A New Method to Derive the EOQ Model with Defective Items and Known Price Increase

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Abstract *- This paper uses a new method, modified quadratic-geometric mean inequality, to derive the optimal EOQ model with defective items and known price increase when the special order can be placed at the regular time for replenishment. This study also uses 100% inspection policy and the known proportion of defective items is removed prior to storage after the screening process. The method is very simple to derive the optimal EOQ model without derivative.*

Keywords *Defective item, modified quadratic-geometric mean inequality, known price increase.*

I. INTRODUCTION

There have been some studies related to basic EOQ (Economic Order Quantity) model with known price increase, which can be found in [5], [7], [8], [10], etc. These studies have yielded useful results in basic deterministic inventory theory. We know that the basic EOQ model with known price increase was adapted by adding the assumption of known price increase to the basic EOQ model. Therefore, the both models have similarity of assumptions. However, this study focuses on the context of known price increase in [10], especially when the special order can be placed at the regular time for replenishment, the inventory level reached the reorder point 0, that can be briefed as follows.

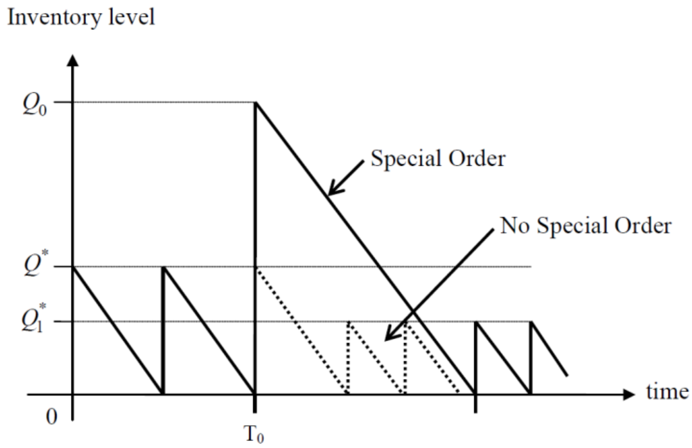

Fig. 1 Known price increase

Assume that, at the present time, the price of an item is c baht and a supplier announces that the price of an item will be increased to be $c+k$ baht on the next time. The known price increase situation is depicted in Fig. 1. From which, it follows that unit purchases before and at T_0 still cost c baht and purchase quantities before price increase are Q^* units. Unit purchases after T_0 will cost $c+k$ baht and purchase quantities after price increase are $Q_1^*(<Q^*)$ units. The special order of Q_0 units is purchased at T_0, when the inventory level reached the reorder point 0. In this situation, [10] used differential calculus to derive the optimal value of Q_0 as follows:

$$Q_0^* = \frac{(Q^*)^2}{2A}\left(\frac{2A}{Q_1^*}+k\right), \tag{1}$$

where $Q^* = \sqrt{\frac{2AD}{ic}}$ and $Q_1^* = \sqrt{\frac{2AD}{i(c+k)}}$, and the maximum total cost saving is of the form

$$G^* = A\left(\frac{Q_0^*}{Q^*} - 1\right)^2. \qquad (2)$$

Consider the EOQ model of [10], see (1), this model is appropriate to the case of all items to be perfect quality. In real situation, it is difficult to purchase items with 100% good quality [4]. Thus, in this study we are interested to improve the EOQ of [10] by adding the assumption of defective items of [4] to this model, and using 100% inspection policy and the known proportion of defective items is removed prior to storage after the screening process. Furthermore, in this study we propose a new alternative optimization method, namely modified quadratic-geometric mean inequality, to derive the desired EOQ model.

II. NOTATION, ASSUMPTIONS AND METHOD

The following notation and assumptions are used to derive the desired model.

A. Notation

- D demand rate for non-defective items in units per time
- A ordering cost per order
- F the fixed inspection cost per lot
- f inspection cost per unit
- c purchase cost before the price increase per unit
- d the known percentage of defective items in each lot
- i holding cost fraction per unit per time
- k known price increase
- Q^* the optimal order quantity before the price increase (including defective items)
- Q_1^* the optimal order quantity after the price increase (including defective items)
- Q_0 special order quantity (including defective items)
- Q_0^* the optimal special order quantity (including defective items)
- C_s total cost when special order is placed
- C_n total cost when no special order is placed
- G total cost saving
- G^* the maximum total cost saving

B. Assumptions ([4])

- Demand rate is known and constant.
- Lead time is equal to zero.
- The proportion of defective items in each lot purchased is known and all defective items are removed prior to storage
- The inspection cost consists of a fixed inspection cost per lot and a fixed inspection cost per unit, and the screening time can be ignored.
- Time period is infinite.
- Shortages are not allowed.

