A Note on Regular Soft Substructures of a Soft Semigroup

Nistala V.E.S. Murthy[1], Emandi Gouthami[2] and Chundru Maheswari[3]
Department of Mathematics
J.V.D. College of Science and Technology
Andhra University, Visakhapatnam-530003
A.P. State, INDIA

Abstract *— The aim of this paper is to study order properties of collections of soft substructures of a soft semigroup.*

Keywords — *Soft (sub) set, soft (sub) semigroup, soft (left, right, quasi, bi) ideal.*

AMS Classification— 06B99, 20M10, 20M12, 06D72

I. INTRODUCTION

As the applications of Mathematics grew more and more in such fields as Economics, Engineering, Social Science, Environmental Sciences etcetera, the need to represent and handle various structurally inherent uncertainties within these fields became clear and the inability of classical tools of Mathematics in these directions paved the way to invent new theories. Until 1999, these uncertainties were addressed with to three theories namely, theory of probability and statistics and the theories of various types of fuzzy sets.

In 1999 Molodtsov[8] introduced the theory of Soft Sets, based on descriptions or parameters. Since then several mathematicians started studying algebraic structures on them. In 2010, Ali-Shabir-Shum[3] introduced the notions of soft semigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) over a semigroup and studied some of their properties.

Coming to this paper, we study some order properties of collections of soft substructures of a soft semigroup.

Throughout the paper, proofs are left for two reasons, namely in most cases they are simple or straight forward but a little involving and secondly to minimize the size of the document, however, in order to make the document more self contained, we recall as many notions and results that are used in subsequent sections, as possible.

II. PRELIMINARIES

In any meet complete poset with the greatest element 1_L, for any subset $\emptyset \neq S \subseteq L$ one can define $\overline{\vee}S = \wedge\{\alpha \in L / \alpha \wedge \beta = \beta \text{ for all } \beta \in S\}$. Then L is a complete lattice, where the join is given by $\overline{\vee}$. For any meet complete poset L with the greatest element 1_L, the join defined as above is called the meet induced join on L and the complete lattice L defined as above is called the associated complete lattice for the meet complete poset L.

Semigroups and Substructures In what follows we recall some conventions, definitions and some lattice theoretic properties of various substructures of a semigroup which are used in the main results later:

(a) The empty set trivially defines a function from the empty set into any set and this function is called the *empty function*. However, the empty set does not define a function from a non-empty set into the empty set.

(b) A *binary operation* on a set A is any function from A^2 into A. Notice that the empty function is the only binary operation on the empty set.

(c) A set S together with a binary operation which is associative is called a *semigroup*. Notice that as in Grillet[6], the empty set is trivially a semigroup with the empty binary operation, called the *empty semigroup*.

Throughout this paper (S,.) is a semigroup and most of the times we write only S instead of (S,.), omitting dot always and almost every where

(d) For any pair of subsets A, B of a semigroup S, the set AB is defined by AB = $\{ab \in S / a \in A \text{ and } b \in B\}$ and it is a subset of S.
(e) For any subsets A, B, C, $(A_i)_{i\in I}$ of a semigroup S, the following are true:
(i) A ⊆ B implies CA ⊆ CB, AC ⊆ BC, AC∩CA ⊆ BC ∩ CB, ACA ⊆ BCB (ii) A($\cap_{i\in I} A_i$) ⊆ $\cap_{i\in I} AA_i$ and A($\cup_{i\in I} A_i$) = $\cup_{i\in I} AA_i$.
(f) For any subset A of a semigroup S, A is a *subsemigroup* of S iff $A^2 \subseteq A$. In other words, A is a subsemigroup of S iff for any $a, a' \in A$, we have $aa' \in A$. Notice that the empty semigroup is trivially a subsemigroup of any semigroup because there are no elements a, a' in the empty set such that their product aa' is not in the empty set.
(g) A is a *left (right) ideal* of S iff SA⊆ A (AS ⊆A). In other words, A is a left (right) ideal of S iff for any a∈A and for any s∈S, we have sa∈A (as∈A). Notice that the empty semigroup is trivially a left (right) ideal of any semigroup S because there is *no* s in S and there is *no* a in the empty set such that sa (as) not in the empty set.
(h) A is an *ideal* of S iff SA∪AS⊆A iff it is both a left and a right ideal of S. In other words, A is an ideal of S iff for any a∈ A and for any s ∈S, we have both sa and as ∈A Notice that the empty semigroup is trivially an ideal of any semigroup.
(i) A is a *quasi ideal* of S iff SA∩AS ⊆A. In other words, A is a quasi ideal of S iff for any $a, a' \in A$ and for any $s, s' \in S$, if t = $sa = a's'$ then t∈A.
Notice that (1) *the* empty semigroup is trivially a quasi ideal of any semigroup (2) every (left, right, quasi) ideal of a semigroup is always a subsemigroup but *not* conversely (3) every (left, right) ideal of a semigroup is always a quasi ideal but not conversely.
(j) A subsemigroup A of a semigroup S is said to be a *bi ideal* of S iff ASA⊆A. In other words, A is a bi ideal of S iff for any $a, a' \in A$ and for any s ∈ S, we have $asa' \in A$.
Notice that (1) the empty semigroup is trivially a bi ideal of any semigroup (2) every (left, right, quasi) ideal of a semigroup is always a bi ideal but *not* conversely (3) every quasi ideal of a semigroup is always a bi ideal but *not* conversely.

