Closed Support of a Graph under Addition II

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Abstract

In this paper we defined closed support of a vertex v under addition and closed support of a graph G under addition. We calculate the closed support for Dutch windmill graph, Butterfly graph and Ladder graph. Also, we generalized the value of closed support under addition for any given graph G.

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I. INTRODUCTION

In this work we consider finite, undirected, simple graphs $G = (V, E)$ with n vertices and m edges. The *n*eighbourhood of a vertex $v \in V(G)$ is the set $N_G(v)$ of all the vertices adjacent to v in G. For a set $X \subseteq V(G)$, the *open neighbourhood*$N_G(X)$ is defined to be $\cup_{v\in X} N_G(v)$ and the closed neighbourhood $N_G[X] = N_G(X) \cup X$. The *degree* of a vertex $v \in V(G)$ is the number of edges of G incident with v and is denoted by $deg_G(v)$ or $deg(v)$. The maximum and the minimum degrees of the vertices of G are respectively denoted by $\Delta(G)$ and $\delta(G)$. A vertex of a degree 0 in G is called an *isolated* vertex and a vertex of degree 1 is called a *pendent* vertex or an *end* vertex of G. A vertex of a graph G is said to be a vertex of full degree if it is adjacent to all other vertices in G. A graph G is said to be *regular of degree* r if every vertex of G has degree r. Such graphs are called *r-regular* graphs.

The *Dutch windmill graph*$D_n^{(m)}$, is the graph obtained by taking m copies of the cycle graph C_n with a vertex in common. The *Butterfly graph* (also called the bowtie graph and the hourglass graph) is a planar undirected graph with 5 vertices and 6 edges. It can be constructed by joining 2 copies of the cycle graph C_3 with a common vertex. It is denoted by B_n. The *ladder graph*L_n is a planar undirected graph with 2n vertices and n+2(n-1) edges. The Ladder graph obtained as the cartesian product of two graphs one of which has only one edge: $L_{n,1} = P_n \times P_1$.

A **closed support of a vertex**, v under addition is defined by $\sum_{u\in N[v]} deg(u)$ and it is denoted by $supp[v]$. A **closed support of a graph**, G under addition is defined by $\sum_{v\in V(G)} supp[v]$ and it is denoted by $supp[G]$.

II. DEFINITIONS

Definition 2.1.*Let G=(V,E) be a graph. A* **closed support of a vertex**, *v under addition is defined by* $\sum_{u\in N[v]} deg(u)$ *and it is denoted by* $supp[v]$.

Definition 2.2.*Let G=(V,E) be a graph. A* **closed support of a graph**, *G under addition is defined by* $\sum_{v\in V(G)} supp[v]$ *and it is denoted by* $supp[G]$.

III. RESULTS

Theorem 3.1.*Let* $G = L_{2n}$. *Then* $supp[G] = 24(n - 1)$.

Proof: Let $G = L_{2n}$ be a Ladder graph with $2n$ vertices.

Let $V(G) = \{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$ and $deg(v_i) = deg(u_i) = 2$ for all $i = 1, n$; $deg(v_i) =$

$deg(u_i) = 3$ for all $i = 2,3, \dots, n-1$. Then

$$supp[v_1] = \sum_{v \in N[v_1]} deg(v)$$
$$= deg(u_1) + deg(v_2) + deg(v_1)$$
$$supp[v_1] = 2 + 2 + 3 = 7$$

Similarly,

$$supp[v_n] = supp[u_1] = supp[u_n] = 7.$$

$$supp[v_2] = \sum_{v \in N[v_2]} deg(v)$$
$$= deg[u_2] + deg[v_1] + deg[v_3] + deg[v_2]$$
$$supp[v_2] = 11$$

Similarly,

$$supp[u_2] = supp[u_{n-1}] = supp[v_{n-1}] = 11.$$

Foreach$i = 3,4, \dots, n-2$,

$$supp[v_i] = \sum_{v \in N[v_i]} deg(v)$$
$$= deg(u_i) + deg(v_{i-1}) + deg(v_{i+1}) + deg(v_i)$$
$$supp[v_i] = 12, \ foralli = 3,4, \dots, n-2.$$