C. Method (the geometric mean and quadratic mean inequality)

It is well known that differential calculus to be a classical optimization method in deterministic inventory theory. In the past few years, some authors have proposed alternative optimization methods, without derivative, to derive their EOQ models such as [3] proposed the algebraic method to derive the basic EOQ model, [6] used the cost comparisons method to derive the basic EOQ model with backorders, [9] used the arithmetic-geometric mean inequality to derive three EOQ models, [1] used the arithmetic-geometric mean inequality together with Cauchy-Bunyakovsky-Schwarz inequality to derive the EOQ model with backorders. In this study, we propose a new alternative optimization method without derivative to derive our EOQ model. The method is defined as follows.

Let a and b are positive real numbers, then the following inequality holds:

$$\sqrt{ab} \le \sqrt{\frac{a^2+b^2}{2}}. \tag{3}$$

The inequality (3) is referred to as the quadratic-geometric mean inequality or the root mean square-geometric mean inequality [2]. It is seen that the inequality (3) can be expressed as

$$-a^2 + 2ab \le b^2 \tag{4}$$

and

$$-a^2 + 2ab = b^2 \tag{5}$$

if and only if $a = b$. In this context, we will call a new inequality (4) that modified quadratic-geometric mean inequality, which uses to derive the desired EOQ model.

III. RESULT

The aim of this study is a use of modified quadratic-geometric mean inequality to derive the optimal EOQ model with defective items and known price increase.

A. Theoretical result

The defective items and known price increase situation is depicted in Fig. 2.

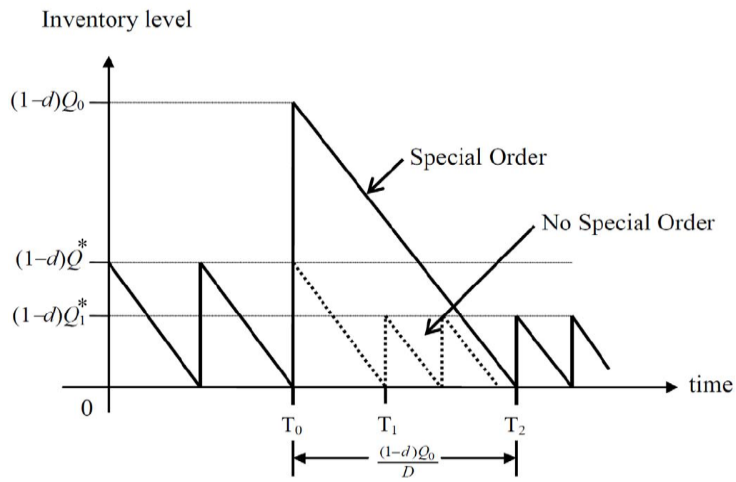

Fig. 2 Defective items and known price increase

The following theorem presents the desired result.

Theorem. *The optimal special order quantity,* Q_0^*, *is of the form*

$$Q_0^* = \frac{(Q^*)^2}{2(A+F)}\left[\frac{2(A+F)}{Q_1^*} + k(1-d)\right], \tag{6}$$

where $Q^* = \frac{1}{1-d}\sqrt{\frac{2(A+F)D}{ic}}$ *and* $Q_1^* = \frac{1}{1-d}\sqrt{\frac{2(A+F)D}{i(c+k)}}$, *and the maximum total cost saving,* G^*, *is*

$$G^* = (A+F)\left(\frac{Q_0^*}{Q^*} - 1\right)^2. \tag{7}$$

Proof. Let us consider Fig. 2, unit purchases before and at T_0 still cost c baht and purchase quantities before price increase are Q^* units, that is,

$$Q^* = \frac{1}{1-d}\sqrt{\frac{2(A+F)D}{ic}}. \tag{8}$$

Unit purchases after T_0 will cost $c+k$ baht and purchase quantities after price increase are Q_1^* units, that is,

$$Q_1^* = \frac{1}{1-d}\sqrt{\frac{2(A+F)D}{i(c+k)}}. \tag{9}$$

Note that, the results in (8) and (9) can be easily derived by applying the formula in [4], and it is clear that $Q^* > Q_1^*$.

