gW– Hausdorffness in Soft topological Spaces

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Abstract: *In this Paper gW- Hausdorffness in soft topological spaces is introduced.*

Keywords : *Soft set, Soft topological Space, Soft gW-Hausdorff space, Soft generalized open set.*

1. INTRODUCTION

Most of the real life problems have various uncertainties .A number of theories have been proposed for dealing with uncertainties in an efficient way. In 1999, Molodstov [8] initiated a novel concept of soft set theory ,which is completely a new approach for modeling vagueness and uncertainity. In 2011, Shabir and Naz [9] defined soft topological spaces and studied seperation axioms. Many Mathematicians threw light on soft separation axioms in soft topological spaces at full length with respect to soft open set, generalized soft open set,soft b-open set, soft semi-open set, soft α - open set and soft β-open set. In this paper gW- Hausdorffness in soft topological spaces is introduced by using generalized soft open sets. In section II of this paper , preliminary definitions regarding soft sets and soft topological spaces are given .In section III of this paper , the concept of gW- Hausdorffness in soft topological spaces is introduced and studied.

Throughout this paper, X denotes initial Universe and E denotes the set of parameters for the universe X.

2.PRELIMINARY DEFINITIONS

Definition: 2.1 [8]

Let X be an initial universe and E be the set of parameters. Let P(X) denotes the power set of X and A be a nonempty subset of E. A pair (F, A) denoted by F_A is called a **soft set** over X, where F is a mapping given by F: A → P(X).The family of all soft sets over X with respect to the parameter set E is denoted by $SS(X)_E$.

Definition 2.2[7]

Let F_A , $G_B \in SS(X)_E$. Then F_A is **soft subset** of G_B , denoted by $F_A \subseteq G_B$, if

(1) $A \subseteq B$, and

(2) $F(e) \subseteq G(e)$, $\forall e \in A$.

In this case, F_A is said to be a soft subset of G_B and

G_B is said to be a soft superset of F_A , denoted as $G_B \supseteq F_A$

Definition 2.3 [7]

Two soft subsets F_A and G_B over a common universe X are said to be **soft equal** if F_A is a soft subset of G_B and G_B is a soft subset of F_A.

Definition 2.4 [7]

The complement of a soft set (F, A) denoted by (F, A)′ is defined by (F, A)′ = (F′, A) , F′ : A ⟶ P(X) is a mapping given by F′(e) =X − F(e); ∀ e ∈ A and F′ is called the **soft complement** function of F. Clearly (F′)′ is the same as F and ((F, A)′)′= (F, A).

Definition 2.5 [7]

A soft set (F, A) over X is said to be a **Null soft set** denoted by $\tilde{\phi}$ or ϕ_A if for all e ∈ A, F(e) = ϕ (null set).

Definition 2.6 [7]

A soft set (F, A) over X is said to be an **absolute soft set** denoted by $\tilde{A}$ or X_A if for all e ∈ A, F(e) = X. Clearly we have $X_A{}' = \phi_A$ and $\phi'_A = X_A$.

Definition 2.7 [7]

The **union** of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where C = A ∪ B and for all e ∈ C,

$$H(e) = \begin{cases} F(e), e \in A - B \\ G(e), e \in B - A \\ F(e) \cup G(e), e \in A \cap B \end{cases}$$

Definition 2.8 [7]

The **intersection** of two soft sets (F, A) and (G, B) over the common universe X is the soft set (H, C), where $C = A \cap B$ and for all $e \in C$, $H(e) = F(e) \cap G(e)$.

Definition 2.9 [9]

Let (X, τ, E) be a soft topological space, $(F, E) \in SS(X)_E$ and Y be a non null subset of X. Then the soft subset of (F, E) over Y denoted by (F_Y, E) is defined as follows:

$F_Y(e) = Y \cap F(e), \forall e \in E$

In other words, $(F_Y, E) = Y_E \cap (F, E)$.

