Modelling Optimal Paint Production using Linear Programming

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Abstract: *The concept of Simplex algorithm is effectively and practically used in this work. A feature of Linear Programming is to allocate raw materials to striving variables (small bucket and big bucket) in a paint industry for the purpose of maximizing company's profit. The analysis was implemented using MS Excel Solver (2007 version) and the result showed that 10 units of small bucket of paint, 0 unit of big bucket of paint should be produced independently in order to make a profit of ₦ 7000.00. Thus, from the analysis, it was discovered that small bucket of paint only contribute objectively to the profit. So, more of small buckets of paint are needed to be produced and sold in order to maximize the company's profit and at same time satisfying the customers' needs.*

Keywords: *Linear programming model, Decision Variables, Simplex Method, MS Excel Solver (2007version),Optimal result.*

I. INTRODUCTION

The origin of Operation Research (OR) situates many decades when attempts were made to apply scientific method in the management of organizations. The investigating work in OR has been linked to the Military services early World War II, because of the war, there was an urgent need to allocate scare resources to the various military commands for maximal result. OR is concerned with efficient allocation of scarce resources which is applied to problems that concern how to conduct and coordinate the activities within an organization for optimum result such areas include: manufacturing, transportation, construction, hospitals, public services, financial planning, telecommunication, military, to mention but a few. There are different kinds of OR models, such models determine the type of solution to apply on a particular problem. (See Chikwendu ,2009).

In this research paper, we considered a linear programming; which is a mathematical programming that is concerned with allocation of scarce or limited resources to several striving activities on the basis of optimality. In optimization, we seek to maximize or minimize a scientific quantity called the objective function which depends on a finite number of input or decision variables which may be independent of one another or may not be related through one or more constraints. Thus, the decisions of production managers are mostly dependent on the decision variables used in the production, together with the output proceeding.

The problem of decision making is based on the use of limited resource which brought about the application of Linear Programming model which is one of the powerful tools decision makers must be employed before reaching a desired result (See Akpan and Iwok, 2016).

II. LITERATURE REVIEW

In this section, we shall review some existing literature done by some researchers.

Akpan and Iwok (2016) investigated on the application of linear programming for optimal use of raw materials in bakery by considering three different sizes of bread (big loaf, giant loaf and small loaf) using simplex algorithm which was ran using TORA software. The optimum result was obtained from the model which indicated that two sizes of bread should be produced.

Lenka (2013) carried a research on process of development of model based on linear programming to solve resource allocation tasks with emphasis on financial aspects. He used two linear programming models such that the results showed that one of the models maximizes the revenue of a company and the other minimizes the cost of operation respectively.

Majeke (2013) worked on incorporating crop rotational requirements in a linear programming model: A case study of rural farmers in Bindura, Zimbabwe. He used computer software (MS Excel) to solve the models and the result obtained was that linear programming model maximizes the income of farmers in rural area.

Balogun *et al* (2012) considered the use of linear programming for optimal production in Coca-Cola Company. They formulated a linear programming model for the production process and solved the model using the simplex algorithm. The result showed that the company should concentrate in the production of the two products in order not to run into high cost.

Joly (2012) investigated on the refinery production planning and scheduling. He used the OR technology, major past and present strategies to deal with present and future challenges. The result obtained was that profitability increase depends on changes in the existing system.

III. LINEAR PROGRAMMING MODEL

A Mathematical Program is an optimization problem in which the objective constraints are given as mathematical functions and functional relationships. Most mathematical model is of the form (See Taha, 2007):

Optimize (max or min) Z = $f(x_1, x_2, x_{3,} ..., x_n)$

Subject to

$$
\begin{aligned}
& g_1(x_1, x_2, x_{3,} ..., x_n) \ (\le, =, \ge) \ b_1 \\
& g_2(x_1, x_2, x_{3,} ..., x_n) \ (\le, =, \ge) \ b_2 \\
& \vdots \qquad\qquad\qquad\qquad \vdots \\
& g_m(x_1, x_2, x_{3,} ..., x_n) \ (\le, =, \ge) \ b_m .
\end{aligned}
\tag{1}
$$

Constraint mathematical programs covered by the formulation given in (1) above are considered in this paper.

A mathematical program as in (1) is linear if $f(x_1, x_2, x_{3,} ..., x_n)$ and each $g_i(x_1, x_2, x_{3,} ..., x_n)$, $(i = 1,2,3,..., m)$ are linear in each of their arguments. That is, if

$$f(x_1, x_2, x_{3,} ..., x_n) = c_1x_1 + c_2x_2 + ... + c_nx_n \tag{2}$$

and

$$g_i(x_1, x_2, x_{3,} ..., x_n) = a_{i1}x_1 + a_{i2}x_2 + ... + a_{in}x_n \tag{3}$$

Where c_i and a_{ij}, $(i = 1,2,3,..., m \ , \ j = 1,2,3,..., n)$ are known constraints, any other mathematical program is nonlinear.

