Behaviour of the Strange Quark Matter with Domain walls in 5D Kaluaza-Klein Theory of Gravitation

Vrishali U Edlabadkar*, G S Khadekar ** and R Radha ***

*Department of Mathematics, PES's Modern College of Engineering,

Pune - 411005 (INDIA) email:edlabadkar.vrishali@gmail.com

**Dept. of Mathematics, Nagpur University,

Mahatma Jyotiba Phule Educational Campus , Amravati Road,

Nagpur - 440033 , e-mail: gkhadekar@yahoo.com

***Dept. of Maths & Stats., Hyderabad University,

Hyderabad-46, email: repakar@yahoo.com

May 21, 2019

Abstract

We have study quark and strange quark matter coupled with domain wall in the context of Kaluaza-Klein theory of gravitation. We also discuss the features of the obtained solution in the presence of decaying cosmological constant Λ.

1 Introduction

A revolutionary development seems to have taken place in cosmology during the last few years. The latest development of super-string theory and super gravitational theory have created interest among scientists to consider higher dimensional space time, for study of the early universe. A number of authors Zeldovich (1980), Sahdev (1984), Emelynor et. al. (1986), Chatterjee and Bhui (1990, 1993) have studied physics of the universe in higher dimensional space time. Overduin and Wesson (1987) have presented an excellent review of higher dimensional unified theories, in which the cosmological and astrophysical implications of extra-dimension have been discussed.

Kaluza-Klein theory has a long and venerable history. However, the original Kaluza version of this theory
suffered from the assumption that the 5-dimensional metric does not depend on the extra coordinate (the cylinder condition). Hence the proliferation in recent years of various versions of Kaluza-Klein theory, supergravity and superstrings. In the last years number of authors Wesson (1992), chatterjee et. al(1994a), Chakraborty and Roy (1994) have considered multi dimensional cosmological model. Kaluza-Klein achievements is shown that five dimensional general relativity contains both Einsteins four-dimensional theory of gravity and Maxwells theory of electromagnetism.

Authors Chtterjee and Banerjee (1993), Banerjee et. al. (1995) have studied Kaluza-Klein inhomogeneous cosmological model with and without cosmological constants respectively. So far there has been many cosmological solution dealing with higher dimensional model containing a variety of matter field. However,there is a few work in a literature where variable G and Λ. have been consider in higher dimension.

Vishwakarma (2001) has studied the magnitude-redshift relation for the type Ia supernovae data and the angular size-redshift relation for the updated compact radio sources data Gurvits (1999) by considering four variable Λ-models:$\Lambda \sim R^{-2}$, $\Lambda \sim H^{-2}$, $\Lambda \sim \rho$ and $\Lambda \sim t^{-2}$.

Some of the discussions on the cosmological constant problem and consequences on the cosmology with a time-varying cosmological constant have been discussed by Dolgov (1997), Sahni and Starobinsky (2000), Peebles and Ratra (2003), Padmanabhan (2003), Vishwakarma (2000, 2001, 2002), Triay (2005) and Kilinc (2006), Bertolami (1986a, 1986b), Chen and Wu (1990), Carvalho et al. (1992), Berman (1991a, 1991b), Abdel-Rahman (1992), Lima and Maia (1993). Al-Rawaf (1998) and Overduin and Cooperstock (1998) have proposed a cosmological model with a cosmological constant of the form $\Lambda \propto (\frac{\ddot{R}}{R})$. Following the same decay law, Arbab (2003) has investigated cosmic acceleration with a positive cosmological constant which is equivalent to $\Lambda \propto H^2$ on dimensional ground. One of the motivations for introducing the Λ-term is to reconcile the age parameter and the density parameter of the universe with recent observational data.

In this study, we will examine quark matter in higher dimensional space time. It is well known that quark gluon plasma existed during on e of the phase transitions of the Universe in the early time when the Universe had higher dimensions than it has today and cosmic temperature was T$\sim$ 200 MeV.

