A Note on Open Support of a Graph under Addition I

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Abstract

In this paper we defined an open support of a vertex v under addition and open support of a graph G under addition. We calculate the open support for some standard graphs.

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I. INTRODUCTION

In this work we consider finite, undirected, simple graphs $G = (V, E)$ with n vertices and m edges. The *n*eighbourhood of a vertex $v \in V(G)$ is the set $N_G(v)$ of all the vertices adjacent to v in G. For a set $X \subseteq V(G)$, the *open neighbourhood*$N_G(X)$ is defined to be $\cup_{v\in X} N_G(v)$ and the closed neighbourhood $N_G[X] = N_G(X) \cup X$. The *degree* of a vertex $v \in V(G)$ is the number of edges of G incident with v and is denoted by $deg_G(v)$ or $deg(v)$. The maximum and the minimum degrees of the vertices of G are respectively denoted by $\Delta(G)$ and $\delta(G)$. A vertex of a degree 0 in G is called an *isolated* vertex and a vertex of degree 1 is called a *pendent* vertex or an *end* vertex of G. A vertex of a graph G is said to be a vertex of full degree if it is adjacent to all other vertices in G. A graph G is said to be *regular of degree* r if every vertex of G has degree r. Such graphs are called *r-regular* graphs.

A **open support of a vertex**, v under addition is defined by $\sum_{u\in N(v)} deg(u)$ and it is denoted by $supp(v)$. A **open support of a graph**, G under addition is defined by $\sum_{v\in V(G)} supp(v)$ and it is denoted by $supp(G)$.

II. DEFINITIONS AND EXAMPLE

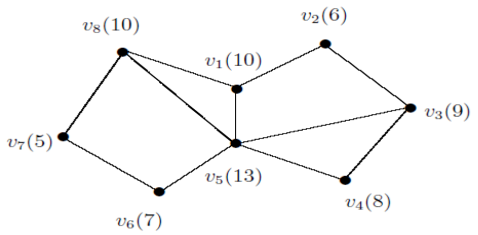
Definition 2.1. *Let G=(V,E) be a graph. A* **open support of a vertex***, v under addition is defined by*$\sum_{u\in N(v)} deg(u)$ *and it is denoted by* $supp(v)$.

Definition 2.2. *Let G=(V,E) be a graph. A* **open support of a graph***, G under addition is defined by* $\sum_{v\in V(G)} supp(v)$ *and it is denoted by* $supp(G)$.

Example 2.3

In the graph $G, deg(v_1) = 3; deg(v_2) = 2; deg(v_3) = 3; deg(v_4) = 2; deg(v_5) = 5; deg(v_6) = 2; deg(v_7) = 2; deg(v_8) = 3;$

Also, $supp(v_1) = 10; supp(v_2) = 6; supp(v_3) = 9; supp(v_4) = 8; supp(v_5) = 13; supp(v_6) = 7; supp(v_7) = 5; supp(v_8) = 10$;. Therefore, $supp(G) = 68$



III. RESULTS

Theorem 3.1. *Let $G = P_n\,(n > 1)$. Then $supp(P_n) = 4n - 6$.*
Proof: Let $G = P_n$ be a path with n vertices. Let $V(G) = \{v_1, v_2, \dots, v_n\}$ and $deg(v_1) = deg(v_n) = 1$; $deg(v_i) = 2$ for all $i = 2,3, \dots, n-1$. Then

$$supp(v_1) = \sum_{v\in N(v_1)} deg(v)$$
$$= deg(v_2)$$
$$supp(v_1) = 2$$

Similarly,

$$supp(v_n) = 2.$$

$$supp(v_2) = \sum_{v\in N(v_2)} deg(v)$$
$$= deg(v_1) + deg(v_3)$$
$$supp(v_2) = 3$$

Similarly,

$$supp(v_{n-1}) = 3.$$

For each $i = 3,4, \dots, n-2$,

$$supp(v_i) = \sum_{v\in N(v_i)} deg(v)$$
$$= deg(v_{i-1}) + deg(v_{i+1})$$
$$supp(v_i) = 4.$$