Algebra of Substructures of a Semigroup:
(k) In any semigroup, arbitrary intersection of subsemigroups (left ideals, right ideals, ideals, quasi ideals, bi ideals) is a subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal).
(l) In any semigroup, arbitrary union of (left, right) ideals is a (left, right) ideal.
(m) In any semigroup and for any subset of it, the intersection of all subsemigroups (left ideals, right ideals, ideals, quasi ideals, bi ideals) containing the given subset is a subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) which is unique and the smallest with respect to the containment of the given subset.
(n) For any subset A of a semigroup S, the unique smallest subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) containing a given subset A defined as in (m) above is called the *subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) generated by* A and is denoted by $(A)_{s,S}$ $((A)_{*,S}$, for $*$ = *l, r, i, q, b respectively*).
(o) For any semigroup S, we have $(\emptyset)_{*,S} = \emptyset$ for $*$ = *s, l, r, i, q, b respectively*.
(p) For any subsets A, B of a semigroup S such that $A \subseteq B$, $(A)_{*,S} \subseteq (B)_{*,S}$ for $*$ = *s, l, r, i, q, b respectively*
(q) For any subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) B of a semigroup S and for any A⊆B, $(A)_{s,S} \subseteq B$ $((A)_{*,S} \subseteq B)$ for $*$ = *l, r, i, q, b respectively*).
(r) For any pair of subsemigroups (left ideals, right ideals, ideals, quasi ideals, bi ideals) A, B of a semigroup S, A is a subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) of B iff $A \subseteq B$.

In what follows, we recall some elementary lattice theoretic study of substructures of a semigroup which are however *not* widely available.
(s) The set of all subsemigroups (quasi ideals, bi ideals) of a semigroup S is a complete lattice with
(1) the partial ordering given by: for any A, B in the set of all subsemigroups (quasi ideals, bi ideals) of S, $A \le B$ iff A is a subsemigroup (quasi ideal, bi ideal) of B iff $A \subseteq B$
(2) the largest element being S and the least element being $\emptyset$
(3) for any family $(A_i)_{i\in I}$ in the set of all subsemigroups (quasi ideals, bi ideals), $\wedge_{i\in I} A_i = \cap_{i\in I} A_i$ and $\vee_{i\in I} A_i = \bar{\vee}_{i\in I} A_i$ where $\bar{\vee}$ is the meet induced join.
Further, $\bar{\vee}_{i\in I} A_i = (\cup_{i\in I} A_i)_{s,S}$ $((\cup_{i\in I} A_i)_{*,S}$ for $* =$ q, b).
(t) The set of all (left, right) ideals of a semigroup S is a complete lattice with
(1) the partial ordering given by: for any A, B in the set of all (left, right) ideals of S, $A \le B$ iff A is a (left, right) ideal of B iff $A \subseteq B$
(2) the largest element being S and the least element being $\emptyset$
(3) for any family $(A_i)_{i\in I}$in the set of all (left, right) ideals, $\wedge_{i\in I} A_i = \cap_{i\in I} A_i$ and $\vee_{i\in I} A_i = \cup_{i\in I} A_i$.
(u) For any pair of subsets A, B of a semigroup S such that $A \subseteq B$, the following are true:
(i) B is a subsemigroup of S implies A is a subsemigroup of B iff A is a subsemigroup of S.