Quark matter is modelled with an equation of state (EOS) based on the phenomenological bag model of quark matter, in which quark confinement is described by an energy term proportional to the volume. In the framework of this model the quark matter is composed of massless u, d quarks, massive s quarks and electrons (Gondek et al. 2006). In the simplified version of the bag model, assuming the quarks are massless and non interacting, we have the quark pressure $p_q = \rho_q/3$ (ρ_q is the quark energy density);

the total energy density is

$$\rho = \rho_q + B \tag{1}$$

while the total pressure is

$$p = p_q - B \tag{2}$$

One then obtains the EOS for strange quark matter (Kapusta 1994; Sontani et al. 2004; Xu 2003),

$$p = \frac{1}{3}(\rho - 4B) \tag{3}$$

where B is the difference between the energy density of the perturbative and non-perturbative QCD vacuum (the bag constant). Equation (3) is essentially the EOS of a gas of massless particles with corrections due to the QCD trace anomaly and perturbative interactions. These corrections are always negative, reducing the energy density at given temperature by about a factor of two (Farhi & Jaffe 1984). For quark stars obeying the bag model EOS (3) the Chandrasekhar limit has been evaluated from simple energy balance relations in Bannerjee et al. (2000). In addition to the fundamental constants, the maximum mass also depends on the bag constant (Harko & Cheng 2002).

Recently, Dey et al. (1998) have obtained new sets of EOSs for strange matter based on a model of interquark potential which has the following features: (a) asymptotic freedom, (b) confinement at zero baryon density and deconfinement at high baryon density, (c) chiral symmetry restoration and (d) gives stable uncharged β-stable strange matter. These EOSs have later been approximated to the following linear form by Gondek et al. (2000),

$$p = \varepsilon(\rho - \rho_0) \tag{4}$$

where ρ_0 denotes the energy density at zero pressure and ε is a constant (Sharma et al. 2006). Recently quark matter and the relations between quark matter and domain walls and also strings have been studied by several authors. Yilmaz et al (2007) studied strange quark matter for Robertson Walker model in the context of general theory of relativity. Also Yilmaz and Yavuz (2006) have obtained higher dimensional Robertson Model cosmological model in the presence of quark gluon plasma in general theory of relativity.

In this paper we studied five dimensional cosmological model in presence of quark matter coupled with domain walls. The paper is organised as follows.

This paper is outlined as follows. In Section 2, the Einstein field equations are solved for quark matter. In Section 3, different cases for strange quark matter coupled with Domain walls and perfect fluid are discussed. Section 4 gives some concluding remarks.

2 Field equations and solutions

The line element for five dimensional Kaluza-Klein model is

$$ds^2 = -dt^2 + a^2(dx_1^2 + dx_2^2 + dx_3^2) + b^2dx_4^2 \tag{5}$$

The Einstein's field equation is given by

$$R_{ab} - \frac{1}{2}Rg_{ab} = -8\pi G T_{ab} + \Lambda g_{ab} \tag{6}$$

The energy momentum tensor of a domain wall is given by

$$T_{ab}^D = \rho U_a U_b + p(g_{ab} + u_a U_b) \tag{7}$$

This perfect fluid form of the domain wall includes quark matter (described by $\rho_m = \rho_q + B_c$ and $p_m = p_q - B_c$) as well as domain wall tension σ_ω i.e. $\rho = \rho_m + \sigma_\omega$ and $p = p_m - \sigma_\omega$. Also p_m and ρ_m are related by equation of state for strange quark matter

$$p_m = \frac{1}{3}(\rho_m - 4B_c) \tag{8}$$

and equation of state

$$p_m = (\gamma - 1)\rho_m \tag{9}$$

Here the five velocity u^a is time like vector such as $u^a u_a = -1$. We use comoving coordinate system $u^a = \delta^a_0$.

