Making an approximations for n^{th} index using Prime number Theorem

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Abstract: *Using of Gauss's Prime number theorem for finding number of primes can also tell us which primes are located for a desired index. But it will give a rough approximation because prime number theorem by Gauss is a rough approximation for number of primes.*

Keywords: Prime number theorem

I. INTRODUCTION

Let us first assume two lines passing through origin with slope m_1 and m_2 and let's assume that these lines makes angle θ and δ with positive direction of X- axis. Now let's make just one more assumption that these two lines intersect curve($y = \frac{x}{\log x}$) at (x_1, y_1) and (x_2, y_2).

Thus from above assumptions we have following equations,

$y = m_1 x$

$y = m_2 x$

Now above lines are intersecting curve at (x_1, y_1) and (x_2, y_2), thus by this we can conclude that these two points are on both lines. Therefore,

$y_1 = m_1 x$ and $y_2 = m_2 x$,

also points are on curve too, therefore we can also conclude that,

$y_1 = x_1/\log x_1$ and $y_2 = x_2/\log x_2$,

Now our one more assumption was that, these lines are making angle θ and δ.

Thus,

$m_1 = \tan\theta$ and $m_2 = \tan\delta$

II. DERIVATION

$x_1 = e^{\cot\theta}$

$y_1 = e^{\cot\theta}.\tan\theta$

$x_2 = e^{\cot\delta}$

$y_2 = e^{\cot\delta}.\tan\delta$

$y_1 / x_1 = \tan\theta$

Now we are finding the line more specifically a secant joining (x_1,y_1) and (x_2, y_2).
So slope of that secant is given as,

$$m = (y_2 - y_1)/(x_1 - x_2) \quad \ldots(\text{Considering } x_1 > x_2)$$

Therefore,

$$m = (\tan\delta\,.\,e^{\cot\delta} - \tan\theta\,.\,e^{\cot\theta})\,/\,(e^{\cot\theta} - e^{\cot\delta})$$

Now let's find equation of that line,

$$\frac{y - y_1}{x - x_1} = m$$

Using above equation we have,

$$y = [x.\left(\tan\delta\,.\,e^{\cot\delta} - \tan\theta\,.\,e^{\cot\theta}\right) - e^{\cot\delta}.\,x_1(\tan\delta + \tan\theta) + 2.\,e^{\cot\theta}.\,x_1.\tan\theta]\ /\ (e^{\cot\theta} - e^{\cot\delta})$$

Now let's consider above equation as a function, on integrating above equation with respect to x,

$$f(x) = [\tfrac{x^2}{2}.\left(\tan\delta\,.\,e^{\cot\delta} - \tan\theta\,.\,e^{\cot\theta}\right) - e^{\cot\delta}.\,x.\,e^{\cot\theta}(\tan\delta + \tan\theta) + 2.\,e^{\cot\theta}.\,x.\,e^{\cot\theta}.\tan\theta]\ /\ (e^{\cot\theta} - e^{\cot\delta})$$

Now, if we consider only first part which is,

$$\frac{x^2}{2}.\left(\tan\delta\,.\,e^{\cot\delta} - \tan\theta\,.\,e^{\cot\theta}\right)/(e^{\cot\theta} - e^{\cot\delta})$$

It provides an approximate prime number.

