Closed Support of a Graph under Addition I

[1]S. Balamurugan , [2]M. Anitha , [3]C. Karnan
[1]PG Department of Mathematics,
Government Arts College, Melur - 625 106, Tamilnadu, India.
[2]Department of Mathematics
Syed Ammal Arts and Science College, Ramanathapuram, Tamilnadu, India..
[3]Department of Mathematics
The Optimus Public School, Erode - 638 455

Abstract

In this paper we defined closed support of a vertex v under addition and closed support of a graph G under addition. We calculate the closed support for some standard graphs.

AMS Subject Code: 05C07

Keywords: Vertex Degree, Closed Support of a Vertex, Closed Support of a Graph.

I. INTRODUCTION

In this work we consider finite, undirected, simple graphs $G = (V, E)$ with n vertices and m edges. The *n*eighbourhood of a vertex $v \in V(G)$ is the set $N_G(v)$ of all the vertices adjacent to v in G. For a set $X \subseteq V(G)$, the *open neighbourhood*$N_G(X)$ is defined to be $\cup_{v\in X} N_G(v)$ and the *c*losed neighbourhood $N_G[X] = N_G(X) \cup X$. The *degree* of a vertex $v \in V(G)$ is the number of edges of G incident with v and is denoted by $deg_G(v)$ or $deg(v)$. The maximum and the minimum degrees of the vertices of G are respectively denoted by $\Delta(G)$ and $\delta(G)$. A vertex of a degree 0 in G is called an *isolated* vertex and a vertex of degree 1 is called a *pendent* vertex or an *end* vertex of G. A vertex of a graph G is said to be a vertex of full degree if it is adjacent to all other vertices in G. A graph G is said to be *regular of degree* r if every vertex of G has degree r. Such graphs are called *r-regular* graphs.

A **closed support of a vertex**, v under addition is defined by $\sum_{u\in N[v]} deg(u)$ and it is denoted by $supp[v]$. A **closed support of a graph**, G under addition is defined by $\sum_{v\in V(G)} supp[v]$ and it is denoted by $supp[G]$.

II. DEFINITIONS

Definition 2.1.*Let G=(V,E) be a graph. A* **closed support of a vertex**, *v under addition is defined by* $\sum_{u\in N[v]} deg(u)$ *and it is denoted by* $supp[v]$.

Definition 2.2.*Let G=(V,E) be a graph. A* **closed support of a graph**, *G under addition is defined by* $\sum_{v\in V(G)} supp[v]$ *and it is denoted by* $supp[G]$.

III. RESULTS

Theorem 3.1.*Let* $G = P_n (n > 1)$. *Then* $supp[P_n] = 6n - 8$.

Proof. Let $G = P_n$ be a path with n vertices. Let $V(G) = \{v_1, v_2, \ldots, v_n\}$ with $v_i v_{i+1} \in E(G)$ for all $i = 1,2, \ldots, n-1$. Then $deg(v_1) = deg(v_n) = 1$; $deg(v_i) = 2$ for all $i = 2,3, \ldots, n-1$. Then

$$supp[v_1] = \sum_{v\in N[v_1]} deg(v)$$
$$= deg(v_1) + deg(v_2)$$
$$supp[v_1] = 1 + 2 = 3$$

Similarly,

$$supp[v_n] = 3.$$

$$supp[v_2] = \sum_{v\in N[v_2]} deg(v)$$
$$= deg(v_1) + deg(v_2) + deg(v_3)$$
$$supp[v_2] = 5$$

Similarly,

$$supp[v_{n-1}] = 5.$$

For each $i = 3,4, \ldots, n-2$,

$$supp[v_i] = \sum_{v\in N[v_i]} deg(v)$$
$$= deg(v_{i-1}) + deg(v_i) + deg(v_{i+1})$$
$$supp[v_i] = 6.$$

Now,

$$supp[G] = \sum_{v\in V[G]} supp[v]$$
$$= supp[v_1] + supp[v_2] + \sum_{i=3}^{n-2} supp[v_i] + supp[v_{n-1}] + supp[v_n]$$
$$= 3 + 5 + \sum_{i=3}^{n-2} (6) + 5 + 3$$
$$= 16 + 6(n-4)$$
$$supp[G] = 6n - 8.$$

