Homology Soft Modules Induced With Interval Valued Q-Fuzzy Soft Level Subset

J.RegalaJebalily[1] , G. Subbiah[2]* and V.Nagarajan[3]

1.Research scholar, Reg.No: 12615, Department of Mathematics, S.T.Hindu College, Nagercoil-629 002,Tamil Nadu, India

*2 *Associate Professor in Mathematics, Sri K.G.S. Arts College, Srivaikuntam-628 619, Tamil Nadu, India.*

3 Assistant Professor in Mathematics, S.T.Hindu College, Nagercoil-629 002, Tamil Nadu, India.

Affiliated to ManonmaniamSundaranar University, Abeishekapatti, Tirunelveli-627 012, Tamil Nadu, India

Abstract - In this paper, we discuss the properties of interval valued bi-cubic homology fuzzy soft sub modules and its arbitrary intersections. Also the level subset of homology soft modules indexed with its interval valued fuzzy soft set has been analysed. Finally,we proved that the inverse image of an interval valued bi-cubic homology fuzzy soft modules is also an interval valued bi-cubic homology fuzzy soft modules.

Keywords:interval number, fuzzy set, soft set, complement, interval valued level set, normal, homology soft module.

AMS Mathematics Subject Classification (2010):06F35, 03G25, 08A72

I. INTRODUCTION

The theory of sets is an indispensable mathematical tool. It describes mathematical models for the class of problems that deal with exactness, precision and certainty. Characteristically, classical set theory is extensional.More often than not, the real life problems inherently involve uncertainties, imprecision and vagueness. In particular, such classes of problems arise in economics, engineering, environmental sciences, medical sciences, social sciences etc., In course of time, a number of mathematical theories such as probability theory [13], fuzzy set theory [14], rough set theory [12], interval mathematical theory [13], vague set theory [3] etc., are formulated to solve such problems, and have been found only partiallysuccessful. For instance, the theory of probability can only deal with stochastically stable systems (or phenomena) where a limit of the sample mean should exist in a long series of trials. Accordingly this can be used to problems of engineering orientationbutnottothatofeconomicorenvironmentalorsocial.The method of interval mathematics takes into account, the errors of calculation cases, but it is not sufficiently adaptable for problems with different sorts of uncertainties.Rough set theory approach can handle problems that involve uncertainties caused by indiscernible elements with different values in decision attributes.The fuzzy set theory approach is found most appropriate for dealing with uncertainties. However, it is short of providing a mechanism on how to set the membership function extremely individualistic.The major reason for these difficulties arising with the above theories is due to the inadequacies of their parameterization tools. In order to overcome these difficulties, in 1999 Molodtsov [7] introduced the concept of soft set as a completelynewmathematicaltoolwithadequateparameterizationfordealingwith uncertainties.we discuss the properties of interval valued bi-cubic homology fuzzy soft sub modules and its arbitrary intersections. Also the level subset of homology soft modules indexed with its interval valued fuzzy soft set has been analysed. Finally, we proved that the inverse image of an interval valued bi-cubic homology fuzzy soft modules is also an interval valued bi-cubic homology fuzzy soft modules.

We first recall some basic concepts which are used to present the paper.
An interval number on [0,1], say $\bar{a}$ is a closed subinterval of [0,1], (ie) $\bar{a} = [a^-, a^+]$ where $0 \le a^- \le a^+ \le 1$.
For any interval numbers $\bar{a} = [a^-, a^+]$ and $\bar{b} = [b^-, b^+]$ on [0,1] , we define

(i) $\bar{a} \le \bar{b}$if and only if $\bar{a} \le \bar{b}$ and $a^+ \le b^+$

(ii) $\bar{a} = \bar{b}$ if and only if $\bar{a} = \bar{b}$ and $a^+ = b^+$

(iii) $\bar{a} + \bar{b} = [a^- + b^-, a^+ + b^+]$, whenever $\bar{a} + \bar{b} \le 1$ and $a^+ + b^+ \le 1$