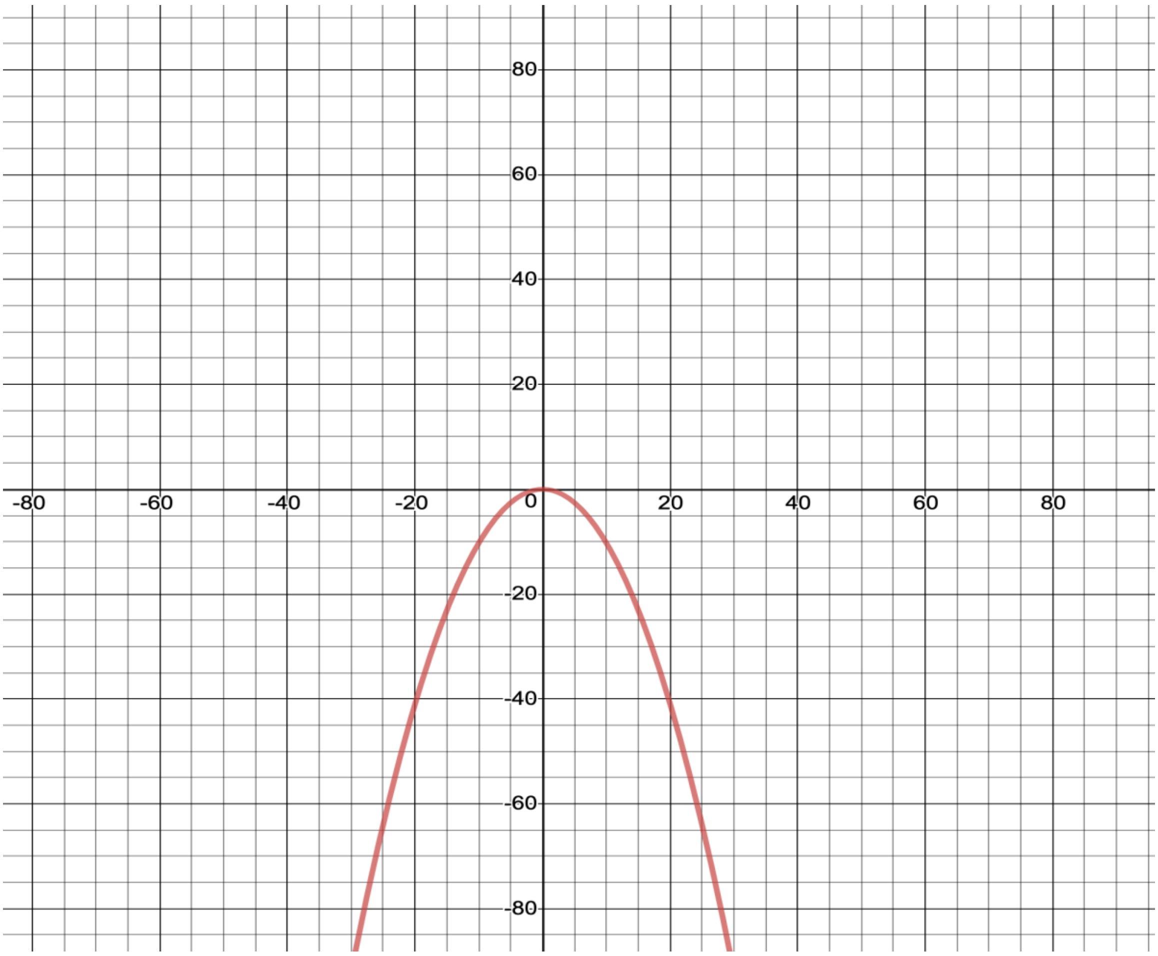
Now here note that although 0, 1 are not considered to be primes but for counting they are counted as primes and so counting starts as follows,