Remark: For any pair of subsets A, B of a semigroup S such that $A \subseteq B$, it may so happen that A is a subsemigroup of both B and S but B is *not* a subsemigroup of S, and in what follows, we give an example for the same:

Example 2.1 Let S = $\{a,b,c,d\}$, B = $\{a,b,c\}$ and A = $\{a,b\}$ be the semigroups with the following Cayley tables:

$\cdot_S$	a	b	c	d
a	a	b	a	b
b	a	b	a	b
c	c	d	c	d
d	c	d	c	d

$\cdot_B$	a	b	c
a	a	b	a
b	a	b	a
c	a	b	c

$.A$	a	b
a	a	b
b	a	b

Thus A is a subsemigroup of both S and B but B is *not* a subsemigroup of S because $c \cdot_s a = c \neq a = c \cdot_B a$.

(ii) B is a left (right, quasi , bi) ideal of S and A is a left (right, quasi, bi) ideal of S implies A is a left (right, quasi , bi) ideal of B but *not* conversely.

The following examples shows that converse of u(ii) above is *not* true:
Example 2.2. Let S = $\{0, a, b, c, d\}$ be a semigroup with the following Cayley table:

.S	0	a	b	c	d
0	0	0	0	0	0
a	0	0	0	a	b
b	0	a	b	0	0
c	0	0	0	c	d
d	0	c	d	0	0

Let B = $\{0, a, c\}$ and A = $\{0, a\}$.Then B is a left ideal of S and A is a left ideal of B but A is *not* a left ideal of S.
Let B = $\{0, a, b\}$ and A = $\{0, a\}$.Then B is a right ideal of S and A is a right ideal of B but A is *not* a right ideal of S.

Example 2.3. Let S =$\{0, a, b, c, d\}$be a semigroup with the following Cayley table:

.S	0	a	b	c	d
0	0	0	0	0	0
a	0	0	a	0	c
b	0	a	b	c	d
c	0	0	c	0	0
d	0	c	d	0	0

Let B = $\{0, a, c\}$ and A = $\{0, a\}$. Then B is an ideal of S and A is a ideal of B but A is *not* an ideal of S.
Let B = $\{0, a, c\}$ and A = $\{0, a\}$. Then B is a quasi ideal of S and A is a quasi ideal of B but A is *not* a quasi ideal of S.

Example 2.4. Let S =$\{0, a, b, c\}$ be a semigroup with the following Cayley table:

.S	0	a	b	c
0	0	0	0	0
a	0	0	0	b
b	b	b	b	b
c	c	c	c	c

Let B = $\{0, a, c\}$ and A = $\{0, a\}$. Then B is a bi ideal of S and A is a bi ideal of B but A is *not* a bi ideal of S.

(v) For any semigroup S and for any pair of subsets A, B of S, B is a subsemigroup of S, A is a subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) of S implies $A \cap B$ is a subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) of B.
(w) For any semigroup S, the following are true:
(i) The intersection of a right ideal and a left ideal in S is a quasi ideal
(ii) For any subsemigroup A of S and for any (left, right, quasi , bi) ideal B of S, $A \cap B$ is a (left, right, quasi , bi) ideal of A.

Internal description of substructures In what follows, we give an internal description for the subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) generated by the given subset:
(x) For any semigroup S and for any A⊆S, the following are true:
(i) $(A)_{s,S} = \cup_{n\geq 0} A_n$ where $A_0 = A \cup AA$, $A_1 = A_0 \cup A_0A_0$ and so on $A_n = A_{n-1} \cup A_{n-1}A_{n-1}$.
(ii) $(A)_{l,S} = \cup_{n\geq 0} A_n$ where $A_0 = A \cup SA$, $A_1 = A_0 \cup SA_0$ and so on $A_n = A_{n-1} \cup SA_{n-1}$.
(iii) $(A)_{r,S} = \cup_{n\geq 0} A_n$ where $A_0 = A \cup AS$, $A_1 = A_0 \cup A_0S$ and so on $A_n = A_{n-1} \cup A_{n-1}S$.
(iv) $(A)_{i,S} = \cup_{n\geq 0} A_n$ where $A_0 = A \cup SA \cup AS$, $A_1 = A_0 \cup SA_0 \cup A_0S$ and so on $A_n = A_{n-1} \cup SA_{n-1} \cup A_{n-1}S$.
(v) $(A)_{q,S} = \cup_{n\geq 0} A_n$ where $A_0 = A \cup (SA \cap AS)$, $A_1 = A_0 \cup (SA_0 \cap A_0S)$ and so on $A_n = A_{n-1} \cup (SA_{n-1} \cap A_{n-1}S)$.
(vi) $(A)_{b,S} = \cup_{n\geq 0} A_n$ where $A_0 = A \cup ASA$, $A_1 = A_0 \cup A_0SA_0$ and so on $A_n = A_{n-1} \cup A_{n-1}SA_{n-1}$.