Theorem 3.2.*Let $G = C_n$. Then $supp[C_n] = 6n$.*
Proof. Let $G = C_n$ be a cycle of order n. Let $V(G) = \{v_1, v_2, \dots, v_n\}$ and $deg(v_i) = 2$ for all $i = 1,2,\dots,n$. Then

$$supp[v_i] = \sum_{v\in N[v_i]} deg(v)$$
$$supp[v_i] = 6.$$

Now,

$$supp[G] = \sum_{v\in V(G)} supp[v]$$
$$= 6 + 6 + \cdots + 6 \quad (ntimes)$$
$$supp[G] = 6n.$$

Theorem 3.3. *Let $G = K_n$. Then $supp[G] = n^2(n-1)$.*
Proof. Let $G = K_n$ be a complete graph with n vertices. Let $V(G) = \{v_1, v_2, \dots, v_n\}$ and $deg(v_i) = n-1$ for all $i = 1,2,\dots,n$. Then

$$supp[v_i] = \sum_{v\in N[v_i]} deg(v)$$
$$= \sum_{v\in N[v_i]} (n-1)$$
$$= n(n-1)$$
$$supp[v_i] = (n-1)n$$

Now,

$$supp[G] = \sum_{v\in V(G)} supp(v)$$
$$= \sum_{v\in V(G)} (n-1)n^2$$
$$supp[G] = (n-1)n^2$$

Theorem 3.4.*Let $G = K_{m,n}$. Then $supp[G] = mn(m+n+2)$.*
Proof. Let $G = K_{m,n}$ be a complete bipartite graph with the bipartition (X, Y) where $X = \{x_1, x_2, \dots, x_m\}$ and $Y = \{y_1, y_2, \dots, y_m\}$. Then $deg(x_i) = n$ for all $i = 1,2,\dots,m$ and $deg(y_j) = m$ for all $j = 1,2,\dots,n$.

$$supp[x_i] = \sum_{v\in N[x_i]} deg(v)$$
$$= \sum_{v\in Y} deg(v) + deg(x_i)$$
$$= \sum_{v\in Y} mn + n$$
$$supp[x_i] = n(m+1), \ foralli = 1,2,\dots,m$$

$$supp[y_j] = \sum_{v\in N[y_j]} deg(v)$$
$$= \sum_{v\in X} deg(v) + deg(y_i)$$
$$= \sum_{v\in X} mn + m$$
$$supp[y_j]) = mn + m, \ forallj = 1,2,\dots,n.$$

Now,

$$supp[G] = \sum_{v\in V(G)} supp[v]$$
$$= \sum_{x\in X} supp[x] + \sum_{y\in Y} supp[y]$$
$$= \sum_{x\in X} n(m+1) + \sum_{y\in Y} m(n+1)$$
$$supp[G] = mn(m+1) + nm(n+1)$$
$$supp[G] = mn(m+n+2)$$

Corollary 3.5.*Let $G = K_{1,n}$. Then $supp[G] = n(n+3)$.*
Proof. Let $G = K_{1,n}$ be a star graph. Put $m = 1$ in theorem 3.4, we get $supp[G] = n(n+3)$.
Theorem 3.6.*Let $G = W_n (n \geq 4)$. Then $supp[W_n] = (n-1)(n+12)$.*
Proof. Let $G = W_n$ be a wheel of order n. Let $V(G) = \{v_0, v_1, v_2, \dots, v_{n-1}\}$ and $deg(v_0) = n-1$; $deg(v_i) = 3$ for all $i = 1,2,\dots,n-1$. Then

$$supp[v_0] = \sum_{v\in N[v_0]} deg(v) + deg(v_0)$$

$$= 3 + 3 + \cdots + 3((n-1)\ times) + (n-1)$$
$$= 4(n-1)$$

For each $i = 1,2,\dots,n-1$,

$$supp[v_i] = \sum_{v\in N[v_i]} deg(v) + deg(v_i)$$
$$= 3 + 3 + 3 + (n-1)$$
$$supp[v_i] = 8 + n.$$