In matrix form (See Brownson, Naadimuthu ,1997), the general form of Linear Programming (LP) model is given as

Optimize $Z = C^T X$

Subject to $AX (\le, \ge, =) b$

$X \ge 0$.

Where $A=(a_{ij})$, $b=(b_i)$ and $C=(c_j)$ are known constants that describe the problem $X=(x_j)$.

The form of a linear programming model for a maximization case is of the form:

Maximize $Z=c_1x_1+c_2x_2+...+c_nx_n$

Subject to

$$a_{11}x_1+a_{12}x_2+...+a_{1n}x_n\le b_1$$

$$a_{21}x_1+a_{22}x_2+...+a_{2n}x_n\le b_2$$

$$\vdots \qquad \qquad \vdots$$

$$a_{m1}x_1+a_{m2}x_2+...+a_{mn}x_n\le b_m$$

and $x_1,x_2,...,x_n\ge 0.$

where $c_1,c_2,..,c_n$ represent the per unit profit (or cost) of decision variables $x_1,x_2,..,x_n$ to the value of the objective function and $a_{11},a_{12},...,a_{1m},a_{21},a_{22},...,a_{2m},...,a_{mn}$ represent the amount of resource consumed per unit of the decision variables. The b_i represents the total availability of the i^{th} resource. Z represents the measure of performance which can be either profit, or cost or reverence etc.

IV. SIMPLEX METHOD

The Simplex method is an algebraic procedure and an iterative process by which one move from a good feasible solution to another better feasible solution until the best optimal solution is reached, that is a solution which cannot be improved upon. The procedure concepts are geometric and it is more general method for solving linear programming problem especially when there are more than two decision variables and many constraints. That is to say it can be used for several large problems.

Linear Programming problems are transformed into the standard form before applying the Simplex method of solution. This transformation will facilitate the calculation for the solution and also ensure that no important element of the problem is overlooked. To do this, after stating the objective function, all the constraints with the inequalities $\ge$ and $\le$ are changed to $=$.

Considering a linear constraint of the form:

$$\sum a_{ij}x_j\le b_i \tag{5}$$

To change the inequality $\le$ to $=$, we add a new nonnegative variable called a slack variable to equalize both the left-hand side with the right-hand of the equation to obtain

$$\sum a_{ij}x_j+s_i=b_i \tag{6}$$

The s_i is the slack variable which represents the waste involved in that phase of the system modeled by the constraint.

For n decision variables and m constraints, the standard form of the linear programming model can be formulated as:

Optimize (max) $Z = \sum_{j=1}^{n} c_j x_i + \sum_{i=1}^{m} 0 s_i$ (7)

Subject to the linear constraints

$$\sum_{j=1}^{n} a_{ij} x_j + s_i = b_i ,\ i = 1,2,...,m$$

For all $x_j, s_i \geq 0$.

V. ASSUMPTIONS

- It is assumed that the qualities of raw materials used in the paint production are standard (not inferior).
- It is assumed that the raw materials required for production of paints are limited (scarce).
- It is assumed that an effective allocation of the raw materials to the variables (small buckets and big buckets) will aid optimal production and at the same time maximizes the profit of the industry.

VI. DATA PRESENTATION AND ANALYSIS

The data for this research work was collected from DE Dove Paint Emulsion Industry, Port Harcourt with eight raw materials (Calcium Carbonate, Titanium dioxide, Polyvinyl Acetate, Ammonium, Nitro-sol, Acticide, Water and Calgon) available for daily production of two different sizes of buckets of paint (small bucket and big bucket) and the profit contribution per unit size of paint produced. The data analysis was carried out with MS Excel Solver (2007 version) software. The individual raw material per each unit product of paint produced is shown as follows.

Calcium Carbonate
Total amount of calcium carbonate available = 50kg
Each unit of small bucket requires 3kg of calcium carbonate
Each unit of big bucket requires 15kg of calcium carbonate

Titanium dioxide
Total amount of Titanium dioxide available = 25kg
Each unit of small bucket requires 0.2kg of Titanium dioxide
Each unit of big bucket requires 0.8kg of Titanium dioxide

Polyvinyl Acetate (PVA)
Total amount of Polyvinyl Acetate available = 240kg
Each unit of small bucket requires 0.3kg of Polyvinyl Acetate
Each unit of big bucket requires 1.5kg of Polyvinyl Acetate

Ammonium
Total amount of Ammonium available = 30kg
Each unit of small bucket requires 0.05kg of Ammonium
Each unit of big bucket requires 0.1 kg of Ammonium

Nitro-sol
Total amount of Nitro-sol available = 25kg
Each unit of small bucket requires 0.03kg of Nitro-sol
Each unit of big bucket requires 0.1kg of Nitro-sol