Soft Sets In what follows we recall the following basic definitions from the Soft Set Theory which are used in the main results:
(y) [9] Let U be a universal set, P(U) be the power set of U and E be a set of parameters. A pair (F, E) is called a *soft set* over U iff F: E → P(U) is a mapping defined by, for each e ∈ E, F(e) is a subset of U. In other words, a soft set over U is a parametrized family of subsets of U.
Notice that a collective presentation of the notions, soft sets and gs-sets raised some serious notational conflicts and to fix the same Murthy-Maheswari[10] deviated from the above notation for a soft set and adapted the following notation for convenience as follows:
Let U be a universal set. A typical soft set over U is an ordered pair S = (σ_s, S), where S is a set of parameters, called the *underlying parameter set* for S, P(U) is the power set of U and σ_s: S → P(U) is a map, called the *underlying set valued map* for S. Sometimes σ_s is also called the soft structure on S.

(z) [2] The *empty soft* set over U is a soft set with the empty parameter set, denoted by **Φ**, is defined by **Φ** = ($\sigma_\emptyset$, ∅). Clearly, it is unique.
(aa) [4] A soft set S over U is said to be a *whole soft set*, denoted by U_S, iff $\sigma_s s$ = U for all s∈S. Notice that $U_\emptyset$= **Φ**.
(ab) [2] A soft set S over U is said to be a *null soft set*, denoted by Φ_s, iff $\sigma_s s = \emptyset$, the empty set, for all s∈S. Notice that $\Phi_\emptyset$ = **Φ**, the empty soft (sub) set.
For any pair of soft sets A, B over U,
(ac) [11] A is a *soft subset* of B, denoted by A ⊆ B, iff (i) A⊆B (ii) σ_A a⊆ σ_B a for all a∈A.
(ad) A is a *d-total* soft subset of B iff A is a soft subset of B and A = B
(ae) The following are easy to see:
(i) Always the empty soft set **Φ** is a soft subset of every soft set A

(ii) A = B iff A ⊆ B and B ⊆ A iff A = B and $\sigma_A = \sigma_B$.
(af) For any family of soft subsets $(A_i)_{i\in I}$ of S,
(i) the *soft union* of $(A_i)_{i\in I}$, denoted by $\cup_{i\in I}A_i$, is defined by the soft set A, where (i) A = $\cup_{i\in I} A_i$, (ii) $\sigma_A\ a = \cup_{i\in I_a} \sigma_{A_i}\ a$, where $I_a = \{i \in I / a \in A_i\}$, for all a∈A
(ii) the *soft intersection* of $(A_i)_{i\in I}$, denoted by $\cap_{i\in I}A_i$, is defined by the soft set A, where (i) A = $\cap_{i\in I} A_i$ (ii) $\sigma_A\ a = \cap_{i\in I} \sigma_{A_i} a$ for all a∈A.
Notice that if $\cap_{i\in I} A_i$ is the empty set then the soft intersection is the empty soft set.

Soft Semigroups In this section we first recall the existing notions of all soft substructures of a soft semigroup.
Ali-Shabir-Shum[3] introduced the notions of soft semigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal). According to them, a soft set (F,A) over a semigroup S which is *neither empty* (cf.2(z)) *nor null* (cf.2(ab)) is a soft semigroup over S iff (F,A)(F,A) ⊆ (F,A) and it is a soft subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) over S iff F(x) is a subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) of S for all x∈A whenever F(x)≠ ∅.

Notice that the definitions of soft (sub) semigroup (left, right, quasi, bi) ideal used in this paper are different from the above ones in two ways. Firstly, the substructure notions defined above are over a crisp semigroup and the substructure notions defined below are a slight generalizations of the above, namely, those of a soft semigroup and secondly, as empty set is trivially a (sub) semigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) for us, in our definitions below, we do not need the two pre-conditions that a soft subset (F, A) be neither empty nor null, as in the above definitions.

