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Mathematical Modelling of Projection for Population in Kayah State

HtayHtay Mar[#1], San San Maw[*2], Lin Lin Naing[**3]

[#]*Department of Engineering Mathematics, Technological University (Loikaw). Kayah State, Myanmar.*
[*]*Department of Engineering Mathematics, Mandalay Technological University, Mandalay, Myanmar.*
[**] *Faculty of Computing, University of Computer Studies, Hinthada, Ayeyarwady Region, Myanmar.*

Abstract — *Censuses and projections for population are the most important activities for a country to provide current and future population sizes needed by government, policy makers and planners of a country. Population censuses were taken at least every 10 years period and based on the data and information; population projection has been done yearly to perform the development of the country in various sectors. Kayah State is Myanmar's smallest state by both geographical area and population. In this paper, the population of Kayah State is projected for the first half of 21st century using two mathematical models. The exponential growth model and logistic growth model are used to forecast the population from 2020 to 2050 based on the data and information of the 2014 Myanmar Population and Housing Census and CEIC (Census and Economic Information Center) data. The accuracies of the projected results of two models are compared using results from CEIC data by Mean Absolute Percentage Error (MAPE). The objective of this paper is to project the population of Kayah State using two mathematical models to the end of the first half of 21st century to support the development of State.*

Keywords - *Exponential growth model, Logistic growth model, Growth rate, Population projection, Carrying capacity, MAPE.*

I. INTRODUCTION

KayahState is located in the eastern part of Myanmar. It is bounded on the north by Shan State, on the east by Thailand's Mae Hong Son Province, and on the south and west by Kayin State. It is Myanmar's smallest state with the area 11,670 km^2 (0.17% of country's area) and only 0.6% of Myanmar's population live in Kayah State [1]. As population history, there were only three censuses (1973, 1983, and 2014) carried out in Myanmar. According to the censuses data, the total populations of Kayah State were 126574 in 1973, 168429 in 1983, and 286627 in 2014 (April). The total number of Kayah State population increased 33.1% in 1983 and 70.1% in 2014 [2]. In this situation, statistical analysis and projection for state's population is very important. According to the CEICestimation, Kayah State population will reach 330356 at the end of 2020. The population size and growth rate in a state directly influence the situation of economy, policy, culture, education, environment of state and determine the cost of natural resources [3]. Projection of any country's population plays a vital role in the planning as well as in the decision making for socioeconomic and demographic development. Every government always requires accurate idea about the future size of population, resources, demands and consumption for their planning activities. To obtain this information, the behavior of the related parameters is analyzed based on the previous data and using the conclusions drawn from the analysis, they make future projection of the targeted data. In this paper, projections of population for Kayah State are done by determining growth rate, vital coefficients, and the maximum size of population known as carrying capacity etc.

II. MATERIALS AND METHODS

The population problem is one of the important indices for a regional sustainable development. The scale of population has a profound impact which is reasonable for social development of a region[4]. To forecast future population sizes of a given country or state, it is important to know the population growth rate, historical and present population. Actually all these data are concerned with demography. Demography is the mathematical way of modeling and statistical analysis of population. In this paper, the historical and forecasted population are explored from CEIC data website [5]. Moreover, research papers on population projection of some countries such as Rwanda [3] and India [6] are also referred for methodology and approaches. A population model is a type of mathematical model that can be applied to the study of population dynamics and two mathematical models, namely, exponential growth model and logistic growth model are used in the projection process.

TABLE I

KAYAH STATE POPULATION HISTORY (FROM CEIC DATA)

Year	Actual Population	Annual Growth Rate	Population Density (P/km^2)
2008	251417	2.57	22
2009	257973	2.51	22
2010	264530	2.45	23
2011	271088	2.39	23
2012	277647	2.34	24
2013	284207	2.28	24
2014	290768	2.18	25
2015	297162	2.16	25
2016	303646	2.14	26
2017	310214	2.12	27
2018	316858	2.10	27
2019	323573	2.07	28