Similarly,

$$supp[u_i] = 9, foralli = 3,4, \dots, n-2.$$

Now,

$$supp[G] = \sum_{v \in V(G)} supp[v]$$
$$= \sum_{i=1,n} supp[v_i] + \sum_{i=1,n} supp[u_i] + \sum_{i=2,n-1} supp[v_i] + \sum_{i=2,n-1} supp[u_i]$$
$$+ \sum_{i=3}^{n-2} supp[v_i] + \sum_{i=3}^{n-2} supp[u_i]$$
$$= \sum_{i=1,n} 7 + \sum_{i=1,n} 7 + \sum_{i=2,n-1} 11 + \sum_{i=2,n-1} 11 + \sum_{i=3}^{n-2} 12 + \sum_{i=3}^{n-2} 12$$
$$= 28 + 44 + 24(n-4)$$
$$= 24n - 96 + 28 + 44$$
$$= 24n - 24$$
$$supp[G] = 24(n-1).$$

Theorem 3.2. *Let $G = D_{r,s} (2 \le r \le s)$. Then $supp[G] = r^2 + 3r + s^2 + 3s - 4$.*

Proof. Let $G = D_{r,s} (2 \le r \le s)$ be a double star with $r + s$ vertices. Let $V(G) = \{x, y, v_1, v_2, \dots, v_{r-1}, u_1, u_2, \dots, u_{s-1}\}$ such that $xv_i \in E(G)$ and $yu_j \in E(G)$ for all $i = 1,2, \dots, r-1$ and $j = 1,2, \dots, s-1$. Then $deg(x) = r$; $deg(y) = s$ and $deg(v_i) = deg(u_i) = 1$ for all i, j.

$$supp[x] = \sum_{v \in N[x]} deg(v)$$
$$= deg(y) + deg(x) + \sum_{i=1}^{r-1} deg(v_i)$$
$$= s + r - 1 + r$$
$$= 2r + s - 1$$
$$supp[x] = 2r + s - 1$$

Similarly,

$$supp[y] = r + 2s - 1.$$

and

$$supp[v_i] = \sum_{v \in N[v_i]} deg(v)$$
$$= deg(x) + deg(v_i)$$
$$supp[v_i] = r + 1, foralli = 1,2, \dots r-1$$

Similarly,

$$supp[u_j] = s + 1, forallj = 1,2, \dots s-1$$

Now,

$$supp[G] = \sum_{v \in V(G)} supp[v]$$
$$= supp[x] + supp[y] + \sum_{i=1}^{r-1} supp[v_i] + \sum_{j=1}^{s-1} supp[u_j]$$
$$= (r-1)(r+1) + (s-1)(s+1) + 2r + s - 1 + r + 2s - 1$$
$$= r^2 - 1 + s^2 - 1 + 3r + 3s - 2$$
$$supp[G] = r^2 + 3r + s^2 + 3s - 4$$

Theorem 3.3. *Let $G = C_n^+ (n \ge 3)$. Then $supp[G] = 14n$.*

Proof. Let $G = C_n^+ (n \geq 3)$ be a corona with $2n$ vertices.

Let $V(G) = \{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$ such that $deg(v_i) = 3$and $deg(u_i) = 1$ for all $i = 1,2, \dots, n$.

$$supp[u_i] = \sum_{v \in N[u_i]} deg(v) + deg(u_i)$$
$$supp[u_i] = 1 + 3 = 4$$
$$supp[v_i] = \sum_{v \in N[v_i]} deg(v) + deg(v_i)$$
$$supp[v_i] = 3 + 3 + 3 + 1 = 10.$$

Now,

$$supp[G] = \sum_{v \in V[G]} supp[v]$$
$$= \sum_{i=1}^{n} supp[v_i] + \sum_{i=1}^{n} supp[u_i]$$
$$= 14n$$
$$supp[G] = 14n.$$

Theorem 3.4. *Let $G = K_n^+$. Then $supp[G] = n^3 + n^2 + 2n$.*

Proof. Let $G = K_n^+$ be a corona of order $2n$. Let $V(G) = \{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$ such that $deg(v_i) = n$and $deg(u_i) = 1$ for all $i = 1,2, \dots, n$.

$$supp[u_i] = \sum_{v \in N[u_i]} deg(v) + deg(u_i)$$
$$supp[u_i] = 1 + n$$

$$supp[v_i] = \sum_{v \in N[v_i]} deg(v) + deg(v_i)$$
$$= n^2 + 1$$
$$supp[v_i] = n^2 + 1$$

Now,

$$supp[G] = \sum_{v \in V(G)} supp[v]$$
$$= \sum_{i=1}^{n} supp[v_i] + \sum_{i=1}^{n} supp[u_i]$$
$$supp[G] = n^3 + n^2 + 2n.$$

Theorem 3.5. *Let $G = K_m(a_1, a_2, \dots, a_m)$.*Then $supp[G] = n(n^2 + n + 2)$.