Acticide
Total amount of Acticide available = 25kg
Each unit of small bucket requires 0.1kg of Acticide

Each unit of big bucket requires 0.5kg of Acticide

Water
Total amount of water available = 20 liters
Each unit of small bucket requires 2 liters of water
Each unit of big bucket requires 10 liters of water

Calgon
Total amount of Calgon available = 25kg
Each unit of small requires 0.1kg of Calgon
Each unit of big bucket requires 0.5kg of Calgon

Profit contribution per unit produced (size) of paint produced
Each unit of small bucket = ₦ 700.00
Each unit of big bucket = ₦ 3000.00

The above data can be summarized in a tabular as shown below

VII. TABLE 1

Raw material	Product		Total available raw material
	Small bucket	Big bucket	
calcium carbonate (kg)	3	15	50
titanium dioxide (kg)	0.2	0.8	25
polyvinyl acetate (pva) (kg)	0.3	1.5	240
ammonium (kg)	0.05	0.1	30
nitro-sol (kg)	0.03	0.1	25
acticide (kg)	0.1	0.5	25
water (l)	2	10	20
calgon (kg)	0.1	0.5	25
profit (₦)	700.00	3000.00	

VIII. MODEL FORMATION

Let the quantity of small bucket to be produced = x_1

Let the quantity of big bucket to be produced = x_2

Let Z denote the profit to be maximized

The linear programming model for the above production data is given by

Max $Z = 700x_1 + 3000x_2$

Subject to:

$$3x_1 + 15x_2 \leq 50$$

$$0.2x_1 + 0.8x_2 \leq 25$$

$0.3x_1 + 1.5x_2 \leq 240$

$0.05x_1 + 0.1x_2 \leq 30$

$0.03x_1 + 0.1x_2 \leq 25$

$0.1x_1 + 0.5x_2 \leq 25$

$2x_1 + 10x_2 \leq 20$

$0.1x_1 + 0.5x_2 \leq 25$

$x_1, x_2 \geq 0$.

Hence, converting the model into its corresponding standard form:

Max $Z = 700x_1 + 3000x_2 + 0s_1 + 0s_2 + 0s_3 + 0s_4 + 0s_5 + 0s_6 + 0s_7 + 0s_8$

Subject to:

$3x_1 + 15x_2 + s_1 = 50$

$0.2x_1 + 0.8x_2 + s_2 = 25$

$0.3x_1 + 1.5x_2 + s_3 = 240$

$0.05x_1 + 0.1x_2 + s_4 = 30$

$0.03x_1 + 0.1x_2 + s_5 = 25$

$0.1x_1 + 0.5x_2 + s_6 = 25$

$2x_1 + 10x_2 + s_7 = 20$

$0.1x_1 + 0.5x_2 + s_8 = 25$

$x_1, x_2, s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8 \geq 0.$

IX. INTERPRETATION OF RESULT

The modeled linear programming was solved using MS Excel solver (2007 version) which gives an optimal solution of $x_1 = 10$ and $x_2 = 0$, with $Z = 7000$ and based on the data collected, the optimum result derived from the model indicates that small size of bucket should be produced which should be 10 units. This will produce a maximum profit of ₦7000.00. The MS Excel solver (2007 version) output is shown below.

	Small bucket	Big Bucket	Total contribution	Max capacity
Decision variables	10	0		
Contribution	700	3000	7000	
Calcium Carbonate	3	15	30	50
Titanium dioxide (kg)	0.2	0.8	2	25
Polyvinyl Acetate (PVA) (kg)	0.3	1.5	3	240
Ammonium (kg)	0.05	0.1	0.5	30
Nitro-sol (kg)	0.03	0.1	0.3	25
Acticide (kg)	0.1	0.5	1	25
Water (L)	2	10	20	20
Calgon (kg)	0.1	0.5	1	25

X. SUMMARY

The research work objective was to apply linear programming for optimal use of raw material in paint production. DE Dove Paint Emulsion Industry, Port Harcourt was used as our case study. Meanwhile, in this research work, the decision variables are the two different sizes of paint (small bucket and big bucket) produced by DE Dove Paint Emulsion Industry, Port Harcourt with eight raw materials (Calcium Carbonate, Titanium dioxide, Polyvinyl Acetate, Ammonium, Nitro-sol, Acticide, Water and Calgon) used in the production and the amount of raw material required of each variable (paint sizes). The result shows that 10 units of small bucket and 0 unit of big bucket should be produced which gives a maximum profit of ₦7000.00.

XI. CONCLUSION

Based on the analysis carried out in this research work and the result shown that DE Dove Paint Emulsion Industry, Port Harcourt should produce the two sizes bucket of paint (small bucket and big bucket) in order to satisfy her customers and at same time attaining a maximum profit, because they contribute mostly to the profit earned by the industry.

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