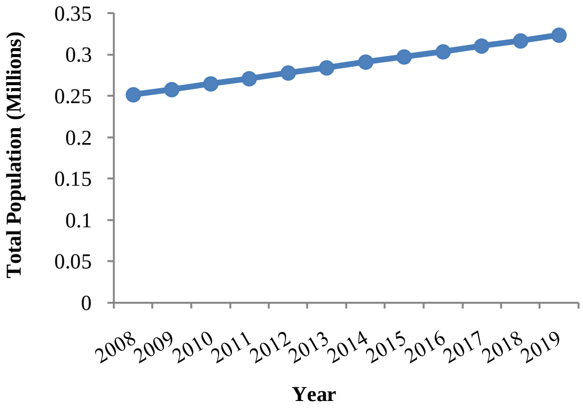

Fig.1 Graph of actual population from CEIC data 2008 to 2019

A. *Problem Definition*

At the end of 20th century, Kayah State population is round about 0.18 million and according to the Myanmar census 2014 which is the country's first national census after three decades [2], population is nearly 0.3 million. A projection of population may be defined as forecast of a future population based on a study of past and present data. The results from population projections are able to be applied for planning of future social security and Medicare obligations, determining of future water demands, estimatingof the needs for new public schools, sites selection for fire stations and predicting the demand for housing [7].

B. *Development of the Models*

The most successful models for explaining the growth of population are exponential growth model and logistic growth model. Exponential growth model was initiated by Thomas Malthus and itdescribes unlimited reproduction and human population growthhas appeared to be exponential considering on infinite amounts of resources. Logistic growth model was the modification of Malthus model and introduced by Verhulst considering limitation of resources so that the population is always less than some number known as carrying capacity. To compare the accuracy of the projection results of these two models, we use Mean Absolute Percentage Error equation [8].

1) ***Exponential Growth Model:***Englishman Thomas Malthus introduced this model in 1798. Suppose we know the population N_0at some given time $t = t_0$and we are interested in projecting the population N at some future time $t = t_1$. In other words, we want to find population function $N(t)$for $t_0 \le t \le t_1$ satisfying $N(t_0) = N_0$. Then considering the initial value problem [6],

$$\frac{dN}{dt} = rN(t), \quad t_0 \le t \le t_1; \ \ N(t_0) = N_0 \,. \tag{1}$$

Integrating by variable separable, we get

$$N(t) = N_0 e^{rt} \tag{2}$$

where r is a constant called growth rate.

$$r = \frac{1}{t}\ln\left(\frac{N}{N_0}\right) \tag{3}$$

$$t = \frac{1}{r}\ln\left(\frac{N}{N_0}\right)$$

the growth rate increases as the population increase.

*2) **Logistic Growth Model:*** Belgian Mathematician Verhulst proposed this model in the 1840s as model for world population growth. His model included the idea of carrying capacity and it was widely used in many fields of modelling and forecasting. Verhulst modified Malthus's exponential model to make a population size proportional to both the previous population and a new term([3],[6]).

$$\frac{a - bN(t)}{a}$$

where a and b are called vital coefficients of the population that reflect how far the population is from its maximum limit. The modified exponential model using this new term is:

$$\frac{dN}{dt} = \frac{aN(t)(a - bN(t))}{a}$$

$$\frac{dN}{dt} = aN - bN^2.$$

Integrating,

$$\int \frac{1}{N(a - bN)} dN = \int dt.$$

Dividing the integrand by partial fraction and using initial condition, $t = 0, N = N_0$,