III. SOFT SUBSTRUCTURES OF A SOFT SEMIGROUP

In what follows we introduce the notions of soft (sub) semigroup, soft (left, right, quasi, bi) ideal of soft semigroup and make a lattice theoretic study of (sub) collections of them.

Definitions and Statements 3.1. (a) A soft set S over a semigroup U is said to be a *soft semigroup over* U iff $\sigma_S\ s$ is a subsemigroup of U for all s∈S. Consequently, for us (i) The empty soft set **Φ** over U is *trivially* a soft semigroup over U because there is *no* s∈ ∅ such that $\sigma_\emptyset\ s$ is *not* a subsemigroup of U and it is called the *empty soft semigroup* over U. Clearly, it is unique. (ii) The null soft set $\mathbf{\Phi}_S$ over U is also *trivially* a soft semigroup over U because $\sigma_S\ s = \emptyset$ is *trivially* a subsemigroup of U for all s∈S and it is called the *null soft semigroup* over U. (iii) A soft semigroup S over U is said to be a *whole soft semigroup* iff $\sigma_S\ s$ = U for all s∈S.

Generalizing the definitions of soft substructures[3], in what follows we introduce the notions of soft product of soft subsets, soft subsemigroup (left, right, quasi, bi) ideal of a soft semigroup, generalizing the corresponding notions for soft semigroup.

(b) For any pair of soft subsets A, B of a soft semigroup S over a semigroup U, the *product* of A by B, denoted by AB, is defined by C where (i) C = A∩ B (ii) $\sigma_C\ s = \sigma_A\ s\, \sigma_B\ s$ for all s∈C.
(c) For any soft semigroup S over a semigroup U and for any soft subset A of S:
(i) A is a *soft subsemigroup* of S iff AA⊆ A
(ii) A is a *soft left ideal* of S iff S A⊆ A
(iii) A is a *soft right ideal* of S iff AS ⊆A
(iv) A is a *soft ideal* of S iff S A∪AS⊆ A

(v) A is a *soft quasi ideal* of S iff S A ∩AS⊆ A
(vi) A is a *soft bi ideal* of S iff, AA ⊆ A and ASA ⊆A.
(d) For any soft subsets A, B, C, $(A_i)_{i\in I}$ of a soft semigroup S, the following are true:
(i) A ⊆ B implies C A⊆ C B, AC⊆ BC, AC ∩ CA ⊆B C∩ CB, ACA ⊆BCB
(ii) A $(\cap_{i\in I} A_i) \subseteq \cap_{i\in I}$ A A_i and A $(\cup_{i\in I} A_i) = \cup_{i\in I}$ A A_i .
(e) For any soft subset A of a soft semigroup S over U, A is a *soft subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal)* of S iff $\sigma_A a$ is a subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) of $\sigma_S a$ for all a ∈ A. Notice that the empty soft subset Φ and a null soft subset Φ_A of E are trivially soft subsemigroups (left ideals, right ideals, ideals, quasi ideals, bi ideals) of S.
Whenever ∗ = s (l, r, i, q, b), the set of *all soft subsemigroups (left ideals, right ideals, ideals, quasi ideals, bi ideals)* of S is denoted by S_s(S) (S_*(S)).
(f) For any soft semigroup S over U and for any soft subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) A of S, A is a *regular soft subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal)* of S iff $\sigma_A a \neq \emptyset$ for all a ∈ A. Notice that the empty soft set Φ is trivially a regular soft subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) of S.
Whenever ∗ = s (l, r, i, q, b), the set of all regular soft subsemigroups (left ideals, right ideals, ideals, quasi ideals, bi ideals) of S is denoted by S_s^r(S) (S_*^r(S)).
(g) For any soft subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi deal) A of a soft semigroup S over U, the *support* of A, denoted by Supp(A), is defined by Supp(A) = {a ∈ A / $\sigma_A a \neq \emptyset$}.
In view of (f), (g), (h), (i) and (j) of Section 2:
(h) For any soft semigroup S over U, the following are true:
(1) The empty soft subset **Φ** and a null soft subset $\mathbf{\Phi}_A$ of S are *trivially* soft subsemigroups (left ideals, right ideals, ideals, quasi ideals, bi ideals) of S
(2) Every soft (left, right, quasi) ideal of S is always a soft subsemigroup of S but *not* conversely
(3) Every soft (left, right) ideal of S is always a soft quasi ideal of S but *not* conversely
(4) Every soft (left, right, quasi) ideal of S is always a soft bi ideal of S but *not* conversely.
Notice that the empty soft subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) **Φ** is trivially the regular soft subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) of S because there is no a∈ ∅ such that $\sigma_A a = \emptyset$.