Definition 2.10 [9]

Let τ be the collection of soft sets over X, then τ is said to be a **soft topology** on X, if

(1) $\phi, X \in \tau$

(2) the union of any number of soft sets in τ belongs to τ

(3) the intersection of any two soft sets in τ belongs to τ

The elements of τ are called **soft open sets**.

The complement of a soft open set is called **soft closed set**.

Definition 2.11 [9]

Let (X, τ, E) be a soft topological space and Y be a non null subset of X. Then $\tau_Y = \{(F_Y, E): (F, E) \in \tau\}$ is said to be the **relative soft topology** on Y and (Y, τ_Y, E) is called a soft subspace of (X, τ, E).

Definition 2.12 [3]

Let $F_A \in SS(X)_E$ and $G_B \in SS(Y)_K$. The **cartesian product** $F_A \otimes G_B$ is defined by $(F_A \otimes G_B)(e, k) = F_A(e) \times G_B(k), \forall (e, k) \in A \times B$. According to this definition $F_A \otimes G_B$ is a soft set over X×Y and its parameter set is $E \times K$.

Definition 2.13[3]

Let (X, τ_X, E) and (Y, τ_y, K) be two soft topological spaces. The **soft product topology** $\tau_X \otimes \tau_Y$ over X×Y with respect to $E \times K$ is the soft topology having the collection $\{ F_E \otimes G_K / F_E \in \tau_x, G_K \in \tau_y \}$ as the basis.

Definition 2.14[10]

A soft topological space (X, τ, E) is said to be **soft W-Hausdorff space of type 1** denoted by $(SW-H)_1$ if for every $e_1, e_2 \in E$, $e_1 \neq e_2$, there exists $F_A, G_B \in \tau$ such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Definition 2.15 [10]

A soft topological space (X, τ, E) is said to be **soft W-Hausdorff space of type 2** denoted by $(SW-H)_2$ if for every $e_1, e_2 \in E$, $e_1 \neq e_2$, there exists $F_E, G_E \in \tau$ such that $F_E(e_1) = X$, $G_E(e_2) = X$ and $F_E \cap G_E = \tilde{\phi}$

Definition 2.16 [6]

A soft set F_A is called a **soft generalized closed** (soft g-closed) in a soft topological space (X, τ, E) if $Cl(F_A) \subseteq G_B$ when ever $F_A \subseteq G_B$ and G_B is soft open in X.

Definition 2.17 [6]

A soft set F_A is called a **Soft generalized open** (soft g- open) in a soft topological space (X, τ, E) if the relative complement F_A' is soft g- closed in X.

Equivalently, a soft set F_A is called a **Soft generalized open** set (soft g-open) in a soft topological space (X, τ, E) if and only if $F_A \subseteq int(G_B)$ whenever $F_A \subseteq G_B$ and G_B is soft closed in X.

3. SOFT gW- HAUSDORFF SPACES

Definition 3.1

A soft topological space (X, τ, E) is said to be **soft gW – Hausdorff space of type 1** denoted by $(sgW-H)_1$ if for every $e_1, e_2 \in E$, $e_1 \neq e_2$,there exists soft g-open sets F_A, G_B such that $F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Theorem 3.2

Soft subspace of a $(sgW\text{-}H)_1$ space is $(sgW\text{-}H)_1$.

Proof:

Let (X, τ, E) be a $(sgW\text{-}H)_1$ space. Let Y be a non null subset of X. Let (Y, τ_y, E) be a soft subspace of X. Let (X, τ, E) where $\tau_y = \{(F_Y, E) : (F, E) \in \tau\}$ is the relative soft topology on Y. Consider $e_1, e_2 \in E$, $e_1 \neq e_2$ there exists soft g- open sets F_A, G_B, such that

$F_A(e_1) = X$, $G_B(e_2) = X$

Here $(F_A)_Y$, $(G_B)_Y$ are soft g – open sets .