Using the line element (5), field equation (6) take the form for the equation (7) as

$$3\frac{\dot{a}^2}{a^2} + 3\frac{\dot{a}\dot{b}}{ab} = \rho + \Lambda \tag{10}$$

$$2\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\dot{a}^2}{a^2} + 2\frac{\dot{a}\dot{b}}{ab} = -p + \Lambda \tag{11}$$

$$3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} = -p + \Lambda \tag{12}$$

where dot denotes differentiation with respect to t. The physical variables, expansion and shear scalar, have the following expressions for the above metric:

$$\theta = 3\frac{\dot{a}}{a} + \frac{\dot{b}}{b} \tag{13}$$

$$\sigma^2 = \frac{3}{8}(\frac{\dot{a}}{a} - \frac{\dot{b}}{b})^2 \tag{14}$$

Equation (10)-(12) are three independent equations in five unknowns ρ, p, Λ, a and b. To get a determinate solution we use supplementary conditions

$$b = \mu a^n \tag{15}$$

where μ amd n are real constants, and the variation of Λ to be of the form

$$\Lambda = \Lambda_0 \left(3\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right)^2 \tag{16}$$

using equation (15), equation (10) to (12) takes the form

$$3(1+n)\frac{\dot{a}^2}{a^2} = \rho + \Lambda \tag{17}$$

$$3\frac{\ddot{a}}{a} + 3\frac{\dot{a}^2}{a^2} = -p + \Lambda \tag{18}$$

$$(n+2)\frac{\ddot{a}}{a} + (n^2+n+1)\frac{\dot{a}^2}{a^2} = -p + \Lambda \tag{19}$$

2.1 When $n \neq 3$ and $n \neq 1$

Using equation (17) to (19) we get

$$a = (\alpha t + \beta)^{\frac{1}{n+3}} \tag{20}$$

$$b = \mu(\alpha t + \beta)^{\frac{n}{n+3}} \tag{21}$$

from equation (16) we get

$$\Lambda = \Lambda_0 \frac{\alpha^2}{(\alpha t + \beta)^2} \tag{22}$$

Energy density ρ and total pressure p will be

$$\rho = \frac{\{3(n+1) - \Lambda_0 (n+3)^2\}\alpha^2}{(n+3)^2(\alpha t + \beta)^2} \tag{23}$$

$$p = \frac{\{\Lambda_0 (n+3)^2 + 3(n+1)\}\alpha^2}{(n+3)^2(\alpha t + \beta)^2} \tag{24}$$

Expansion θ and shear scalar σ^2 takes the form

$$\theta = \frac{\alpha}{\alpha t + \beta} \tag{25}$$

$$\sigma^2 = \frac{3}{8}\frac{(n-1)^2}{(n+3)^2}\frac{\alpha^2}{(\alpha t + \beta)^2} \tag{26}$$

2.2 When $n = -3$

Using equation (17) to (19) we get

$$a = \mathrm{e}^{(\alpha t + \beta)} \tag{27}$$

$$b = \mu \mathrm{e}^{-3(\alpha t + \beta)} \tag{28}$$

from equation (16) we get

$$\Lambda = 0 \tag{29}$$

Energy density ρ and total pressure p will be

$$\rho = -6\alpha^2 \tag{30}$$

$$p = -6\alpha^2 \tag{31}$$

Expansion θ and shear scalar σ^2 takes the form

$$\theta = 0 \tag{32}$$

$$\sigma^2 = 6\alpha^2 \tag{33}$$

3 Different cases

3.1 Strange Quark matter coupled with the domain walls

$$\rho_q = \frac{9}{2}\frac{n\alpha^2}{(n+3)^2}\frac{1}{(\alpha t+\beta)^2} \tag{34}$$

$$p_q = \frac{3}{2}\frac{n\alpha^2}{(n+3)^2(\alpha t+\beta)^2} \tag{35}$$

$$\sigma_\omega = -B_c - \frac{n\alpha^2}{(n+3)^2(\alpha t+\beta)^2}\left\{3 + \Lambda_0(n+3)^2 + \frac{3n}{2}\right\} \tag{36}$$

3.2 Solution of the field equations using equation of state

$$p_m = (\gamma - 1)\rho_m$$

$$\rho_m = \rho_q + B_c = \frac{6n\alpha^2}{\gamma(n+3)^2(\alpha t+\beta)^2} \tag{37}$$

$$p_m = \frac{6n(\gamma-1)\alpha^2}{\gamma(n+3)^2(\alpha t+\beta)^2} \tag{38}$$

$$\sigma_\omega = \frac{\alpha^2}{(n+3)^2(\alpha t+\beta)^2}\left\{-3 - \Lambda_0(n+3)^2 - \frac{6n}{\gamma} + 3n\right\} \tag{39}$$

4 Discussion

In this paper, we have obtained exact solutions of string cloud and domain walls with quark mater in the context of Kaluaza-Klein theory of Gravitation.