Let X be a set. Amapping A : X → [0,1] is called a fuzzy set in X. Let A be a fuzzy set in X and $\alpha \in [0,1]$. Define L(A : α) as follows:
$L(A: \alpha) = \{ x \in X / A(x) \leq \alpha \}$. Then L (A:α) is called the lower level cut of A.
Let X be a set. A mapping $\bar{A}$: X → D[0,1] is called on interval-valued fuzzy set (briefly i-v fuzzy set) of X, where D[0,1] denotes the family of all closed sub intervals of [0,1], and $\bar{A}(x) = [A^-(x), A^+(x)]$, ∀ x∈ X, where $\bar{A}$ and A^+ are fuzzy sets in X.For an i-v fuzzy set $\bar{A}$ of a set X and $(\alpha, \beta) \in D[0,1]$ define $L(\bar{A}: [\alpha, \beta])$ as follows $\bar{L}(A:[\alpha, \beta])$ which is called the level sub set of $\bar{A}$.

II. PRELIMINARIES AND BASIC DEFINITIONS

In this section, we recall some basic definitions for the sake of completeness.

***A. Definition[Yang et.al]*:** An interval valued fuzzy set $\tilde{F}$ (over a basic set X) is specified by a function$T_{\tilde{F}}$: X → D([0,1]) , where D([0,1]) is the set of all intervals within [0,1], i.e. for all x ∈ X, $T_{\tilde{F}}(x)$ is an interval $[\mu_1, \mu_2]$,$0 \leq \mu_1 \leq \mu_2 \leq 1$.

B. Definition[Jun et.al]:A bi fuzzy set $\tilde{V}$, in a basic set X, is characterized by a truth membership function $t_{\tilde{V}}$, $t_{\tilde{V}}$: X → [0,1] and a false membership function $f_{\tilde{V}}$, $f_{\tilde{V}}$: X → [0,1]. If the generic element of X is denoted by x_i,then the lower bound on the membership grade of x_i derived from evidence for x_i is denoted by $t_{\tilde{V}}(x_i)$ and the lower bound on the negation of $x_i$is denoted by $f_{\tilde{V}}(x_i)$ associate a real number in the interval [0,1] with each point in X, where $t_{\tilde{V}}(x_i) + f_{\tilde{V}}(x_i) \leq 1$.

When X is continuous, a bi-fuzzy set $\tilde{V}$ can be written as

$\tilde{V} = \int_X [t_{\tilde{V}}(x_i), 1 - f_{\tilde{V}}(x_i)] / x_i, x_i \in X$.

When X is discrete , a bi-fuzzy$\tilde{V}$ can be written as

$\tilde{V} = \sum_{i=1}^{n} [t_{\tilde{V}}(x_i), 1 - f_{\tilde{V}}(x_i)] / x_i, x_i \in X$.

C. Definition[Aktas et.al.]:Let A $=[a_{1,} a_2]$ and B=$[b_{1,} b_2]$ be two arbitrary intervals. Then the minimum of A and B is represented by "MIN [A,B]" and is defined byMIN ($[a_{1,} a_2]$; $[b_{1,} b_2]$)=[min$(a_{1,} b_1)$ $(a_{2,} b_2)$].

***D. Definition[Aktas et.al]*:** The complement of an interval A $=[a_{1,} a_2]$ is denoted by $\bar{A}$ and is defined by $\bar{A}$= [1- $a_{2,}$1- a_1].The definition of interval valued fuzzy set and definitions related to interval valued level fuzzy set are introduced here.

E. Definition[Yang et.al] :An interval valued level fuzzy set $\tilde{V}$ over a basic set X is defined as an object of the form $\tilde{V} =< [x_i; T_{\tilde{V}}(x_i); 1 - f_{\tilde{V}}(x_i)] > x_i \in X$, Where $T_{\tilde{V}}$: X → D[0,1] and $f_{\tilde{V}}$: X → D[0,1] are called "Truth membership function" and "False membership function" respectively and where D([0,1]) is the set of all intervals within [0,1].
F. Definition[Molodtsov]: A pair K_A is called a soft set over U, where F is a mapping given by K : A ⟶ P(U). In other words, a soft set over U is a parameterized family of subsets of the universe U.

G. Definition[Molodtsov]:For two soft sets K_A and G_B over U, K_A is called a soft subset of G_B, if

(1) A ⊆ B and
(2) For all eε A ; K(e) and G(e) are identical approximations.

It is denoted by $K_A \subseteq G_B$. K_A is called a soft super set of G_B if G_B is a subset of K_A. It is denoted by $K_A \subseteq G_B$.