Proof. Let $G = K_m(a_1, a_2, \dots, a_m)$ be a multistar graph of order $m + a_1 + a_2 + \cdots + a_m$. Let $V(G) = \{v_1, v_2, \dots, v_n, u_1, u_2, \dots, u_n\}$ such that $deg(v_i) = n$and $deg(u_i) = 1$ for all $i = 1,2, \dots, n$.

$$supp[u_i] = \sum_{v \in N[u_i]} deg(v)$$
$$supp[u_i] = n + 1$$

$$supp[v_i] = \sum_{v \in N[v_i]} deg(v)$$
$$supp[v_i] = n^2 + 1$$

Now,

$$supp[G] = \sum_{v \in V(G)} supp[v]$$
$$= \sum_{i=1}^{n} supp[v_i] + \sum_{i=1}^{n} supp[u_i]$$
$$= \sum_{i=1}^{n} (n^2 + 1 + n + 1)$$
$$= \sum_{i=1}^{n} (n^2 + n + 2)$$
$$supp[G] = n(n^2 + n + 2).$$

Theorem 3.6.*Let $G = P_n^+$ be a corona graph. Then $supp[G] = 2(7n - 6)$.*

Proof. Let $G = P_n^+$ be a corona graph. Let $V(P_n^+) = \{v_1, v_2, v_3, \dots, v_n, u_1, u_2, \dots, u_n\}$ such that $deg(v_1) = deg(v_n) = 2$; $deg(v_i) = 3$ for all $i = 2,3, \dots, n - 1$; $deg(u_j) = 1$ for all $j = 1,2, \dots, n$.

Now,

$$supp[v_1] = \sum_{v \in N[v_1]} deg(v)$$
$$= deg(v_1) + deg(v_2) + deg(u_1)$$
$$= 2 + 3 + 1 = 6$$

Similarly,

$$supp[v_n] = 6;$$

$$supp[v_2] = \sum_{v \in N[v_2]} deg(v)$$
$$= deg(v_2) + deg(v_1) + deg(v_3) + deg(u_2)$$

$= 3 + 2 + 3 + 1 = 9$

Similarly,

$supp[v_{n-1}] = 9;$

For $i = 3,4, \ldots, n-2$

$supp[v_i] = \sum_{v \in N[v_i]} deg(v)$
$= deg(v_i) + deg(v_{i-1}) + deg(v_{i+1}) + deg(u_i)$
$= 3 + 3 + 3 + 1 = 10$
$supp[u_1] = \sum_{v \in N[u_1]} deg(v)$
$= deg(u_1) + deg(v_1)$
$= 1 + 2 = 3$

Similarly,

$supp[u_n] = 3;$

For$i = 2,3,4, \ldots, n-1$

$supp[u_i] = \sum_{v \in N[u_i]} deg(v)$
$= deg(u_i) + deg(v_i)$
$= 4$

Now,

$supp[G] = \sum_{v \in V(G)} supp[v]$
$= \sum_{i=1}^{n} supp[v_i] + \sum_{i=1}^{n} supp[u_i]$
$= supp[v_1] + supp[v_2] + \sum_{i=3}^{n-2} supp[v_i] + supp[v_{n-1}] + supp[v_n]$
$+ supp[u_1] + supp[u_n] + \sum_{i=2}^{n-1} supp[u_i]$
$= 6 + 9 + \sum_{i=3}^{n-2} 10 + 9 + 6 + 3 + \sum_{i=2}^{n-1} 4 + 3$
$supp[G] = 36 + 10(n-4) + 4(n-2)$
$= 14n - 12 = 2(7n - 6)$

Theorem 3.7.*Let G=(m,m,...,m) be a caterpillar graph.*
Then $supp[G] = 4mn + 2m^2 + m^2n + 4n - 4m - 8$

Proof. Let G=$(m, m, \ldots, m)$ be a caterpillar graph.