Algebra of Soft Substructures of a Soft Semigroup In this section first we show that arbitrary intersection of soft substructures of a given type is a soft substructure again of the same type and we use it to introduce the external descriptions of substructures generated by a soft subset of a soft subsemigroup and in the next section we give an internal description of various soft substructures generated by a soft subset of the soft subsemigroup.

Lemma 3.2. (a) For any soft semigroup S over U, the following are true:
(i) For any pair of soft subsemigroups (left ideals, right ideals, ideals, quasi ideals, bi ideals) A, B of S, A is a soft subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) of B iff A is a soft subset of B
(ii) Arbitrary union of soft (left, right) ideals is a soft (left, right) ideal of S
(iii) Arbitrary intersection of soft subsemigroups (left ideals, right ideals, ideals, quasi ideals, bi ideals) is a soft subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) of S
(iv) The intersection of all soft subsemigroups (left ideals, right ideals, ideals, quasi ideals, bi ideals) containing a given soft subset is a soft subsemigroup (left ideal, right ideal, ideal,

quasi ideal, bi ideal) which is unique and smallest with respect to the containment of the given soft subset.
(b) For any soft subset A of a soft semigroup S over U, the unique smallest soft subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) containing A defined as in (iii) of (a) is called the *soft subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) generated by* A and is denoted by $(A)_{s,s}$ ($(A)_{*,s}$ for * = *l, r, i, q, b respectively*).

Proof: It is straight forward.

Lemma 3.3. For any soft subset A of a soft semigroup S over a semigroup U, $(A)_{*,s}$ = B, where B = A and $\sigma_B s = (\sigma_A s)_{*,\sigma_S s}$ (cf.2(m)) for all s∈B, for * = s, l, r, i, q, b.

Proof: It follows from Lemma 3.2(a)(iii) and (b).

Lemma 3.4. For any soft semigroup S over U and for any pair of soft subsets A, B of S such that A ⊆ B, the following are true:
(a) If B is a soft subsemigroup of S then A is a soft subsemigroup of B iff A is a soft subsemigroup of S but *not* conversely
(b) If B is a soft (left, right, quasi , bi) ideal of S then A is a soft (left, right, quasi , bi) ideal of S implies A is a soft (left, right, quasi, bi) ideal of B but *not* conversely.

Proof: It is straight forward.

The Examples to show that the converses of the above Lemma are not true, follow from that of Examples 2.1, 2.2, 2.3 and 2.4 of 2(u).

Lemma 3.5. For any soft semigroup S over U, the following are true:
(a) The intersection of a soft left ideal and a soft right ideal in S is a soft quasi ideal
(b) For any soft subsemigroup A of S and for any soft (left, right, quasi, bi) ideal B of S, A∩ B is a soft (left, right, quasi, bi) ideal of A.

Proof: It is straight forward.

Notation: Whenever ∗ = s (l, r, i, q, b), for any soft semigroup S over U and for any pair of soft subsemigroups (left ideals, right ideals, ideals, quasi ideals, bi ideals) A, B of S, A $\le_s$ B (A $\le_*$ B) iff A is a soft subsemigroup (left ideal, right ideal, ideal, quasi ideal, bi ideal) of B.

Theorem 3.6. For any soft semigroup S over U, whenever ∗ = s, q, b, l, r, i, the set $\mathcal{S}_*$(S) is a complete lattice with
(1) the partial ordering defined by: for any A, B ∈ $\mathcal{S}_*$(S), A ≤ B iff A $\le_*$ B
(2) the largest and the least elements in $\mathcal{S}_*$(S) are S and Φ respectively
(3) for any family $(A_i)_{i\in I}$ in $\mathcal{S}_*$(S), $\wedge_{i\in I} A_i = \cap_{i\in I} A_i$
(4) for any family $(A_i)_{i\in I}$ in $\mathcal{S}_*$(S), however,
(i) for ∗ = s, q, b, $\vee_{i\in I} A_i = \overline{\vee}_{i\in I} A_i$, where $\overline{\vee}$ is the meet induced join in $\mathcal{S}_*$(S) and $\overline{\vee}_{i\in I} A_i$ = A, where A = $\cup_{i\in I} A_i$ and $\sigma_A s = (\cup_{i\in I_s} \sigma_{A_i} s)_{*,\sigma_S s}$ for all s ∈ A, where I_s = {i ∈ I / s ∈ A_i} and
(ii) for ∗ = l, r, i, $\vee_{i\in I} A_i = \cup_{i\in I} A_i$.

