F_\mu \bar{q} G_\delta$. A GFS $q-T_1$ and GFS $q-$ regular is a generalized fuzzy soft $q-T_3-$ space (GFS $q-T_3-$space for short).

Theorem 3.17. A $GFST-$space (X, T, E) is a GFS $q-$regular if and only if for a GFS point (x_α, e_λ) and any GFS open G_δ in $GFSS(X, E)$ such that $(x_\alpha, e_\lambda) q G_\delta$ there is an GFS open set F_μ such that $(x_\alpha, e_\lambda) q F_\mu$ and $cl(F_\mu) \tilde{\sqsubseteq} G_\delta$.

Proof. Let (X, T, E) be GFS $q-$regular. On the other hand, a GFS point (x_α, e_λ) and GFS open G_δ in $GFSS(X, E)$ are given such that $(x_\alpha, e_\lambda) q G_\delta$. Then G_δ^c is GFS closed set and $(x_\alpha, e_\lambda) \tilde{\not\in} G_\delta^c$.

Case I. When $G^c(e)(x)=0$, $\delta^c(e)=0$, since (X,T,E) is GFS $q-$regular, there are GFS open sets F_μ and H_ν such that $(x_\alpha,e_\lambda)\tilde{\in}F_\mu$, $G^c_\delta\tilde{\subseteq}H_\nu$ and $F_\mu\bar{q}H_\nu$. Then $H^c_\nu\tilde{\subseteq}G_\delta$ and $F_\mu\tilde{\subseteq}H^c_\nu$. Since $cl(F_\mu)$ is smallest GFS closed set which containing F_μ, we have $F_\mu\tilde{\subseteq}cl(F_\mu)\tilde{\subseteq}H^c_\nu\tilde{\subseteq}G_\delta$.

Case II. When $G^c(e)(x)\neq 0$, $\delta^c(e)\neq 0$, there exist GFS open sets F_μ and H_ν such that $(x_\alpha,e_\lambda)qF_\mu$, $G^c_\delta\tilde{\subseteq}H_\nu$ and $F_\mu\bar{q}H_\nu$. Since $cl(F_\mu)$ is smallest GFS closed set which containing F_μ, we have $F_\mu\tilde{\subseteq}cl(F_\mu)\tilde{\subseteq}H^c_\nu\tilde{\subseteq}G_\delta$ and $(x_\alpha,e_\lambda)qF_\mu$.

Conversely, let any GFS closed set H_ν be in $GFSS(X,E)$ and any GFS point (x_α,e_λ) be in $GFSP(X,E)$ such that $(x_\alpha,e_\lambda)\tilde{\notin}H_\nu$. Then H^c_ν is GFS open set and $(x_\alpha,e_\lambda)qH^c_\nu$.

Case I. When $H(e)(x)=0$, $\nu(e)=0$, if $\alpha,\lambda\leq 0.5$, then there exists GFS open set F_μ such that $(x_\alpha,e_\lambda)\tilde{\in}F_\mu$ and $cl(F_\mu)\tilde{\subseteq}H^c_\nu$.

Therefore, we have $(x_\alpha,e_\lambda)\tilde{\in}F_\mu$, $H_\nu\tilde{\subseteq}[cl(F_\mu)]^c$ and $F_\mu\bar{q}[cl(F_\mu)]^c$.

If $\alpha,\lambda\succ 0.5$, then exists GFS open set F_μ such that $(x_{1-\alpha},e_{1-\lambda})\tilde{\in}F_\mu$ and $cl(F_\mu)\tilde{\subseteq}H^c_\nu$. Therefore, we have $(x_\alpha,e_\lambda)\tilde{\in}F_\mu$, $H_\nu\tilde{\subseteq}[cl(F_\mu)]^c$ and $F_\mu\bar{q}[cl(F_\mu)]^c$.

Case II. When $H(e)(x)\neq 0$, $\nu(e)\neq 0$, there exists GFS open set F_μ such that $(x_\alpha,e_\lambda)qF_\mu$ and $cl(F_\mu)\tilde{\subseteq}H^c_\nu$. So, we have $(x_\alpha,e_\lambda)qF_\mu$, $H_\nu\tilde{\subseteq}[cl(F_\mu)]^c$ and $F_\mu\bar{q}[cl(F_\mu)]^c$. Then (X,T,E) is GFS $q-$regular.

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REFERENCES

[1] D. Molodtsov, "*Soft set theory-First results*," Comput. Math. Appl., vol. 37, pp. 19-31, 1999.

[2] F. H. Khedr, and M. S. Malfi, "*Generalized fuzzy soft separation axioms in generalized fuzzy soft topological* spaces." Submitted.

[3] F. H. Khedr and M. S. Malfi, "*Generalized fuzzy soft quasi separation axioms in generalized fuzzy soft topological space*," Submitted.

[4] F. H. Khedr, S. A. Abd El-Baki and M. S. Malfi, "*Results on generalized fuzzy soft topological spaces*," African Journal of Mathematics and Computer Science Research., vol. 11, no. 3, pp. 35-45, 2018.

[5] F. H. Khedr, S. A. Abd El-Baki and M. S. Malfi, "*Generalized fuzzy soft continuity*," Appl. Math. Inf. Sci., vol. 12, no. 5, pp. 1001-1011, 2018.

[6] F. H. Khedr, S. A. Abd El-Baki and M. S. Malfi, "*Fuzzy soft separation axioms in Fuzzy soft topological spaces*," Submitted.

[7] H. Al. Qahtani and AbdulGawad Al-Qubati, "*On b-Separation Axioms in intuistionistic Fuzzy Topological Spaces*," International Journal of Mathematics Trends and Technology., vol. 21, no. 2, pp. 83-93, 2015.

[8] J. Mahanta and P.K. Das, "*Fuzzy soft topological spaces*," Journal of Intelligent & Fuzzy Systems., vol. 32, pp. 443–450, 2017.

[9] K. Borgohain, "*Separation Axioms in Intuitionistic Fuzzy Soft Topological space*," International Journal of Mathematics Trends and Technology., vol. 5, pp. 176-181, 2014.

[10] L. A. Zadeh, "*Fuzzy sets*," Inform and control., vol. 8, pp. 338-353, 1965.

[11] P. K. Maji, R. Biswas and A. R. Roy, "*Fuzzy soft sets*," J. Fuzzy Math., vol. 9, pp. 589-602, 2001.

[12] P. Majumdar and S. K. Samanta, "*Generalised fuzzy soft sets*," Comput. Math. Appl., vol. 59, pp. 1425-1432, 2010.

[13] P. Mukherjee, *Some operators on generalised fuzzy soft topological spaces*," Journal of New Results in Science., vol. 9, pp. 57-65, 2015.

[14] R. P. Chakraborty and P. Mukherjee, "*On generalized fuzzy soft topological spaces*," Afr. J. Math. Comput. Sci. Res., vol. 8, pp. 1-11, 2015.

[15] S. Ganguly and S. Saha, "*On separation axioms and T_i-fuzzy continuity*," Fuzzy Sets and Systems., vol. 16, pp. 265-275, 1985.

[16] S. Roy and T. K. Samanta, "*A note on fuzzy soft topological spaces*," Ann. Fuzzy Math. Inform., vol. 3, no. 2, pp. 305-311, 2011.