To determinate a solution, we used two conditions $\Lambda = \Lambda_0\left(3\frac{\dot{a}}{a} + \frac{\dot{b}}{b}\right)^2$ (Arbab 1997) and a relation between gravitational potentials a and b as $b = \mu a^n$. It shows that the solution of the above model is obtained when $n \neq 1$. When $n \neq -3, 1$, it is observed that, for a fixed $n > -3$, the behaviour of a and b (respectively; Λ, θ, ρ, p and σ^2) increases (respectively; decrease) as t increases. Also observe that Λ and θ are independent of n. However, ρ (respectively; p) turns out to be zero if Λ_0 is chosen as $\Lambda_0 = \frac{3(n+1)}{(n+3)^2}$ (respectively; $\Lambda_0 = -\frac{3(n+1)}{(n+3)^2}$).

When $n = -3$, the behaviour of a and b is increasing as t increases to infinity where as the remaining variables Λ, θ, p, ρ and σ^2 are turn out to be constant.

In section 3.1, we found that the strange quark solution coupled with domain wall. Here p_q and ρ_q tends to 0 as $t \to \infty$ and tends to a constant as $t \to 0$. In this case, from the equations (27) and (28), we get $p_q = \frac{\rho_q}{3}$. If the vacuum energy density B_c(bag constant)is absorbed into tension of the wall, then these solutions corresponds to $\gamma = 4/3$ to (i.e. radiation case)
section 3.2. In our model σ/θ tends to a constant. In both the cases, the parameter θ and σ^2 tends to infinity as $t \to -\beta/\alpha$, and they vanish as $t \to \infty$.

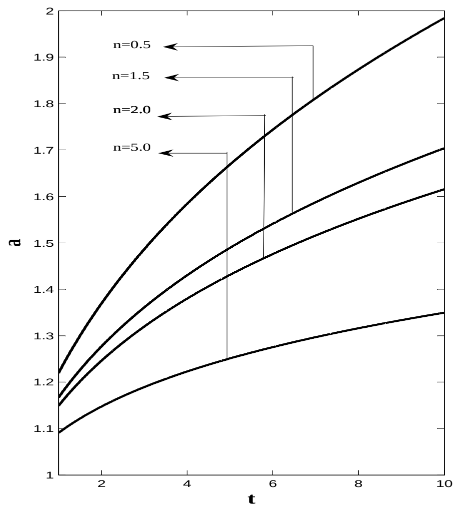

Fig:1a

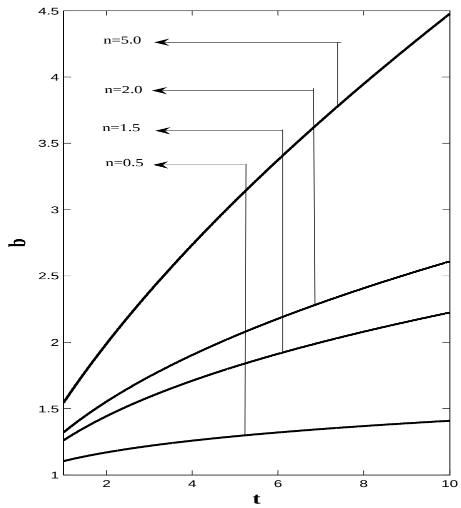

Fig:1b

Fig.1: The behavior of a and b for different values of n with respect to t are depicted in Figures 1a and 1b.

References

[1] Ya. B. Zeldovich, *Mon. Not. R. Astron. Soc.* **192**,663 (1980).

[2] Sahdev D, *Phys Rev D* **30**,2495 (1984)

[3] Emelyanov V M , Nikitin Yu P,Rozental J L and Berkov A V, *Phys. Rep.* **143**,1 (1986).

[4] Chatterjee S and Bhui B, *Mon. Not. R. Astron. Soc.* **247**,57 (1990).

[5] Chatterjee S and Bhui B, *Int. J Theo. Phys* **32**,671 (1993).

[6] Overduin J M and Wesson P S, *Phys. Rep.* **283**,303 (1987).