If special order is placed at T_0 of Q_0 units then total cost in this case, in time T_0 to T_2 as shown in Fig. 2, consists of ordering cost, the inspection cost, items cost and holding cost, that is, total cost when special order is placed

$$C_s = A + F + fQ_0 + c(1-d)Q_0 + \frac{ic(1-d)^2 Q_0^2}{2D}$$
$$= A + F + fQ_0 + c(1-d)Q_0 + \frac{(A+F)Q_0^2}{(Q^*)^2} \quad \left(\text{by } \frac{ic(1-d)^2}{2D} = \frac{A+F}{(Q^*)^2}\right) \tag{10}$$

If there is no special order, but all regular orders occur in time T_0 to T_2 of Q_0 units then total cost in time T_0 to T_2, see dashed lines in Fig. 2, can be considered to be two parts. The first part, T_0 to T_1, considers when the first regular order is placed at T_0 of Q^* units (c baht). Thus, total cost in this part is equal to

$$A + F + fQ^* + c(1-d)Q^* + \frac{ic(1-d)^2 (Q^*)^2}{2D} = 2(A+F) + fQ_0 + c(1-d)Q_0 \quad (\text{by } (8)). \tag{11}$$

The second part, T_1 to T_2, it is seen that purchase quantities in time T_1 to T_2 of $Q_0 - Q^*$ units ($c+k$ baht) and number of orders is $\frac{Q_s - Q^*}{Q_1^*}$. Thus, total cost in this part is equal to

$$\frac{Q_0 - Q^*}{Q_1^*}(A+F) + f(Q_0 - Q^*) + (c+k)(1-d)(Q_0 - Q^*) + \left(\frac{Q_0 - Q^*}{Q_1^*}\right)\frac{i(c+k)(1-d)^2 (Q_1^*)^2}{2D}$$
$$= 2\left(\frac{Q_0 - Q^*}{Q_1^*}\right)(A+F) + (c+k)(1-d)(Q_0 - Q^*) + f(Q_0 - Q^*) \quad (\text{by } (9)). \tag{12}$$

Therefore, by combining (11) and (12), total cost when no special order is placed

$$C_n = 2(A+F) + fQ^* + c(1-d)Q^* + 2\left(\frac{Q_0 - Q^*}{Q_1^*}\right)(A+F) + f(Q_0 - Q^*) + (c+k)(1-d)(Q_0 - Q^*)$$
$$= 2(A+F) + fQ_0 + c(1-d)Q_0 + 2\left(\frac{Q_s - Q^*}{Q_1^*}\right)(A+F) + k(1-d)(Q_0 - Q^*) \tag{13}$$

Hence, total cost saving when special order is placed, G,

$$G = C_n - C_s$$
$$= 2(A+F) + fQ_0 + c(1-d)Q_0 + 2\left(\frac{Q_s - Q^*}{Q_1^*}\right)(A+F) + k(1-d)(Q_0 - Q^*) - A - F - fQ_0 - c(1-d)Q_0 - \frac{(A+F)Q_0^2}{(Q^*)^2}$$
$$= A + F + 2\left(\frac{Q_0 - Q^*}{Q_1^*}\right)(A+F) + k(1-d)(Q_0 - Q^*) - \frac{(A+F)Q_0^2}{(Q^*)^2}$$
$$= -\frac{(A+F)Q_0^2}{(Q^*)^2} + \left[\frac{2(A+F)}{Q_1^*} + k(1-d)\right](Q_0 - Q^*) + A + F$$
$$= -\frac{(A+F)Q_0^2}{(Q^*)^2} + 2\left[\frac{A+F}{Q_1^*} + \frac{k(1-d)}{2}\right]Q_0 - \left[\frac{2(A+F)}{Q_1^*} + k(1-d)\right]Q^* + A + F \tag{14}$$
$$\leq \frac{(Q^*)^2}{4(A+F)}\left[\frac{2(A+F)}{Q_1^*} + k(1-d)\right]^2 - \left[\frac{2(A+F)}{Q_1^*} + k(1-d)\right]Q^* + A + F. \tag{15}$$

The inequality (15) is obtained by looking $\frac{(A+F)Q_0^2}{(Q^*)^2}$ and $2\left[\frac{A+F}{Q_1^*} + \frac{k(1-d)}{2}\right]Q_0$ in (14) to be a^2 and $2ab$ in (4), respectively. From (14) and (15), it can be seen that the maximum value of G in (14) is obtained if and only if G is equals to the result in (15). By (5), we then have

$$\frac{\sqrt{A+F}Q_0}{Q^*} = \frac{Q^*}{2\sqrt{A+F}}\left[\frac{2(A+F)}{Q_1^*}+k(1-d)\right].$$