$$\int \frac{1}{a}\left(\frac{1}{N} + \frac{b}{a - bN}\right) dN = \int dt$$

$$\frac{1}{a}[\ln N - \ln(a - bN)] = t + c.$$

Using initial condition, $t = 0, N = N_0$,

$$c = \frac{1}{a}[\ln N_0 - \ln(a - bN_0)]$$

$$\frac{1}{a}[\ln N - \ln(a - bN)] = t + \frac{1}{a}[\ln N_0 - \ln(a - bN_0)]$$

$$\frac{1}{a}\left[\ln\frac{N}{a - bN} - \ln\frac{N_0}{a - bN_0}\right] = t.$$

Simplification yields,

$$N = \frac{a\,N_0 e^{at}}{a + bN_0(e^{at} - 1)}.$$

Multiplying numerator and denominator by $\frac{1}{bN_0 e^{at}}$,

$$N = \frac{a/b}{\frac{a}{bN_0 e^{at}} + \frac{1}{e^{at}}(e^{at} - 1)}$$

$$N(t) = \frac{a/b}{1 + (\frac{a/b}{N_0} - 1)e^{-at}} \tag{4}$$

as $t \to \infty$, $(a > 0)$

$$N_{max} = \lim_{t\to\infty} N(t) = \frac{a}{b}. \tag{5}$$

To find the value of $\frac{a}{b}$, suppose that the values of N are N_1 and N_2 at time $t = 1$ and $t = 2$ respectively. From equation (4) we obtain,

$$N_1 = \frac{a/b}{1 + (\frac{a/b}{N_0} - 1)e^{-a}}$$

$$= \frac{1}{\frac{b(1-e^{-a})}{a} + \frac{1}{N_0}e^{-a}}$$

$$\frac{b}{a}(1-e^{-a}) = \frac{1}{N_1} - \frac{e^{-a}}{N_0}. \tag{6}$$

Similarly,

$$N_2 = \frac{a/b}{1+(\frac{a/b}{N_0}-1)e^{-2a}}$$

$$= \frac{1}{\frac{b(1-e^{-2a})}{a} + \frac{1}{N_0}e^{-2a}}$$

$$\frac{b}{a}(1-e^{-2a}) = \frac{1}{N_2} - \frac{e^{-2a}}{N_0}. \tag{7}$$

Dividing equation (7) by equation (6),

$$\frac{\frac{b}{a}(1-e^{-2a})}{\frac{b}{a}(1-e^{-a})} = \frac{\frac{1}{N_2} - \frac{e^{-2a}}{N_0}}{\frac{1}{N_1} - \frac{e^{-a}}{N_0}}.$$

Simplification yields,

$$1 + e^{-a} = \frac{\frac{1}{N_2} - \frac{e^{-2a}}{N_0}}{\frac{1}{N_1} - \frac{e^{-a}}{N_0}}$$

$$e^{-a} = \frac{N_0(N_2 - N_1)}{N_2(N_1 - N_0)} . \tag{8}$$

Equation (6) can also be written as

$$\frac{b}{a}(1-e^{-a}) = \frac{N_0 - N_1 e^{-a}}{N_1 N_0}$$

$$\frac{b}{a} = \frac{N_0 - N_1 e^{-a}}{N_1 N_0} \times \frac{1}{1-e^{-a}}.$$

By substituting the value of e^{-a} from equation (8), we get

$$\frac{b}{a} = \frac{N_0(N_1^2 - N_0 N_2)}{N_0 N_1(N_1 N_2 - 2N_0 N_2 + N_0 N_1)}$$

$$= \frac{N_1^2 - N_0 N_2}{N_1(N_1 N_2 - 2N_0 N_2 + N_0 N_1)}.$$

Thus the limiting value in equation (5) becomes

$$N_{max} = \frac{N_1(N_1 N_2 - 2N_0 N_2 + N_0 N_1)}{N_1^2 - N_0 N_2}. \tag{9}$$

3) ***Mean Absolute Percentage Error (MAPE):*** It is a measure of prediction accuracy as a percentage, and is defined by the formula:

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{A_t - P_t}{A_t}\right| \times 100\,\%$$

whereA_t is the actual value and P_t is the projected value. The absolute value is summed for every projected point in time and divided by the number of fitted point *n*. Multiplying by 100% makes it a percentage error. It can only be used based on the actual population and predicted population, so it cannot be calculated without actual population in projection process. In this paper, MAPE is used to compare the accuracy of two or more projection models. A model with smaller MAPE gives better forecast results than other models [8].

III. RESULTS AND DISCUSSION

The two mathematical models, exponential growth model and logistic growth model, are used to forecast future population of Kayah State. For exponential model, we need to determine the growth rate of Myanmar's population based on the actual population from CEIC data 2019 and carrying capacity and vital coefficients are required for logistic model.