H. Definition [Maji et.al]: Union of two soft sets of K_A and G_B over U is the soft set H_C, where C = A U B and e ε C,

H_C= K(e) , if e ε A-B,
G(e) , if e ε B-A,

K(e)U G(e) , if e ε A∩B.It is denoted by $K_A U G_B = H_C$.

I. Definition [Maji et.al]: Intersection of two soft sets of K_A and G_B over U is the soft set H_C, where C = A ∩ B and e ε C,

H_C = { K(e) , if e ε A-B,
G(e) , if e ε B-A,
K(e) ∩ G(e) if e ε AUB.It is denoted by $K_A \cap G_B = H_C$.

J. ***Definition [Maji et.al]:***Let G be a non empty set. A Q-fuzzy soft subset μ on G is defined by $\mu : G\times Q \to [0,1]$ for all x ∈ G.

K. Definition[Maji et.al]: Letμ be a Q-fuzzy soft subset in a group G. Then μ is called a Q-fuzzy soft submodule (QFSM)of G if

(i) $\mu(x+y,q) \geq \min\{ \mu(x,q), \mu(y,q) \}$ for all $x, y \in G$
(ii) $\mu(x^m, q) \geq \mu(x,q)$ for all $x \in$ G.

L. Definition[Yang et.al]:Let G be a set. An interval valued Q-fuzzy soft set A defined on G is given by A = ($x, \mu_A^-(x,q), \mu_A^+(x,q)$) for all $x \in$ G. Briefly denote A by A=[μ_A^-,μ_A^+] where μ_A^- and μ_A^+ are lower and upper fuzzy soft sets in G such that $\mu_A^-(x,q) \leq \mu_A^+(x,q)$ for all $x \in$ G.

2.13Definition[Yang et.al]:Let G be a non empty set. An interval valued bi-cubic soft set (IVBSS) 'A' in a set G is a structure $\mathscr{A} = \{(x, \tilde{\mu}_A(x), \tilde{V}_A(x),), x\in G\}$which is briefly denoted by $\mathscr{A}$=<$\tilde{\mu}_A$,$\tilde{V}_A$> where $\tilde{\mu}_A$=[μ_A^-,μ_A^+] is an IVFSS is G ,$\tilde{V}_A$=[$t_{A,}$1- f_A] is an interval valued vague soft set (IVVSS) in G.

M. Definition:An IVBFSS$\mathscr{A}$ is said to be an interval valued bi-cubic homology fuzzy soft submodule (IVBHFSM) if

(i) $\tilde{\mu}_A(x+ym,q) \geq r\min\{ \tilde{\mu}_A(xm,q), \tilde{\mu}_A(ym,q)\}$
(ii) $\tilde{\mu}_A(x^m,q) \geq \tilde{\mu}_A(xm,q)$
(iii) $\tilde{V}_A(x+ym,q) \leq r\max\{ \tilde{V}_A(xm,q), \tilde{V}_A(ym,q)\}$
(iv) $\tilde{V}_A(x^m,q) \leq \tilde{V}_A(xm,q)$, for all values of $x, y \in X$, $q \in Q$.

III. PROPERTIES OF INTERVAL VALUED HOMOLOGY FUZZY SOFT SUBMODULE

A. Definition: Let (M,A) = { (M_n , A_n), r_n }$_{n \in z}$ be a fuzzy soft chain complex. The condition ř .ř = 0 implies that Im$ř_{n+1}$ ⊆ker$ř_n$, n ∊ z. Hence, we can associate with (M, A) the fuzzy grade module H (M ,A) = { H_n(M, A)}, where H_n(M, A) is the fuzzy soft quotient or ker$ř_n$ by Im$ř_{n+1}$, n ∊ z. Then H (M ,A) = { H_n(M, A)} is called the n^{th} fuzzy soft homology module of (M, A).

B. Definition:Let $\mathscr{A} = \{\tilde{\mu}_A, \tilde{V}_A\}$ be a IVBHFS'A' in a group G.Let [α,β] & [γ,δ] ∈[0,1].The set ∪{ $\mathscr{A}$:[α,β] ,[γ,δ]}={x ∈ G /$\tilde{\mu}_A(x,q) \geq$[α,β] &$\tilde{V}_A(x,q) \leq$ [γ,δ] }is called cubic level set of $\mathscr{A}$.