Let $V(G) = \{v_1, v_2, \ldots, v_n, u_n, u_{11}, \ldots, u_{1m}, \ldots, u_{n1}, \ldots, u_{nm}\}$ such that $deg(v_1) = deg(v_n) = m + 1;$ $deg(v_i) = m + 2$,for all i=2,3,...,n-1 and $deg(u_{ij}) = 1$ for all $i = 1,2, \ldots, n$ and $j = 1,2, \ldots, m$

$supp[v_1] = \sum_{i=1}^{n} deg(u_{1i}) + deg(v_1) + deg(v_2)$
$= \sum_{i=1}^{m} 1 + m + 1 + m + 2$
$= 3m + 3$

Similarly,

$supp[v_n] = 3m + 3;$

$supp[v_2] = \sum_{i=1}^{n} deg(u_{2i}) + deg(v_1) + deg(v_3) + deg(v_2)$
$= \sum_{i=1}^{m} 1 + m + 1 + m + 2 + m + 2$
$= 4m + 5$

Similarly,

$supp[v_{n-1}] = 4m + 5;$

For $i = 3,4, \ldots, n-2$

$supp[v_i] = \sum_{v \in N[v_i]} deg(v)$
$= deg(v_{i-1}) + \sum_{j=1}^{n} deg(u_{ij}) + deg(v_{i+1}) + deg(v_i)$
$= m + 2 + \sum_{j=1}^{n} 1 + m + 2 + m + 2$
$= 4m + 6$

For $j = 1,2, \ldots, m$

$supp[u_{1j}] = deg(v_1) + deg(u_{1j})$
$= m + 2$

Similarly,

$supp[u_{nj}] = m + 2;$

For $i = 2,3, \ldots, n-1$ and $j = 1,2, \ldots, m$

$supp[u_{ij}] = deg(v_i) + deg(u_{ij})$

$= m + 3$

Similarly,

$supp[u_{nj}] = m + 3;$

Now,

$supp[G] = \sum_{v \in V(G)} supp[v]$
$= \sum_{i=1}^{n} supp[v_i] + \sum_{i=1}^{n} \sum_{j=1}^{m} supp[u_{ij}]$
$= supp[v_1] + supp[v_2] + \sum_{i=3}^{n-2} supp[v_i] + supp[v_{n-1}] + supp[v_n]$
$+ \sum_{j=1}^{m} supp[u_{1j}] + \sum_{j=1}^{m} supp[u_{nj}] + \sum_{i=2}^{n-1} \sum_{j=1}^{m} supp[u_{ij}]$
$= 3m + 3 + 4m + 5 + \sum_{i=3}^{n-2} (4m + 6) + 3m + 3 + 4m + 5$
$+ \sum_{j=1}^{m} (m + 2) + \sum_{j=1}^{m} (m + 2) + \sum_{i=2}^{n-1} \sum_{j=1}^{m} (m + 3)$
$= 14m + 16 + (4m + 6)(n - 4) + m(m + 2) + m(m + 2)$
$+(n - 2)m(m + 3)$
$supp[G] = 4mn + 2m^2 + m^2 n + 4n - 4m - 8.$

Theorem 3.8. *Let G be a tadpole graph. Then* $supp[G] = 6m + 6n + 2$.

Proof. Let G be a tadpole graph. Let $V(G) = \{x, v_1, v_2, \dots, v_{m-1}, u_1, \dots, u_n\}$ such that $deg(x) = 3; deg(v_i) = 2$,for all $i = 1,2,3, \dots, m - 1$; $deg(u_n) = 1$ and $deg(u_i) = 2$ for all $i = 1,2, \dots, n - 1$

$supp[x] = \sum_{v \in N[x]} deg(v)$
$= deg(v_1) + deg(v_{m-1}) + deg(u_1) + deg(x)$
$= 2 + 2 + 2 + 3 = 9$

$supp[v_1] = \sum_{v \in N[v_1]} deg(v)$
$= deg(x) + deg(v_2) + deg(v_1)$
$= 3 + 2 + 2 = 7$

Similarly,

$supp[v_{m-1}] = 7 \ and supp[u_1] = 7;$

For $i = 2,3, \dots, m - 2$

$supp[v_i] = \sum_{v \in N[v_i]} deg(v)$
$= deg(v_{i-1}) + deg(v_{i+1}) + deg(v_i)$
$= 2 + 2 + 2 = 6$

For $i = 2,3, \dots, n - 2$

$supp[u_i] = \sum_{v \in N[u_i]} deg(v)$
$= deg(u_{i-1}) + deg(u_{i+1}) + deg(u_i)$
$= 2 + 2 + 2 = 6$

$supp[u_{n-1}] = \sum_{v \in N[u_{n-1}]} deg(v)$
$= deg(u_{n-2}) + deg(u_{n-1}) + deg(u_n)$
$= 2 + 2 + 1 = 5$