A. Growth Rate for Exponential Growth Model

In this paper, the growth rate for exponential model is determined using with $t = 0$ in year 2008 as N_0 and the current year 2019 as N with $t = 11$ in equation (3). The respective population of these two years can be seen in Table I, i.e., $N_0 = 251417$and $N = 323573$.

$$r = \frac{1}{t}\ln\left(\frac{N}{N_0}\right)$$

$$= \frac{1}{11}\ln\left(\frac{323573}{251417}\right) = 0.0229374 = 2.29\%.$$

By substituting $N_0,\ r$ in equation (2), exponential growth model for projected population is

$$N(t) = N_0 e^{rt} = 251417\, e^{0.0229374\ t}.$$

By substituting the value of t, the projected populations can be obtained for respective year with the same average growth rate $r = 0.0229374$ year by year until 2050 and respective projection of population values can be seen in Table II.

B. Carrying Capacity for Logistic Growth Model

The maximum number of individuals that can be supported sustainably by a given environment is known as its ‘carrying capacity’.Carrying capacity for the human population of a state is the maximum numberof people who can live using the resources available from that state [9]. It is not a fixed number and it depends on available resources and per capita consumption [10]. If the number of people exceeds over carrying capacity, the population declines because its environment can no longer support the excess numbers [11]. Three consecutive populations are required to calculate the carrying capacity and in this work, we used$t = 0$ in year 2008 as N_0,$t = 1$ in year 2009 as N_1, and $t = 2$ in year 2010as N_2. The populations of these three years are 251417,257973, and 264530 respectively. The maximum number of population that people can live in Kayah State can be estimated based on above years using equation (9),

$$N_{max} = \frac{a}{b} = \frac{N_1(N_1N_2 - 2N_0N_2 + N_0N_1)}{N_1^2 - N_0N_2}$$

$$= \frac{2.211278 \times 10^{13}}{42729719}$$

$$= 517503 \quad \text{people}.$$

From equation (8),

$$e^{-a} = \frac{N_0(N_2 - N_1)}{N_2(N_1 - N_0)}$$

$$= \frac{251417(264530 - 257973)}{264530(257973 - 251417)}$$

$$= 0.950574.$$

The values of vital coefficients are

$$a = -\ln(0.95) = 0.05129 = 5.12\%$$

$$b = \frac{a}{517503} = \frac{0.05129}{517503} = 9.911689 \times 10^{-8}.$$

The value of a denotes the difference between birth rate and death rate. The smaller value of coefficient $b$implies the higher value of carrying capacity.

By substituting $N_0,\ e^{-a}$and$\frac{a}{b}$in equation (4), the logistic growth model for projected population is

$$N(t) = \frac{517503}{1 + \left(\frac{517503}{251417} - 1\right)(0.95)^t}.$$

By substituting the value of t, the projected populations can be obtained for respective years.

TABLE II
KAYAH STATE POPULATION (HISTORICAL AND PROJECTION)

Year	**CEIC data**		**Projected Population**	
	Actual	**Projection**	**Exponential growth Model**	**Logistic growth model**
2008	251417		251417	251417
2009	257973		257250	258051
2010	264530		263219	264686
2011	271088		269326	271313
2012	277647		275575	277924
2013	284207		281969	284509
2014	290768		288512	291062
2015	297162		295206	297572
2016	303646		302055	304032
2017	310214		309064	310435
2018	316858		316235	316773
2019	323573		323572	323038
2020		330356	331080	329223
2021		337195	338762	335323
2022		344089	346622	341331
2023		351034	354665	347242
2024		358028	362894	353050
2025		365066	371314	358750
2026		372139	379929	364338
2027		379243	388744	369811
2028		386370	397764	375164
2029		393513	406993	380396
2030		400663	416437	385502
2031		407057	426099	390482
2032		413541	435986	395334
2033		420109	446102	400056
2034		426503	456452	404648
2035		432987	467043	409109
2036		439555	477880	413439
2037		445949	488968	417638
2038		452433	500313	421707
2039		459001	511922	425646
2040		465395	523799	429458
2041		471879	535953	433142
2042		478447	548388	436702
2043		484841	561112	440138
2044		491325	574131	443453
2045		497893	587453	446648
2046		504287	601083	449727
2047		510771	615030	452691
2048		517339	629300	455544
2049		523733	643901	458287
2050		530217	658841	460924
MAPE			5.92%	4.41%