Now,

$$supp(G) = \sum_{v\in V(G)} supp(v)$$
$$= supp(v_1) + supp(v_2) + \sum_{i=3}^{n-2} supp(v_i) + supp(v_{n-1}) + supp(v_n)$$
$$= 2 + 3 + \sum_{i=3}^{n-2} (4) + 3 + 2$$
$$= 10 + 4(n-4)$$
$$supp(G) = 4n - 6.$$

Theorem 3.2 *Let $G = C_n$. Then $supp(C_n) = 4n$.*
Proof: Let $G = C_n$ be a cycle of order n. Let $V(G) = \{v_1, v_2, \dots, v_n\}$ and $deg(v_i) = 2$ for all $i = 1,2, \dots, n$. Then

$$supp(v_i) = \sum_{v\in N(v_i)} deg(v)$$
$$supp(v_i) = 4.$$

Now,

$$supp(G) = \sum_{v\in V(G)} supp(v)$$
$$= 4 + 4 + \cdots + 4 \quad (ntimes)$$
$$supp(G) = 4n.$$

Theorem 3.3 *Let $G = K_n$. Then $supp(G) = n(n-1)^2$.*
Proof: Let $G = K_n$ be a complete graph with n vertices. Let $V(G) = \{v_1, v_2, \dots, v_n\}$ and $deg(v_i) = n - 1$ for all $i = 1,2, \dots, n$. Then

$$supp(v_i) = \sum_{v\in N(v_i)} deg(v)$$
$$= \sum_{v\in N(v_i)} (n-1)$$
$$= (n-1)(n-1)$$
$$supp(v_i) = (n-1)^2$$

Now,

$$supp(G) = \sum_{v\in V(G)} supp(v)$$
$$= \sum_{v\in V(G)} (n-1)^2$$
$$supp(G) = n(n-1)^2$$

Theorem 3.4. *Let $G = K_{m,n}$. Then $supp(G) = mn(m+n)$.*
Proof: Let $G = K_{m,n}$ be a complete bipartite graph with the bipartition (X, Y) where $X = \{x_1, x_2, \dots, x_m\}$ and $Y = \{y_1, y_2, \dots, y_m\}$. Then $deg(x_i) = n$ for all $i = 1,2, \dots, m$ and $deg(y_j) = m$ for all $j = 1,2, \dots, n$.

$$supp(x_i) = \sum_{v\in N(x_i)} deg(v)$$
$$= \sum_{v\in Y} deg(v)$$
$$= \sum_{v\in Y} m$$
$$supp(x_i) = mn, \text{for all} i = 1,2, \dots, m$$

$supp(y_j) = \sum_{v\in N(y_j)} deg(v)$
$= \sum_{v\in X} deg(v)$
$= \sum_{v\in X} n$
$supp(y_j) = mn$, for all $j = 1,2,\dots,n$.

Now,

$supp(G) = \sum_{v\in V(G)} supp(v)$
$= \sum_{x\in X} supp(x) + \sum_{y\in Y} supp(y)$
$= \sum_{x\in X} mn + \sum_{y\in Y} mn$
$supp(G) = mn(m+n)$

Corollary 3.5. *Let $G = K_{1,n}$. Then $supp(G) = n(n+1)$.*
Proof: Let $G = K_{1,n}$ be a star graph. Put $m = 1$ in theorem 3.4, we get $supp(G) = n(n+1)$.

Theorem 3.6 *Let $G = W_n (n \geq 4)$. Then $supp(W_n) = (n-1)(n+8)$.*
Proof: Let $G = W_n$ be a wheel of order n. Let $V(G) = \{v_0, v_1, v_2, \dots, v_{n-1}\}$ and $deg(v_0) = n-1$; $deg(v_i) = 3$ for all $i = 1,2,\dots,n-1$. Then

$supp(v_0) = \sum_{v\in N(v_0)} deg(v)$
$= 3 + 3 + \cdots + 3 \ (n-1 \quad times)$
$= 3(n-1)$

For each $i = 1,2,\dots,n-1$,

$supp(v_i) = \sum_{v\in N(v_i)} deg(v)$
$= 3 + 3 + (n-1)$
$supp(v_i) = 5 + n$.