C. Proposition:Let $\mathscr{A} = \{\tilde{\mu}_A, \tilde{V}_A\}$ be aIVBHFSM 'A' in a group G. Then $\tilde{\mu}_A(x^m,q) = \tilde{\mu}_A(x,q)$ and $\tilde{V}_A(x^m,q) = \tilde{V}_A(x,q)$ for all of $x \in G$, $q \in Q$.

Proof:For all x ∈ G, we have

$\tilde{\mu}_A((x^m,q) = \tilde{\mu}_A((x^m)^{-1},q) \geq \tilde{\mu}_A((x^m,q) \geq \tilde{\mu}_A(x,q)$&
$\tilde{V}_A((x^m,q)$=$\tilde{V}_A((x^m)^{-1},q) \leq \tilde{V}_A(x^m,q) \leq \tilde{V}_A(x,q)$.
Hence $\tilde{\mu}_A(x^m,q)$=$\tilde{\mu}_A(x,q)$&$\tilde{V}_A(x^m,q)$=$\tilde{V}_A(x,q)$.

D. Proposition: An IVBHFS$\mathscr{A} = \{\tilde{\mu}_A, \tilde{V}_A\}$ is IVBHFSM of G if and only if

(i) $\tilde{\mu}_A(x+ym \;\; ,q) \geq r\min\{ \tilde{\mu}_A(x,q), \tilde{\mu}_A(y,q)\}$&
(ii) $\tilde{V}_A(x+ym \;\; ,q) \leq r\max\{ \tilde{V}_A(x,q), \tilde{V}_A(y,q)\}$ for$x, y \in X$, $q \in Q$.

Proof: Assume that $\mathscr{A} = \{\tilde{\mu}_A, \tilde{V}_A\}$ is a IVBHFSM of G and $x, y \in G$.

Then $\tilde{\mu}_A(x+ym\ ,q) \geq$ rmin $\{\tilde{\mu}_A(xm,q), \tilde{\mu}_A(ym\ ,q)\}$ (By definition)

$=$ rmin $\{\tilde{\mu}_A(x,q), \tilde{\mu}_A(y,q)\}$ (By Proposition 3.2)

Also $\tilde{V}_A(x+ym\ ,q) \leq$ rmax $\{\tilde{V}_A(xm,q), \tilde{V}_A(ym\ ,q)\}$ (By definition)

$=$ rmax $\{\tilde{V}_A(x,q), \tilde{V}_A(y,q)\}$ (By Proposition 3.2)

Conversely, suppose (i) & (ii) are valid.

If we take $y = x^m$ in (i) & (ii), then

$\tilde{\mu}_A(e,q)=\tilde{\mu}_A(x+x^m,q) \geq$ rmin $\{\tilde{\mu}_A(x,q), \tilde{\mu}_A(x^m,q)\}$

$=$rmin $\{\tilde{\mu}_A(x,q), \tilde{\mu}_A(x,q)\}$ (By Proposition 3.2)

$\tilde{\mu}_A(e,q) \geq \tilde{\mu}_A(x,q)$and

$\tilde{V}_A(e,q)=\tilde{V}_A(x+x^m,q) \leq$ rmax $\{\tilde{V}_A(x,q), \tilde{V}_A(x^m,q)\}$

$=$rmax $\{\tilde{V}_A(x,q), \tilde{V}_A(x,q)\}$ (By Proposition 3.2)

$\tilde{V}_A(e,q) \geq \tilde{V}_A(x,q)$

It follows from (i) & (ii), that

$\tilde{\mu}_A(y^m,q) = \tilde{\mu}_A(e+y^m,q) \geq$ rmin $\{\tilde{\mu}_A(e,q), \tilde{\mu}_A(y^m,q)\}$

$\geq$ rmin $\{\tilde{\mu}_A(e,q), \tilde{\mu}_A(y,q)\}$ (By Proposition 3.2)

$\tilde{\mu}_A(y^m,q) \geq \tilde{\mu}_A(y,q)$

Also $\tilde{V}_A(y^m,q) = \tilde{V}_A(e+y^m,q) \leq$ rmax $\{\tilde{V}_A(e,q), \tilde{V}_A(y^m,q)\}$