$supp[u_n] = \sum_{v \in N[u_n]} deg(v)$
$= deg(u_{n-1}) + deg(u_n)$
$= 2 + 1 = 3$

Now,

$supp[G] = \sum_{v \in V(G)} supp[v]$
$= \sum_{i=1}^{m-1} supp[v_i] + supp[x] + \sum_{i=1}^{n} supp[u_i]$
$= supp[v_1] + supp[v_{m-1}] + \sum_{i=2}^{m-2} supp[v_i] + supp[x]$
$+supp[u_1] + \sum_{i=2}^{n-2} supp[u_i] + supp[u_{n-1}] + supp[u_n]$
$= 7 + 7 + \sum_{i=2}^{m-2} 6 + 9 + 7 + \sum_{i=2}^{n-2} 6 + 3 + 5$
$= 38 + (m - 3)6 + 6(n - 3)$
$supp[G] = 6m + 6n + 2.$

Theorem 3.9. *Let G be a butterfly graph. Then* $supp(G) = 44$

Proof. Let G be a butterfly graph. Let $V(G) = \{x, v_1, v_2, v_3, v_4\}$ such that $deg(x) = 4; deg(v_i) = 2$,for all $i = 1,2,3,4$;

$$supp[x] = \sum_{v\in N[x]} deg(v)$$
$$= deg(v_1) + deg(v_2) + deg(v_3) + deg(v_4) + deg(x)$$
$$= 12$$

For $i = 1,2,3,4$

$$supp[v_i] = \sum_{v\in N[v_i]} deg(v)$$
$$= 2 + 2 + 4$$
$$= 8$$

Now,

$$supp[G] = \sum_{v\in V(G)} supp[v]$$
$$= supp[v_1] + supp[v_2] + supp[v_3] + supp[v_4] + supp[x]$$
$$= 44$$

Theorem 3.10. *Let $G = D_{(n)}^{(m)}$ be a Dutch windmill graph.* Then $supp[G] = 2m(2m + 3n - 2)$

Proof. Let $G = D_{(n)}^{(m)}$ be a Dutch windmill graph. Let $V(G) = \{x, v_1^i, v_2^i, v_3^i, \dots, v_{n-1}^i\}$ for $i = 1,2,\dots,m$ such that $deg(x) = 2m; deg(v_j^i) = 2$, for all $i = 1,2,\dots,m$ and $j = 1,2,\dots,n-1$;

$$supp[x] = \sum_{v\in N[x]} deg(v)$$
$$= deg(x) + \sum_{i=1}^{m} (deg(v_1^i) + deg(v_{n-1}^i))$$
$$= 2m + \sum_{i=1}^{m} (4))$$
$$= 6m$$

For $i = 1,2,\dots,m$

$$supp[v_1^i] = \sum_{v\in N[v_1^i]} deg(v)$$
$$= 2m + 4$$

Similarly,

$$supp[v_{n-1}^i] = 2m + 4.$$

For $j = 2,3,\dots,n-2$,

$$supp[v_j^i] = \sum_{v\in N[v_j^i]} deg(v)$$
$$= 2 + 2 + 2 = 6$$

Now,

$$supp[G] = \sum_{v\in V(G)} supp[v]$$
$$= supp[x] + \sum_{i=1}^{m} \sum_{j=1}^{n-1} supp[v_j^i]$$
$$= 6m + 2m(2m + 4) + 6m(n - 3)$$
$$= 4m^2 - 4m + 6mn$$
$$= 2m(2m + 3n - 2)$$

Theorem 3.11. *For any graph G, $supp[G] = \sum_{v\in V(G)} (deg(v))^2 + 2q$*

Theorem 3.12. *Let $G \circ K_1$ be a corona product of G and K_1. Then $supp[G \circ K_1] = supp[G] + 4(m + n)$, where n and m are the order and size of G respectively.*

Proof. Let G be an any graph of order n and size m. Let $G \circ K_1$ be a corona product of G and K_1. By Theorem (3.11), $supp[G \circ K_1] = \sum_{v\in V(G\circ K_1)} (deg(v))^2 + 2m = \sum_{v\in V(G)} (deg(v) + 1)^2 + n + 2m$. Since degree of each vertex, v in $G \circ K_1$ increases one from its corresponding vertex in G.

Therefore, $supp[G \circ K_1] = supp[G] + 4(m + n)$.

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