Now,

$supp(G) = \sum_{v\in V(G)} supp(v)$
$= supp(v_0) + \sum_{i=1}^{n-1} supp(v_i)$
$= 3(n-1) + (n-1)(5+n)$
$supp(G) = (n-1)(n+8)$.

Theorem 3.7 *Let $G = F_n (n \geq 4)$. Then $supp(F_n) = (n-2)(n+9)$.*
Proof: Let $G = F_n$ be a fan of order n. Let $V(G) = \{v_0, v_1, v_2, \dots, v_{n-1}\}$ and $deg(v_0) = n-1$; $deg(v_1) = deg(v_{n-1}) = 2$; $deg(v_i) = 3$ for all $i = 2,3,\dots,n-2$. Then

$supp(v_0) = \sum_{v\in N(v_0)} deg(v)$
$= degv_1 + degv_{n-1} + \sum_{i=2}^{n-2} supp(v_i)$
$= 2 + 2 + 3(n-3)$
$= 3n - 5$

$supp(v_1) = \sum_{v\in N(v_1)} deg(v)$
$= degv_0 + degv_2$
$= n - 1 + 3$
$= n + 2$

$supp(v_{n-1}) = n + 2$

$supp(v_2) = \sum_{v\in N(v_2)} deg(v)$
$= degv_0 + degv_1 + degv_3$
$= n - 1 + 2 + 3$
$= n + 4$

$supp(v_{n-2}) = n + 4$.

For each $i = 3,4,\dots,n-3$,

$supp(v_i) = \sum_{v\in N(v_i)} deg(v)$
$= degv_{i-1} + degv_{i+1} + degv_0$
$= 3 + 3 + (n-1)$
$supp(v_i) = n + 5$.

Now,

$supp(G) = \sum_{v\in V(G)} supp(v)$
$= supp(v_0) + supp(v_1) + supp(v_{n-1}) + supp(v_{n-2}) + \sum_{i=3}^{n-3} supp(v_i)$
$= 3n - 5 + 2(n+2) + 2(n+4) + (n-5)(n+5)$
$supp(G) = (n-2)(n+9)$.

Theorem 3.8 *Let* $G = D_{r,s}(2 \le r \le s)$. *Then* $supp(G) = r(r+1) + s(s+1) - 2$.

Proof: Let $G = D_{r,s}(2 \le r \le s)$ be a double star with $r+s$ vertices. Let $V(G) = \{x, y, v_1, v_2, \dots, v_{r-1}, u_1, u_2, \dots, u_{s-1}\}$ such that $xv_i \in E(G)$ and $yu_j \in E(G)$ for all $i = 1,2,\dots,r-1$ and $j = 1,2,\dots,s-1$. Then $deg(x) = r$; $deg(y) = s$ and $deg(v_i) = deg(u_i) = 1$ for all i, j.

$$supp(x) = \sum_{v \in N(x)} deg(v)$$
$$= deg(y) + \sum_{i=1}^{r-1} supp(v_i)$$
$$supp(x) = s + r - 1$$

Similarly,

$$supp(y) = s + r - 1.$$

and

$$supp(v_i) = \sum_{v \in N(v_i)} deg(v)$$
$$= deg(x)$$
$$supp(v_i) = r, \text{for all } i = 1,2,\dots,r-1$$

Similarly,

$$supp(u_j) = s, \text{for all } j = 1,2,\dots,s-1$$

Now,

$$supp(G) = \sum_{v \in V(G)} supp(v)$$
$$= supp(x) + supp(y) + \sum_{i=1}^{r-1} supp(v_i) + \sum_{j=1}^{s-1} supp(u_j)$$
$$= r + s - 1 + r + s - 1 + (r-1)r + (s-1)s$$
$$supp(G) = r(r+1) + s(s+1) - 2.$$

Theorem 3.9 *Let* $G = C_n^+(n \ge 3)$. *Then* $supp(G) = 10n$.