$\leq$rmax$\{\tilde{V}_A(e,q), \tilde{V}_A(y,q)\}$ (By Proposition 3.1)

$\therefore \tilde{V}_A(y^m,q) \leq \tilde{V}_A(y,q)$

$\tilde{\mu}_A(x+ym,q) = \tilde{\mu}_A(x+(y^m)^{-1},q) \geq$ rmin $\{\tilde{\mu}_A(xm,q), \tilde{\mu}_A(ym\ ,q)\}$

$\geq$ rmin $\{\tilde{\mu}_A(x,q), \tilde{\mu}_A(y,q)\}$

$\tilde{V}_A(x+ym,q) = \tilde{V}_A(x+(y^m)^{-1},q) \leq$ rmax $\{\tilde{V}_A(xm,q), \tilde{V}_A(ym\ ,q)\}$

$\leq$ rmax $\{\tilde{V}_A(x,q), \tilde{V}_A(y,q)\}$.

$\therefore \{\tilde{\mu}_A, \tilde{V}_A\}$ is IVBHFSM of G.

E. Proposition: Let $\mathscr{A} = \{\tilde{\mu}_A, \tilde{V}_A\}$ be an IVBHFSM of G. Then the following conditions one equivalent:

(i) $\mathscr{A} = \{\tilde{\mu}_A, \tilde{V}_A\}$ be a IVBHFSM of G,

(ii) The non empty cubic level set of $\mathscr{A} = \{\tilde{\mu}_A, \tilde{V}_A\}$ is a subgroup of G.

Proof: Assume that $\mathscr{A} = \{\tilde{\mu}_A, \tilde{V}_A\}$ is a IVBHFSM of G,

Let $x, y \in \cup\{\mathscr{A}: [\alpha,\beta], [\gamma,\delta]\}$ for all $[\alpha,\beta]$ & $[\gamma,\delta] \in D[0,1]$

Then $\tilde{\mu}_A(x,q) \geq [\alpha,\beta]$, $\tilde{V}_A(x,q) \leq [\gamma,\delta]$

$\tilde{\mu}_A(y,q) \geq [\alpha,\beta]$, $\tilde{V}_A(y,q) \leq [\gamma,\delta]$

It follows that

$\tilde{\mu}_A(x+y^{-1},q) \geq$ r min $\{\tilde{\mu}_A(x,q), \tilde{\mu}_A(y,q)\} \geq [\alpha,\beta]$

$\tilde{V}_A(x+y^{-1},q) \leq$r max $\{\tilde{V}_A(x,q), \tilde{V}_A(y,q)\} \leq [\gamma,\delta]$

So that $x+y^{-1} \in \cup\{\mathscr{A}: [\alpha,\beta], [\gamma,\delta]\}$.

The non empty cubic level set $\mathscr{A} = \{\tilde{\mu}_A, \tilde{V}_A\}$ is IVBHFSM of G.

Conversely, $[\alpha,\beta]$ & $[\gamma,\delta] \in D[0,1]$ such that

$\cup(\mathscr{A}: [\alpha,\beta], [\gamma,\delta]) \neq \emptyset$& $\cup(\mathscr{A}: [\alpha,\beta], [\gamma,\delta])$ is a subgroup of G.

Suppose that Proposition 3. 4 (i) is not true & Proposition 3. 4 (ii) is valid. Then there exist

$[\alpha_0, \beta_0] \in D[0,1]$ & $a, b \in G$ such that

$\tilde{\mu}_A(a+b^m,q) \leq [\alpha_0,\beta_0] \leq$ r min $\{\tilde{\mu}_A(a,q), \tilde{\mu}_A(b,q)\}$

$\tilde{V}_A(ab^m,q) \geq [\gamma_0,\delta_0] \geq$ r max $\{\tilde{V}_A(a,q), \tilde{V}_A(b,q)\}$

F. Proposition: Let f: $G \to G^{l}$ is a homomorphism of groups.If $\mathscr{A} = \{\tilde{\mu}_A, \tilde{V}_A\}$ is an IVBHFSMof G^{l}, then $\mathscr{A} = \{\tilde{\mu}_A{}^f, \tilde{V}_A{}^f\}$ is IVBHFSM of G.