Thus, the optimal values of Q_0 and G^* are

$$Q_0^* = \frac{(Q^*)^2}{2(A+F)}\left[\frac{2(A+F)}{Q_1^*}+k(1-d)\right]$$

and

$$G^* = \frac{(Q^*)^2}{4(A+F)}\left[\frac{2(A+F)}{Q_1^*}+k(1-d)\right]^2 - \left[\frac{2(A+F)}{Q_1^*}+k(1-d)\right]Q^* + A+F.$$

$$= \left\{\frac{Q^*}{2\sqrt{A+F}}\left[\frac{2(A+F)}{Q_1^*}+k(1-d)\right]-\sqrt{A+F}\right\}^2$$

$$= (A+F)\left\{\frac{Q^*}{2(A+F)}\left[\frac{2(A+F)}{Q_1^*}+k(1-d)\right]-1\right\}^2$$

$$= (A+F)\left(\frac{Q_0^*}{Q^*}-1\right)^2,$$

which gives the desired results. □

Remark. 1. Consider $G^* = (A+F)\left(\frac{Q_0^*}{Q^*}-1\right)^2$, because $Q^* > Q_1^*$, we have

$$Q_0^* = \frac{(Q^*)^2}{2(A+F)}\left[\frac{2(A+F)}{Q_1^*}+k(1-d)\right] = Q^*\left[\frac{Q^*}{Q_1^*}+\frac{k(1-d)Q^*}{2(A+F)}\right] > Q^*, \quad (16)$$

this implies that

$$G^* = (A+F)\left(\frac{Q_0^*}{Q^*}-1\right)^2 > A+F. \quad (17)$$

Thus, the special order of Q_0^* units should be purchased when the inventory level is equals to 0 unit.

2. From the result in (16), it is observed that

$$\frac{Q_0^*}{Q^*} = \frac{Q^*}{Q_1^*}+\frac{k(1-d)Q^*}{2(A+F)}$$
$$= \sqrt{\frac{c}{c+k}}+k\sqrt{\frac{D}{2ic(A+F)}}, \quad (18)$$

this implies that $G^* = (A+F)\left(\frac{Q_0^*}{Q^*}-1\right)^2$ does not depend on d, the known percentage of defective items.

3. Consider the result in the Theorem, if $d=0$, that is, all items are perfect quality. Therefore, the results in the Theorem are the same results in [10] as mentioned in (1) and (2).

4. Because the term of f is not in both formulas of Q_0^* and G^*, thus we can ignore it for computing numerical results.

B. Numerical results

We give two numerical examples to illustrate numerical results of (6) and (7) in the Theorem.

• Example 1. Let demand rate $D = 2{,}000$ units per year, ordering cost $A = 2{,}000$ baht per order, the fixed inspection cost $F = 1{,}000$ baht per lot, holding cost fraction $i = 5\%$ of an item price per year, item price $c = 1{,}000$ baht per unit, known price increase $k = 200$ baht per unit and the known percentage of defective items $d = 1\%,\ 5\%$ and 10%.

The optimal special order quantity and the maximum total cost saving for giving $d = 1\%,\ 5\%$ and 10% are as follows:

d	Q^*	Q_1^*	Q_0^*	G^*
1%	494.8464	451.7309	8,622.8852	
5%	515.6821	470.7512	8,985.9540	809,379.00
10%	544.3311	496.9040	9,485.1737	

- Example 2. Let demand rate $D = 3{,}000$ units per year, ordering cost $A = 3{,}000$ baht per order, the fixed inspection cost $F = 2{,}000$ baht per lot, holding cost fraction $i = 10\%$ of an item price per year, item price $c = 1{,}500$ baht per unit, known price increase $k = 300$ baht per unit and the known percentage of defective items $d = 5\%,\ 10\%$ and 20%.

The optimal special order quantity and the maximum total cost saving for giving $d = 5\%,\ 10\%$ and 20% are as follows:

d	Q^*	Q_1^*	Q_0^*	G^*
5%	470.7512	429.7350	6,831.4715	
10%	496.9040	453.6092	7,210.9977	912,850.85
20%	559.0170	510.3104	8,112.3724	

From Examples 1 and 2, it is seen that the optimal special order quantity, Q_0^*, changes along the known percentage of defective items, d. That is, the optimal special order quantity depends directly on the known percentage of defective items. However, the maximum total cost saving, G^*, does not depend on the known percentage of defective items as mentioned in the remark.

IV. CONCLUSION

The new method, modified quadratic-geometric mean inequality, was used to derive the optimal EOQ with defective items and known price increase by assuming that the special order can be placed at the regular time for replenishment, when the inventory level reached the reorder point 0. In addition, the assumption of defective items in [4] was also added to this model by using 100% inspection policy and the known proportion of defective items is removed prior to storage after the screening process. In view of the method of this study, it is a simple alternative optimization method to derive the desired EOQ model without derivative.

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