Proof: It is straight forward.

Theorem 3.7. For any soft semigroup S over U, whenever ∗ = s, q, b, the set $\mathcal{S}_*^r$(S) is a join complete subposet of the complete lattice $\mathcal{S}_*$(S) and also itself a complete lattice with

(1) the induced partial ordering defined from the super poset S_*(S)
(2) the largest and the least elements in S_*^r(S) are L, where L = Supp(S) and $\sigma_L s = \sigma_S s$ for all s ∈ L, and Φ respectively
(3) for any family $(A_i)_{i\in I}$ in S_*^r(S), $\wedge_{i\in I} A_i = \sqcap_{i\in I} A_i$, where $\sqcap_{i\in I} A_i$ = A, where A = Supp($\cap_{i\in I} A_i$) and $\sigma_A s = \cap_{i\in I} \sigma_{A_i} s$ for all s ∈ A
(4) for any family $(A_i)_{i\in I}$ in S_*^r(S), $\vee_{i\in I} A_i = \bar{\sqcup}_{i\in I} A_i = \bar{\vee}_{i\in I} A_i$, where $\bar{\sqcup}$ and $\bar{\vee}$ are ⊓ and ∧ induced joins in S_*^r(S) and S_*(S) respectively.

Proof: It is straightforward.

Theorem 3.8. For any soft semigroup S over U, whenever ∗ = l, r, i, the set S_*^r(S) is a join complete subposet of the complete lattice S_*(S) with
(1) the induced partial ordering defined from the super poset S_*(S)
(2) the largest and the least elements in S_*^r(S) are L (cf.3.7(2)) and Φ respectively
(3) for any family $(A_i)_{i\in I}$ in S_*^r(S), $\vee_{i\in I} A_i = \cup_{i\in I} A_i$
(4) Further, S_*^r(S) is a complete lattice with the join induced meet $\bar{\wedge}$ given by for any family $(A_i)_{i\in I}$ in S_*^r(S), $\bar{\wedge}_{i\in I} A_i$ = A, where A = Supp($\cap_{i\in I} A_i$) and $\sigma_A s = \cap_{i\in I} \sigma_{A_i} s$ for all s ∈A.

Proof: It is straight forward.

Remark: Whenever ∗ = s, q, b, the set S_*^r(S) is not a meet complete subposet of S_*(S). However, with the join induce by the meet, ⊓ in S_*^r(S), S_*^r(S) is a join complete subposet of S_*(S) or equivalently the join induced by ⊓ in S_*^r(S), and the restriction of the join induced by intersection in S_*(S) onto S_*^r(S) are the same.

Lemma 3.9. For any soft semigroup S over U, whenever ∗ = s, q, b, for any family $(A_i)_{i\in I}$ in S_*^r(S), $\bar{\bar{\wedge}}_{i\in I} A_i$ = A, where A = Supp($\cap_{i\in I} A_i$) and $\sigma_A s = \cap_{i\in I} \sigma_{A_i} s$ for all s ∈A.

Proof: It is straight forward.

Corollary 3.10. For any soft semigroup S over U, whenever ∗ = s, q, b, for any family $(A_i)_{i\in I}$ in S_*^r(S), $\sqcap_{i\in I} A_i = \bar{\bar{\wedge}}_{i\in I} A_i$, where $\bar{\bar{\wedge}}$ is the meet induced by $\bar{\vee}$ in S_*^r(S), which is the $\bar{\vee}$ restricted from S_*(S) to S_*^r(S).

Proof: It follows from 3.7(3) and 3.9.

Remark: Whenever ∗ = s, q, b, the set S_*(S) is a meet complete poset which induces $\bar{\vee}$. The restriction of $\bar{\vee}$ onto S_*^r(S) makes S_*^r(S) a $\bar{\vee}$-complete sub-poset of S_*(S). $\bar{\vee}$ on S_*^r(S) induces $\bar{\bar{\wedge}}$ on S_*^r(S). By the above Corollary, $\bar{\bar{\wedge}}$ is equal to ⊓. The join $\bar{\sqcup}$ induced by ⊓ is the restriction of $\bar{\vee}$ onto S_*^r(S). In other words, the join induced by $\bar{\bar{\wedge}}$ is the restriction of $\bar{\vee}$ onto S_*^r(S).