Proof:

(i) $\tilde{\mu}_A{}^f(x+ym,q) = \tilde{\mu}_A(f(x+ym,q)) = \tilde{\mu}_A(f(xm,q), f(ym,q))$ [since f is homo]

$\geq$ rmin$\tilde{\mu}_A(f(xm,q), \tilde{\mu}_A(f(ym,q))$ = rmin $\{\tilde{\mu}_A{}^f(xm,q), \tilde{\mu}_A{}^f(ym,q)\}$

(ii) $\tilde{\mu}_A{}^f(x^m,q) = \tilde{\mu}_A(f(x^m,q)) \geq \tilde{\mu}_A(f(x,q)) = \tilde{\mu}_A{}^f(x,q)$

(iii) $\widetilde{V}_A{}^f(x+ym,q) = \widetilde{V}_A\big(f(x+ym,q)\big) = \widetilde{V}_A(f(xm,q), f(ym,q))$ [since f is homo]
$\leq r\max\{\widetilde{V}_A(f(xm,q), \tilde{v}_A(f(ym,q))\}$
$= r\max\{\widetilde{V}_A{}^f(xm,q), \widetilde{V}_A{}^f(ym,q)\}$
(iv) $\widetilde{V}_A{}^f(x^m,q) = \widetilde{V}_A\big(f(x^m,q)\big) \leq \widetilde{V}_A(f(x,q)) = \widetilde{V}_A{}^f(x,q)$.

G. Proposition: Let $\mathscr{A}$ be an IVBHFSM of G and A(e)=1 is normal defined by $\widetilde{A}^+(x)=\widetilde{A}(x)+1-\widetilde{A}(e), \forall\, x, e \in G$. Then $\widetilde{A}^+$ is IVBHFSM of G.
Proof: Let $\mathscr{A} = \{\tilde{\mu}_A, \widetilde{V}_A\}$ is IVBHFSM for x, y ∈ G & $e \in G$ such that $\widetilde{A}^+(e) = \widetilde{A}(e)+1-\widetilde{A}(e)=1$
Now (i) $\tilde{\mu}_A{}^+(x+ym,q) = \tilde{\mu}_A(x+ym,q)+1-\tilde{\mu}_A(e,q)$
$\geq r\min\{\tilde{\mu}_A(xm,q), \tilde{\mu}_A(ym,q)\}+1-\tilde{\mu}_A(e,q)$
$\geq r\min\{\tilde{\mu}_A(xm,q)+1-\tilde{\mu}_A(e,q), \tilde{\mu}_A(ym,q)+1-\tilde{\mu}_A(e,q)\}$
$= r\min\{\tilde{\mu}_A{}^+(xm,q), \tilde{\mu}_A{}^+(ym,q)\}$
(ii) $\tilde{\mu}_A{}^+(x^m,q) = \tilde{\mu}_A(x^{-1},q)+1-\tilde{\mu}_A(e,q) \geq \tilde{\mu}_A(xm,q)+1-\tilde{\mu}_A(e,q)$
$\geq \tilde{\mu}_A{}^+(xm,q)$
(iii) $\widetilde{V}_A{}^+(x+ym,q) = \widetilde{V}_A(x+ym,q)+1-\widetilde{V}_A(e,q)$
$\leq r\max\{\widetilde{V}_A(xm,q), \tilde{\mu}_A(ym,q)\}+1-\widetilde{V}_A(e,q)$
$\leq r\max\{\widetilde{V}_A(xm,q)+1-\widetilde{V}_A(e,q), \widetilde{V}_A(ym,q)+1-\widetilde{V}_A(e,q)\}$
$= r\max\{\widetilde{V}_A{}^+(xm,q), \widetilde{V}_A{}^+(ym,q)\}$
(iv) $\widetilde{V}_A{}^+(y^m,q) = \widetilde{V}_A(y^m,q)+1-\widetilde{V}_A(e,q) \leq r\max\{\widetilde{V}_A(ym,q)+1-\widetilde{V}_A(e,q)\}$
$\leq \widetilde{V}_A{}^+(ym,q)$.