Also $(F_A)_Y(e_1) = Y \cap F_A(e_1)$

$= Y \cap X$

$= Y$

$(G_B)_Y(e_2) = Y \cap G_B(e_2)$

$= Y \cap X$

$= Y$

$((F_A)_Y \cap (G_B)_Y)(e) = ((F_A \cap G_B)_Y)(e)$

$= Y \cap (F_A \cap G_B)(e)$

$= Y \cap \tilde{\phi}(e)$

$= \phi$

$(F_A)_Y \cap (G_B)_Y = \tilde{\phi}$

Hence (Y, τ_y, E) is $(sgW-H)_1$.

Theorem 3.3

Product of two $(sgW\text{-}H)_1$ spaces is $(sgW\text{-}H)_1$.

Proof:

Let (X, τ_x, E) and (Y, τ_y, K) be two spaces. Consider two distinct points $(e_1, k_1), (e_2, k_2) \in E \times K$.
Either $e_1 \neq e_2$ or $k_1 \neq k_2$.
Assume $e_1 \neq e_2$. Since (X, τ_x, E) is $(sgW\text{-}H)_1$, there exist g -soft open sets F_A, G_B such that
$F_A(e_1) = X$, $G_B(e_2) = X$ and $F_A \cap G_B = \tilde{\phi}$.

Here $F_A \otimes Y_K$, $G_B \otimes Y_K$ are soft g -open sets

$(F_A \otimes Y_K)(e_1, k_1) = F_A(e_1) \times Y_K(k_1)$
$= X \times Y$

$(G_B \otimes Y_K)(e_2, k_2) = G_B(e_2) \times Y_K(k_2)$
$= X \times Y$

If for any $(e, k) \in (E \times K)$, $(F_A \otimes Y_K)(e, k) \neq \phi$

$\Rightarrow F_A(e) \times Y_K(k) \neq \phi$

$\Rightarrow F_A(e) \times Y \neq \phi$

$\Rightarrow F_A(e) \neq \phi$

$\Rightarrow G_B(e) = \phi$ (since $F_A \cap G_B = \phi \Rightarrow F_A(e) \cap G_B(e) = \phi$)

$\Rightarrow G_B(e) \times Y_K(k) = \phi$

$\Rightarrow (G_B \otimes Y_K)(e, k) = \phi$

$\Rightarrow (F_A \otimes Y_K) \cap (G_B \otimes Y_K) = \tilde{\phi}$

Assume $k_1 \neq k_2$. Since (Y, τ_y, K) is $(sgW\text{-}H)_1$, there exist soft g -open sets F_A, G_B, such that

$F_A(k_1) = Y$, $G_B(k_2) = Y$ and $F_A \cap G_B = \tilde{\phi}$.

Here $X_E \otimes F_A$, $X_E \otimes G_B$ are soft g - open sets

$(X_E \otimes F_A)(e_1, k_1) = X_E(e_1) \times F_A(k_1)$
$= X \times Y$

$(X_E \otimes G_B)(e_2, k_2) = X_E(e_2) \times G_B(k_2)$
$= X \times Y$

If for any $(e, k) \in E \times K$, $(X_E \otimes F_A)(e, k) \neq \phi$

$\Rightarrow X_E(e) \times F_A(k) \neq \phi$

$\Rightarrow X \times F_A(k) \neq \phi$

$\Rightarrow F_A(k) \neq \phi$

$\Rightarrow G_B(k) = \phi$ (Since $F_A \cap G_B = \tilde{\phi} \Rightarrow F_A(k) \cap G_B(k) = \phi$)

$\Rightarrow (X_E)(e) \times G_B(k) = \phi$

$\Rightarrow (X_E \otimes G_B)(e, k) = \phi$

$\Rightarrow (X_E \otimes F_A) \cap (X_E \otimes G_B) = \tilde{\phi}$

Hence $(X \times Y, \tau_X \otimes \tau_y, E \times K)$ is $(sgW\text{-}H)_1$.