Internal description of soft substructures In what follows we give an internal description of a soft substructure generated by a soft subset of a soft semigroup.

Theorem 3.9. For any soft semigroup S and for any A ⊆ S, the following are true:
(i) $(A)_{s,S} = \cup_{n\geq 0} A_n$, where $A_0 = A \cup AA$, $A_1 = A_0 \cup A_0 A_0$ and so on $A_n = A_{n-1} \cup A_{n-1} A_{n-1}$.
(ii) $(A)_{l,S} = \cup_{n\geq 0} A_n$, where $A_0 = A \cup S A$, $A_1 = A_0 \cup S A_0$ and so on $A_n = A_{n-1} \cup S A_{n-1}$.
(iii) $(A)_{r,S} = \cup_{n\geq 0} A_n$, where $A_0 = A \cup A S$, $A_1 = A_0 \cup A_0 S$ and so on $A_n = A_{n-1} \cup A_{n-1} S$.
(iv) $(A)_{i,S} = \cup_{n\geq 0} A_n$, where $A_0 = A \cup S A \cup A S$, $A_1 = A_0 \cup S A_0 \cup A_0 S$ and so on $A_n = A_{n-1} \cup S A_{n-1} \cup A_{n-1} S$.

(v) $(A)_{q,S} = \cup_{n\geq 0} A_n$, where $A_0 = A\cup(SA\cap AS)$, $A_1 = A_0\cup(SA_0\cap A_0S)$ and so on $A_n = A_{n-1}\cup(SA_{n-1}\cap A_{n-1}S)$.
(vi) $(A)_{b,S} = \cup_{n\geq 0} A_n$, where $A_0 = A\cup ASA$, $A_1 = A_0\cup A_0SA_0$ and so on $A_n = A_{n-1}\cup A_{n-1}SA_{n-1}$.
Proof: It follows from Lemma 3.2 and 2(v).

ACKNOWLEDGMENTS

Acknowledgments: The second and third authors express their heart full thanks to the first author for his constant encouragement and help throughout the preparation of this paper.

REFERENCES

[1] Aktas, H., Cagman, N., 2000 "Soft sets and soft groups", Inf. Sci. 177, 2726-2735.
[2] Ali, M.I., Shabir, M., 2010"Comments on De Morgan's law in fuzzy soft sets", J. Fuzz Math. 18, 679-686.
[3] Ali, M.I., Shabir, M., and Shum, K.P., 2010 "On Soft Ideals over Semigroups", Southeast Asian Bulletin of Mathematics 34,595610.
[4] Feng, F., Ali, M.I. and Shabir, M., 2013 "Soft relations applied to semigroups", Published by Faculty of Sciences and Mathematics,
[5] University of Ni's, Serbia Filomat 27,1183,1196.
[6] Feng, F., Jun, Y.B. and Zhao, X., 2008 "Soft semirings", Comput. Math. Appl. 56(10), 2621-2628.
[7] Grillet, P.A., Semigroups "An Introduction to the Structure Theory", Tulane University New Orleans, Louisiana
[8] Jayalakshmi, D. and Subbiah, G., 2019 "Soft T-ideals of Soft BCI-Algebras" International Journal of Mathematics Trends and
[9] Technology, 65(1), 31-39.
[10] Maji, P.K., Biswas, R. and Roy, A.R., 2003 "Soft set theory", Comput. Math. Appl. 45(4-5), 555-562.
[11] Molodtsov, D. "Soft set theory first results", Comput. Math. Appl. 37(4-5), 19-31, 1999.
[12] Murthy, N.V.E.S. and Maheswari, Ch., 2017 "A First Study of f-(Fuzzy) Soft τ –Algebras and their f-(Fuzzy) Soft ω-Subalgebras",
[13] Global Journal of Pure and Applied Mathematics 13(6), 2503-2526.
[14] Pei, D. and Miao, D., 2005 "From soft sets to information systems", Proceedings of Granular Computing, IEEE International
[15] Conference 2, 617- 621.
[16] Szasz, G., 1963 "An Introduction to Lattice Theory", Academic Press, New York.
[17] Zadeh, L.A., 1965 "Fuzzy sets", Information and Control, Vol.8, pp.338-353.