H. Definition: Let θ be a mapping from X to Y. If A & B are IVBHFSM's in X & Y respectively, then the inverse image of B under θ denoted by $\theta^{-1}(B)$ is defined by $\theta^{-1}(B) = \tilde{\mu}_\theta{}^{-1}(B)$ where $\tilde{\mu}_\theta{}^{-1}{}_{(B)}(x,q) = \tilde{\mu}_B(\theta(x,q))$ and $\tilde{\mu}_\theta{}^{-1}{}_{(B)}(x^m,q) = \tilde{\mu}_B(\theta(x^m,q)) \forall\, x \epsilon Y, q \epsilon Q$.

I. Proposition: The inverse image of an IVBHFSM is also IVBHFSM
Proof: Let G and $\overline{G}$ be two groups and $\theta: G \to \overline{G}$ a homomorphism. Let B is IVBHFSM of $\overline{G}$.
We have to prove that $\theta^{-1}(B)$ is IVBHFSM of G.
Let x, y ϵ G, q ϵ Q.
(i) $\tilde{\mu}_\theta{}^{-1}{}_{(B)}(x+ym,q) = \tilde{\mu}_B\big(\theta(x+ym,q)\big)$
$= \tilde{\mu}_B\big(\theta(xm)\theta(ym),q\big)$
$\geq r\min\{\tilde{\mu}_B(\theta(xm),q), \tilde{\mu}_B(\theta(ym),q)\}$
$\geq r\min\{\tilde{\mu}_\theta{}^{-1}{}_{(B)}(xm,q), \tilde{\mu}_\theta{}^{-1}{}_{(B)}(ym,q)\}$
(ii) $\tilde{\mu}_\theta{}^{-1}{}_{(B)}(x^m,q) = \tilde{\mu}_B(\theta(x^m,q))$
$= \tilde{\mu}_B(\theta x^m,q) = \tilde{\mu}_B(\theta xm,q) = \tilde{\mu}_\theta{}^{-1}{}_{(B)}(xm,q)$
(iii) $\widetilde{V}_\theta{}^{-1}{}_{(B)}(x+ym,q) = \widetilde{V}_B\big(\theta(x+ym,q)\big)$
$= \widetilde{V}_B\big(\theta(xm)\theta(ym),q\big)$
$\leq r\max\{\widetilde{V}_B(\theta(xm),q), \widetilde{V}_B(\theta(ym),q)\}$
$\leq r\max\{\widetilde{V}_\theta{}^{-1}{}_{(B)}(xm,q), \widetilde{V}_\theta{}^{-1}{}_{(B)}(ym,q)\}$
(iv)) $\widetilde{V}_\theta{}^{-1}{}_{(B)}(x^m,q) = \widetilde{V}_B(\theta(x^m,q))$
$= \widetilde{V}_B(\theta x^m,q) = \widetilde{V}_B(\theta xm,q) = \widetilde{V}_\theta{}^{-1}{}_{(B)}(xm,q)$
∴ $\theta^{-1}(B)$ is IVBHFSM of G.

J. Proposition: If $\{A_i\}_i \in A$ is a family of IVBHFSM's of G, then $\bigcap_{i\in A} A_i$ is IVBHFSM of G, where $\bigcap_{i\in A} A_i = \{((x,q), \wedge\tilde{\mu}_{A_i}(x,q)) / x \in G, q \in Q\}$.
Proof: Let x, y ϵ G, q ϵ Q.
(i) $(\bigcap_{i\in A}\tilde{\mu}_{A_i})(x+ym,q) = \wedge_{i\in A}\tilde{\mu}_{A_i}(x+ym,q)$
$\geq \wedge_{i\in A} r\min\{\tilde{\mu}_{A_i}(xm,q), \tilde{\mu}_{A_i}(ym,q)\}$
$= r\min\{\wedge_{i\in A}\tilde{\mu}_{A_i}(xm,q), \wedge_{i\in A}\tilde{\mu}_{A_i}(ym,q)\}$
$= r\min\{(\bigcap_{i\in A}\tilde{\mu}_{A_i})(xm,q), (\bigcap_{i\in A}\tilde{\mu}_{A_i})(ym,q)\}$