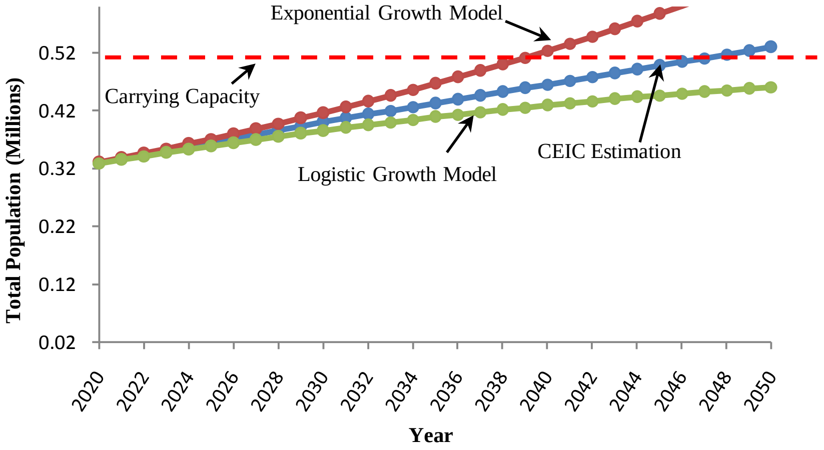

Fig. 2 Comparison of population projection values from exponential growth model, logistic growth model, and CEIC data.

C. Comparison and Recommendation

Two mathematical models have been applied to forecast the population of Kayah State from 2020 to 2050. Census and Economic Information Center (CEIC)forecasts results are used for reference data. Mean Absolute Percentage Error (MAPE) equation has been used to compare the accuracy of growth models by using actual populations and predicted population values. Moreover, the projected populations from CEIC and two mathematical models have also been compared visually by using graph. According to the percentages of MAPE from the last row of table II, logistic growth model gives the more accurate projection results than exponential growth model. Based on the above comparison of results, we recommend that the logistic growth model has a good projection result as compared to the exponential growth model for some period of years. Kayah State is the smallest state in area and has the lowest population in Myanmar. Furthermore, Kayah State is one of the developing States and the State government should try to improve carrying capacity of population of the state. In this case, industrialization of the state is one of the best ways to increase carrying capacity. The more industrialized a country is the more living space and food it has, thus raising the carrying capacity.

IV. CONCLUSIONS AND FUTURE WORKS

Based on the actual populations from the 2014 Myanmar census, estimated populations from CEIC dataand forecasted populations from 2020 to 2050 by the two growth models have been projected and visualized by using XY scatter graphs. The exponential growth model forecasted a growth rate of approximately 2.29% and projected Kayah State's population to 658841 in the year 2050 with a MAPE 5.92% while logistic growth model projected the population of Kayah State to 460924 in the year 2050 with a MAPE 4.41% and carrying capacity 517503. Based on this model we also find out that the population of Kayah State is expected to reach its maximum limit (carrying capacity) in the year 2100. The predicted results of CEIC data estimated the Kayah State's population to 530217 in the year 2050. According to the MAPE precents, we can conclude that logistic growth model gives more accurate results than exponential growth model. The population density of Kayah State will be 40 people per squarekilometre in 2050. The applications of these models can also be extended to predict the growth rates of some population dynamics such as colonies of bacteria, animal populations, and other biological species with constrained or unconstrained resources. Decaying of radioactive substances and other monotonically decreasing processes can also be estimated by exponential decay model [12]. In economic sector, these models can also be applied to estimate the simple or compound interest of investments and other sectors with dynamic populations such as education, industrial, policy, culture, and environmental management and demographic development etc.

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