Proof: Let $G = C_n^+(n \ge 3)$ be a corona with $2n$ vertices. Let $V(G) = \{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$ such that $deg(v_i) = 3$ and $deg(u_i) = 1$ for all $i = 1,2,\dots,n$.

$$supp(u_i) = \sum_{v \in N(u_i)} deg(v)$$
$$supp(u_i) = 3$$

$$supp(v_i) = \sum_{v \in N(v_i)} deg(v)$$
$$supp(v_i) = 7$$

Now,

$$supp(G) = \sum_{v \in V(G)} supp(v)$$
$$= \sum_{i=1}^{n} supp(v_i) + \sum_{i=1}^{n} supp(u_i)$$
$$= 3n + 7n$$
$$supp(G) = 10n.$$

Theorem 3.10 *Let* $G = K_n^+$. *Then* $supp(G) = n(n^2+1)$.

Proof: Let $G = K_n^+$ be a corona of order $2n$. Let $V(G) = \{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$ such that $deg(v_i) = n$and $deg(u_i) = 1$ for all $i = 1,2,\dots,n$.

$$supp(u_i) = \sum_{v \in N(u_i)} deg(v)$$
$$supp(u_i) = n$$

$$supp(v_i) = \sum_{v \in N(v_i)} deg(v)$$
$$= n(n-1) + 1$$
$$supp(v_i) = n^2 - n + 1$$

Now,

$$supp(G) = \sum_{v \in V(G)} supp(v)$$
$$= \sum_{i=1}^{n} supp(v_i) + \sum_{i=1}^{n} supp(u_i)$$
$$= \sum_{i=1}^{n} n^2 - n + 1 + n$$
$$= \sum_{i=1}^{n} n^2 + 1$$
$$supp(G) = n(n^2+1).$$

Theorem 3.11 *Let* $G = K_m(a_1, a_2, \dots, a_m)$. *Then* $supp(G) = n(n^2+1)$.

Proof: Let $G = K_m(a_1, a_2, \dots, a_m)$ be a multistar graph of order $m + a_1 + a_2 + \cdots + a_m$. Let $V(G) = \{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$ such that $deg(v_i) = n$and $deg(u_i) = 1$ for all $i = 1,2,\dots,n$.

$$supp(u_i) = \sum_{v \in N(u_i)} deg(v)$$
$$supp(u_i) = n$$

$$supp(v_i) = \sum_{v \in N(v_i)} deg(v)$$
$$= n(n-1) + 1$$
$$supp(v_i) = n^2 - n + 1$$

Now,

$$supp(G) = \sum_{v \in V(G)} supp(v)$$
$$= \sum_{i=1}^{n} supp(v_i) + \sum_{i=1}^{n} supp(u_i)$$
$$= \sum_{i=1}^{n} n^2 - n + 1 + n$$
$$= \sum_{i=1}^{n} n^2 + 1$$
$$supp(G) = n(n^2 + 1).$$

REFERENCES

[1] M. Anitha, S. Balamurugan: Strong (Weak) Efficient Open Domination on Product Graphs, International Journal of Pure and Applied Mathematics, (J.No. 23425), IISN 1311 - 8080, communicated.

[2] R. Balakrishnan, K. Ranganathan, *A Textbook of Graph Theory*, Springer,2011.

[3] S. Balamurugan, *A study on chromatic strong domination in graphs*, Ph.D Thesis, Madurai Kamaraj University, India 2008.

[4] J.A. Bondy, U.S.R. Murthy, *Graph theory with Applications*, North-Holland, 1982.

[5] F. Harary : *Graph Theory*, Addsion Wesley, Reading, Mass, (1972).

[6] T. W. Haynes, S. T. Hedetniemi, P. J. Slater: *Fundamentals of Domination in Graphs*, Marcel Dekker, New York, (1998).

[7] C. Y. Ponnappan: *Studies in Graph Theory Support Strong Domination in Graphs*, Ph.D thesis, Madurai Kamaraj University (2008).