0, 1, 2, 3, 5, 7, 11, …

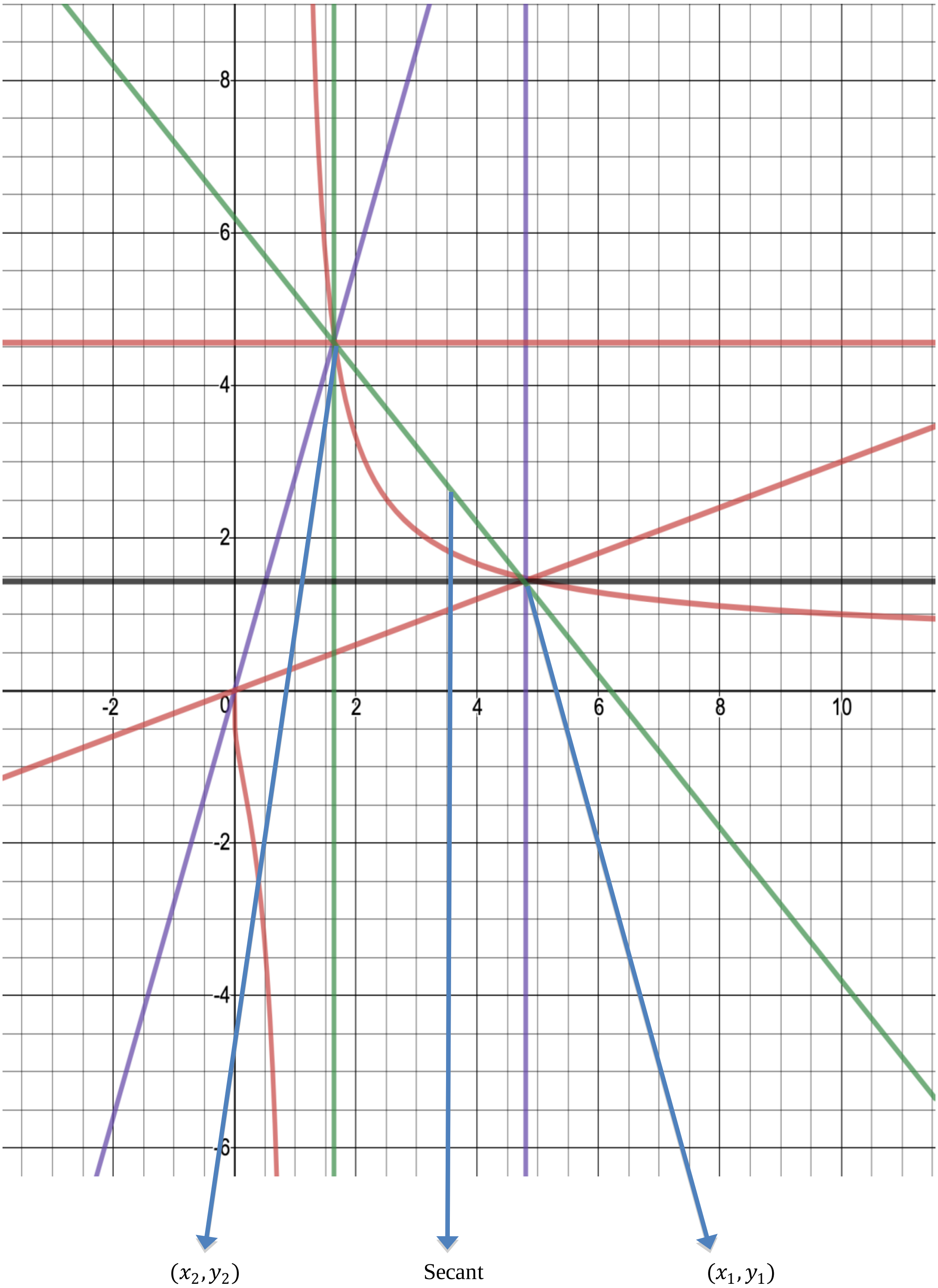
Now suppose you want the eleventh prime just replace x by 11 in above equation and take value of $\theta = 60^\circ\ and\ \delta = 45^\circ$. Now, when function get's break at some point where it stops giving prime values just increase value of θ by 1 degree or 2 degree.

III. VISUAL REPRESENTATION

$\theta = 60^\circ\ and\ \delta = 45^\circ$



IV. CONCEPT VISUAL REPRESENTATION



V. USES

→You can determine which prime is at which index just by one equation.
→Many further improvements can be done in methods like RSA Encryption, …
→Also further development at atomic scale can be done by this theory.

VI. CONCLUSION

Using my method which has been inspired from Gauss, now you can have value with relative error of less than 0.01% for n^{th} prime number.

VII. REFERENCES

[1] Prime number theorem(https://faculty.math.illinois.edu/~r-ash/CV/CV7.pdf)
[2] Newman's short proof(https://people.mpim-bonn.mpg.de/zagier/files/doi/10.2307/2975232/fulltext.pdf)