Now,

$$supp[G] = \sum_{v\in V(G)} supp[v]$$
$$= supp[v_0] + \sum_{i=1}^{n-1} supp[v_i]$$
$$= 4(n-1) + (n+8)(n-1)$$
$$supp[G] = (n-1)(n+12).$$

Theorem 3.7. *Let $G = F_n\,(n \geq 4)$. Then $supp[F_n] = n^2 + 11n - 40$.*

Proof. Let $G = F_n$ be a fan of order n. Let $V(G) = \{v_0, v_1, v_2, \dots, v_{n-1}\}$ and $deg(v_0) = n-1$; $deg(v_1) = deg(v_{n-1}) = 2$; $deg(v_i) = 3$ for all $i = 2,3,\dots,n-2$. Then

$$supp[v_0] = \sum_{v\in N[v_0]} deg(v)$$
$$= deg(v_1) + deg(v_{n-1}) + \sum_{i=2}^{n-2} deg(v_i) + deg(v_0)$$
$$= 2 + 2 + 3(n-3) + n - 1$$
$$= 4n - 6$$

$$supp[v_1] = \sum_{v\in N[v_1]} deg(v)$$
$$= deg(v_0) + deg(v_2) + deg(v_1)$$
$$= n - 1 + 3 + 2$$
$$= n + 4$$

Similarly,

$supp[v_{n-1}] = n + 4.$

$$supp[v_2] = \sum_{v\in N[v_2]} deg(v)$$
$$= deg(v_0) + deg(v_1) + deg(v_3) + deg(v_2)$$
$$= n - 1 + 2 + 2 + 3$$
$$= n + 7$$

Similarly,

$supp[v_{n-2}] = n + 7.$

For each $i = 3,4,\dots,n-3$,

$$supp[v_i] = \sum_{v\in N[v_i]} deg(v)$$
$$= deg(v_{i-1}) + deg(v_{i+1}) + deg(v_0) + deg(v_i)$$
$$= 3 + 3 + 3 + (n-1)$$
$$supp[v_i] = n + 8.$$

Now,

$$supp[G] = \sum_{v\in V(G)} supp[v]$$
$$= supp[v_0] + supp[v_1] + supp[v_{n-1}] + supp[v_{n-2}] + \sum_{i=3}^{n-3} supp[v_i]$$
$$= 4n - 6 + 2n - 8 + 2n + 14 + (n+8)(n-5)$$
$$= 4n + 2n + 2n + n^2 + 3n - 40$$
$$supp[G] = n^2 + 11n - 40.$$

REFERENCES

[1] M. Anitha, S. Balamurugan: Strong (Weak) Efficient Open Domination on Product Graphs, International Journal of Pure and Applied Mathematics, (J.No. 23425), IISN 1311 - 8080, communicated.

[2] S. Balamurugan, M. Anitha , C. Karnan and P. Aristotle: *A Note on Open Support of a Graph under Addition I*, Communicated

[3] S. Balamurugan, M. Anitha , C. Karnan and P. Aristotle: *A Note on Open Support of a Graph under Addition II*, Communicated

[4] R. Balakrishnan, K. Ranganathan, *A Textbook of Graph Theory*, Springer,2011.

[5] S. Balamurugan, *A study on chromatic strong domination in graphs*, Ph.D Thesis, Madurai Kamaraj University, India 2008.

[6] J.A. Bondy, U.S.R. Murthy, *Graph theory with Applications*, North-Holland, 1982.

[7] F. Harary : *Graph Theory*, Addsion Wesley, Reading, Mass, (1972).

[8] T. W. Haynes, S. T. Hedetniemi, P. J. Slater: *Fundamentals of Domination in Graphs*, Marcel Dekker, New York, (1998).

[9] C. Y. Ponnappan: *Studies in Graph Theory Support Strong Domination in Graphs*, Ph.D thesis, Madurai Kamaraj University (2008).