(ii) $(\bigcap_{i\in A}\ \tilde{\mu}_{A_i})\ (x^m, q) = \wedge_{i\in A}\ \tilde{\mu}_{A_i}\ (x^m, q)$

$$\geq \bigwedge_{i\in A} \tilde{\mu}_{A_i}\ (xm, q)$$

$= (\bigcap_{i\in A}\ \tilde{\mu}_{A_i})\ (xm, q)$

(iii) $(\bigcap_{i\in A}\ \tilde{V}_{A_i})\ (x + ym, q) = \wedge_{i\in A}\ \tilde{V}_{A_i}\ (x + ym, q)$

$\geq \wedge_{i\in A}\ rmin\{\ \tilde{V}_{A_i}(xm, q), \tilde{V}_{A_i}(ym, q)\}$

$= rmax\ \{\ \wedge_{i\in A}\ \tilde{V}_{A_i}\ (xm, q), \wedge_{i\in A}\ \tilde{V}_{A_i}\ (ym, q)\ \}$

$= rmax\ \{\ (\bigcap_{i\in A}\ \tilde{V}_{A_i})\ (xm, q), (\bigcap_{i\in A}\ \tilde{V}_{A_i})\ (ym, q)\ \}$

(iv) $(\bigcap_{i\in A}\ \tilde{V}_{A_i})\ (x^m, q) = \wedge_{i\in A}\ \tilde{V}_{A_i}\ (x^m, q)$

$\geq \wedge_{i\in A}\ \tilde{V}_{A_i}\ (xm, q) = (\bigcap_{i\in A}\ \tilde{V}_{A_i})\ (xm, q)$

Hence $\bigcap_{i\in A}\ A_i$ is IVBHFSM of G.

CONCLUSION

The application of soft sets is not limited to these areas only but it also motivated people working in more abstract areas of mathematics to apply soft sets in their areas.In this paper, we discuss the properties of an interval valued bi-cubic homology fuzzy soft sub module [IVBHFSM] and its arbitrary intersections. Also the level subset of homology soft modules indexed with its interval valued fuzzy soft set has been discussed. Finally, the inverse image of an interval valued bi -cubic homology fuzzy soft modules is also an interval valued bi -cubic homology fuzzy soft modules is proved.

REFERENCES

[1] Acar, U., Koyuncu, F. and Tanay, B. (2010). Soft sets and soft rings. Computers and Mathematics with Applications, 59: 3458- 3463.
[2] Aktas, H. and Cagman, N. (2007). Soft sets and soft groups. Information Sciences, 1: 2726-2735.
[3] Ali, M. I., Feng, F., Lui, X., Min, W. K. and Shabir, M. (2009). On some new operations in soft set theory. Computers and Mathematics with Applications, 57: 1547- 1553.
[4] Atagun, A. O. and Sezgin, A. (2011). Soft substructures of rings, fields and modules. Computers and Mathematics with Applications, 61: 592- 601.
[5] Jun, Y.B. (2008). Soft BCK/BCI-algebras. Computers and Mathematics with Applications, 56: 1408-1413.
[6] .Jun, Y.B. and Park, C.H. (2008). Applications of soft sets in ideals theory of BCK/BCI- algebras. Information Sciences, 178: 2466-2475.
[7] Molodtsov, D. (1999). Soft set Theory-First results. Computers and Mathematics with Applications, 37: 19-31.
[8] Maji, P.K., Biswas, R. and Roy, A.R. (2003). Soft set theory. Computers and Mathematics with Applications, 45: 555-562.
[9] Ozturk, M.A. and Inna, E. (2011). Soft -rings and idealistic soft –rings. Annuals of Fuzzy Mathematics. and Informatics, 1(1): 71-80
[10] Sezgin, A. and Atagun, A.O. (2011A). On operations of soft sets. Computers and Mathematics with Applications, 60: 1840-1849.
[11] Sezgin, A. and Atagun, A.O. (2011B). Soft groups and normalistic soft group. Computers and Mathematics with Applications, 62: 685-698.
[12] Sun, Q.M., Zhang, Z. and Liu, J. (2008). Soft sets and soft modules. Proceedings of Rough sets and knowledge Technology, Third international Conference RSKT, Chengdu, China, 403-409.
[13] Yang, C. (2008). A note on ''Soft set Theory''. Computers and Mathematics with Applications, 56: 1899-1900.
[14] Zadeh, L.A.(1965). Fuzzy sets. Information and Control, 8: 338-353.