[7] Wesson P S, *Astrophys. J* **394**,19 (1992).

[8] Chatterjee S , Panigrahi D, Banerjee A, *Class Quantum Grav* **11**,371 (1994a).

[9] Chatterjee S, Bhui B, Basu M B, banerjee A, *Phys Rev D* **50**,2924 (1994a).

[10] Chakraborty S, Roy A, *Int. J Mod. Phys D* **8**,645 (1999).

[11] Chatterjee S , Banerjee A, *Class. Quantum Grav* **10**,L1 (1993).

[12] BAnerjee A, Panigrahi D, Chatterjee S, *J Math Phys* **36**,3619 (1995).

[13] Vishwakarma R G, *Class. Quantum Gravity* **18**,1159 (1993).

[14] Gurvits L I, Kellermann K I and Frey S, *Astron Astro Phys* **342**,378 (1999).

[15] Dolgov A D, *Phys Rev D* **55**,5881 (1997).

[16] Sahni Vand Starobinsky A, *Int. J MOd. Phys D* **9**,373 (2000).

[17] Peebles P J E and Ratra B, *Rev Mod Phys* **75**,559 (2003).

[18] Padmanabhan T, *Phys Rep* **380**,235 (2003).

[19] Vishwakarma R G, *Class Quantum Grav.* **17**,3833 (2000).

[20] Vishwakarma R G, *Class Quantum Grav.* **18**,1159 (2001).

[21] Vishwakarma R G, *Mon. Not. R Astron. Soc.* **331**,776 (2002).

[22] Triay R, *Int. J Mod. Phys D* **14**,1667 (2005).

[23] Kilinc C B , *Astro. Phys Space Science* **301**,83 (2006).

[24] Bertolami O, *Nuovo Cimento* **93**,36 (1986a).

[25] Bertolami O, *Fortschr. Phys.* **34**,829 (1986b).

[26] Chen W and Wu Y S, *Phys Rev D* **41** ,695 (1990).

[27] Carvalho J C, Lima J A S and Waga I, *Phys Rev D* **46**,2404 (1992).

[28] Berman M S , *Phys Rev D* **43**,1075 (1991).

[29] Berman M S, *Gen Relativ. Gravitaiton* **23**,465 (1991).

[30] Abdel Rahman A M M, *Phys Rev D* **45**,3497 (1992).

[31] Lima J A S and Maia J M F, *Mod Phys Lett A* **48**,591 (1993).

[32] Al-rawaf A S , *Mod Phys Lett A* **13**,429 (1998).

[33] Overduin J M and Cooperstock F I , *Phys Rev D* **58**,043506 (1998).

[34] Arbab A I, *Class. Quantum Gravitation* **20**,93 (2003).

[35] Gondek-Rosinska D, Gourgoulhon E, Haensel P , *Astron Astrophysics* **412**,777 (2003).

[36] Kapusta J , *Finite temperature Field theory, Cambridge U Press 2nd edition* , (2011).

[37] Sotani H, Hahri K, Harada T, *Phys Rev D* **69**,084008 (2004).

[38] Xu R X, *Chin J Astron Astrophysics* **3**,33 (2003).

[39] Farhi E and Jaffe R L, *Phys Rev D* **30**,2379 (1984).

[40] Bannerjee S, Ghosh S K, Raha S, *J Phys G Nucl.* **26**,L1 (2000).

[41] Harko T and Cheng K S , *Astron Astrophysics* **385**,947 (2002).

[42] Dey M, Bambaci I Dey J, Ray S, Samanta B C, *Phys. Lett B* **438**,123 (1998).

[43] Gondek-Rosinska D, Bulik T, Zdunik L , *Astron Astrophysics* **363**,1005 (2000).

[44] Sharma R,Karmarkar S , Mukherjee S, *Int Journal of Mod. Phys. D* **15**,405 (2006).

[45] Arbab A I , *Gen Relativity and Gravitation* **29**,61 (1997).

[46] Yilmaz I, Kucukarslan A.and Serhat ozder, *Int. Jou. Mor. Phys. A* **22**,2283 (2007)

[47] Yilmaz I.,Yahus A. A., *Int. Jou. Mor. Phys. D* **15**,477 (2006).