Definition 3.4

A soft topological space (X, τ, E) is said to be **soft gW-Hausdorff space of type 2** denoted by $(sgW\text{-}H)_2$ if for every $e_1, e_2 \in E$, $e_1 \neq e_2$ there exists soft g-open sets F_E , G_E such that $F_E(e_1) = X$, $G_E(e_2) = X$ and $F_E \cap G_E = \tilde{\phi}$

Theorem 3.5

Soft subspace of a $(sgW\text{-}H)_2$ space is $(sgW\text{-}H)_2$.

Proof :

Let (X, τ, E) be a $(sgW\text{-}H)_2$ space. Let Y be a non-null subset of X. Let (Y, τ_y, E) be a soft subspace of (X, τ, E) where $\tau_y = \{ (F_Y, E) : (F, E) \in \tau\}$ is the relative soft topology on Y. Consider $e_1, e_2 \in E$, $e_1 \neq e_2$ there exist soft g-open sets F_E, G_E, such that $F_E(e_1) = X$, $G_E(e_2) = X$ and $F_E \cap G_E = \tilde{\phi}$.

Here $((F_E)_Y, E)$, $((G_E)_Y, E)$ are soft g-open sets.

Also $(F_E)_Y (e_1) = Y \cap F_E (e_1)$
$= Y \cap X$
$= Y$

$(G_E)_Y (e_2) = Y \cap G_E (e_2)$
$= Y \cap X$
$= Y$

$((F_E)_Y \cap (G_E)_Y) (e) = ((F_E \cap G_E)_Y) (e)$
$= Y \cap (F_E \cap G_E) (e)$
$= Y \cap \tilde{\phi} (e)$
$= Y \cap \phi$
$= \phi$

$(F_A)_Y \cap (G_B)_Y = \tilde{\phi}$

Hence (Y, τ_y, E) is $(sgW\text{-}H)_2$.

Theorem 3.6

Product of two $(sgW\text{-}H)_2$ spaces is $(sgW\text{-}H)_2$.

Proof:

Let (X, τ_X, E) and (Y, τ_y, K) be two $(sgW\text{-}H)_2$ spaces. Consider two distinct points $(e_1, k_1), (e_2, k_2) \in E\times K$.

Either $e_1 \neq e_2$ or $k_1 \neq k_2$.

Assume $e_1 \neq e_2$. Since (X, τ_X, E) is $(sgW\text{-}H)_2$, there exist soft g-open sets F_E, G_E, such that $F_E (e_1) = X$, $G_E (e_2) = X$ and $F_E \cap G_E = \tilde{\phi}$.

Here $F_E \otimes Y_K$, $G_E \otimes Y_K$ are soft g-open sets

$(F_E \otimes Y_K) (e_1, k_1) = F_E (e_1) \times Y_K (k_1)$
$= X \times Y$

$(G_E \otimes Y_K) (e_2, k_2) = G_E (e_2) \times Y_K (k_2)$
$= X \times Y$

If for any $(e, k) \in (E\times K)$, $(F_E \otimes Y_K) (e, k) \neq \phi$

$\Rightarrow F_E (e) \times Y_K (k) \neq \phi$

$\Rightarrow F_E (e) \times Y \neq \phi$

$\Rightarrow F_E (e) \neq \phi$

$\Rightarrow G_E (e) = \phi$ (Since $F_E \cap G_E = \tilde{\phi} \Rightarrow F_A (e) \cap G_E (e) = \phi$)

$\Rightarrow G_E (e) \times Y_K (k) = \phi$

$\Rightarrow (G_E \otimes Y_K) (e, k) = \phi$

$\Rightarrow (F_E \otimes Y_K) \cap (G_E \otimes Y_K) = \tilde{\phi}$

Similarly, one can prove the case when $k_1 \neq k_2$.

Hence $(X \times Y, \tau_X \otimes \tau_y, E \times K)$ is $(sgW\text{-}H)_2$.

4. CONCLUSION

In this paper the concept of gW –Hausdorffness in soft topological spaces is introduced and some basic properties regarding this concept are studied .

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