Binary Regular ^ Generalized Continuous And Irresolute Functions

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ABSTRACT: *The authors [9] introduced the concept of binary regular ^ generalized closed sets in binary topological spaces and studied its basic properties. In this paper we introduce the concept of binary regular ^ generalized continuous function, totally and strongly binary regular^generalized continuous function and study the relationship with other sets.*

KEYWORDS: *Binary r^g-continuous (μ_br^g-continuous) function, strongly and totally μ_br^g-continuous functions and μ_br^g-irresolute functions.*

1. INTRODUCTION

The concept of regular continuous functions was first introduced by Arya. S.P. and Gupta.R.[1]. Later Palaniappan. N. and Rao.K.C[7] studied the concept of regular generalized continuous functions. Recently the authors S. Nithyanantha Jothi and P. Thangavelu[4] introduced the concept of binary topology between two sets and investigate some of the basic properties, where a binary topology from X to Y is a binary structure satisfying certain axioms that are analogous to the axioms of topology. Throughout the paper P(X) represents the power set of X.

The purpose of this paper is to introduce the concept of binary regular ^ generalized continuous functions and study their relationship. Section 2 deals with the basic concepts. Binary r^g continuity is discussed in section 3. Section 4 deals with binary r^g irresolute functions.

2. PRELIMINARIES

Definition 2.1[3]: Let X and Y be any two non empty sets. A binary generalized topology from X to Y is a binary structure $\mu_b \subseteq P(X) \times P(Y)$ that satisfies the following axioms:

(i) $(\phi,\phi) \in \mu_b$ and $(X,Y) \in \mu_b$.

(ii) $(A_1 \cap A_2, B_1 \cap B_2) \in \mu_b$ whenever (A_1,B_1) and $(A_2,B_2) \in \mu_b$

(iii) If $\{(A_\alpha,B_\alpha) : \alpha \in \Delta\}$ is a family of members of μ_b, then $(\cup A_\alpha, \cup B_\alpha) \in \mu_b$.

If μ_b is a binary generalized topology from X to Y then the triplet (X, Y, μ_b) is called a binary generalized topological space and the members of μ_b are called binary generalized open sets.

The compliment of an element of $P(X) \times P(Y)$ is defined component wise. That is the binary compliment of (A, B) is (X – A, Y – B).

Definition 2.2[3]: Let (X, Y, μ_b) be a binary generalized topological space and $A \subseteq X$, $B \subseteq Y$. Then (A, B) is called binary generalized closed if (X – A, Y – B) is binary generalized open.

Definition 2.3[3]: Let (A,B), (C,D) $\in P(X) \times P(Y)$. Then

(i) $(A, B) \subseteq (C, D)$ if $A \subseteq C$ and $B \subseteq D$.

(ii) $(A, B) \cup (C, D) = (A \cup C, B \cup D)$.

(iii) $(A, B) \cap (C, D) = (A \cap C, B \cap D)$.

Definition 2.4[3]: Let (X, Y, μ_b) be a binary generalized topological space and $(x, y) \in X \times Y$, then a subset (A, B) of (X, Y) is called a binary generalized neighbourhood of (x, y) if there exists a binary generalized open set (U, V) such that $(x, y) \in (U, V) \subseteq (A, B)$.

Definition 2.5[3]: Let (X, Y, μ_b) be a binary generalized topological space, $(A,B) \subseteq (X,Y)$.
(i) $(A, B)^{1^\circ} = \cup\{A_\alpha : (A_\alpha, B_\alpha)$ is binary generalized open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$
(ii) $(A, B)^{2^\circ} = \cup\{B_\alpha : (A_\alpha, B_\alpha)$is binary generalized open and $(A_\alpha, B_\alpha) \subseteq (A, B)\}$.
Then the pair $((A, B)^{1^\circ},(A, B)^{2^\circ})$ is called the binary generalized interior of (A,B) and denoted by $\mu_b Int(A,B)$.

Remark 2.6[3]: The set (A,B) is binary generalized open in (X, Y, μ_b) if and only if $\mu_b Int(A,B) = (A,B)$.

Definition 2.7[3]: Let (X, Y, μ_b) be a binary generalized topological space, $(A, B) \subseteq (X,Y)$.
(i) $(A, B)^{1^*} = \cap\{A_\alpha :(A_\alpha, B_\alpha)$is binary generalized closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$
(ii) $(A,B)^{2^*} = \cap\{B_\alpha :(A_\alpha, B_\alpha)$is binary generalized closed and $(A, B) \subseteq (A_\alpha, B_\alpha)\}$.
Then the pair $((A, B)^{1^*},(A, B)^{2^*})$ is called the binary generalized closure of (A,B) and denoted by $\mu_b Cl(A,B)$.

Remark 2.8[3]: The set (A,B) is binary generalized closed in (X, Y, μ_b) if and only if $\mu_b Cl(A,B) = (A,B)$.

Definition 2.9[3]: A subset (A,B) of a binary topological space is said to be clopen if it is both open and closed.

Definition 2.10[3]: A subset (A,B) of topological space (X, Y, μ_b) is called a

(i) $\mu_b$semiclosed set if $\mu_b Int(\mu_b Cl(A,B)) \subseteq (A,B)$.

(ii) $\mu_b$semipreclosed set if $\mu_b Int(\mu_b Cl (\mu_b Int (A,B))) \subseteq (A,B)$.

(iii) $\mu_b$gclosed set if $\mu_b Cl(A,B) \subseteq (U,V)$ whenever $(A,B) \subseteq (U,V)$ and (U,V) is open in (X, Y, μ_b).

(iv) μ_bg*closed set $\mu_b Cl(A,B) \subseteq (U,V)$ whenever $(A,B) \subseteq (U,V)$ and (U,V) is μ_b g-open in (X, Y, μ_b).

(v) μ_br^gclosed set if $\mu_b gCl(A,B) \subseteq (U,V)$ whenever $(A,B) \subseteq (U,V)$ and (U,V) is binary regular open in (X, Y, μ_b).

Definition 2.11[3]: Let (Z,η) be a topological space and (X,Y, μ_b) be a binary topological space. Then the map $f:Z \to X\times Y$ is called a binary continuous function if $f^{-1}(A,B)$ is open(closed) in (Z,η) for every open(closed) set (A,B) in (X, Y, μ_b).

Definition 2.12[3]: Let (Z,η) be a topological space and (X,Y, μ_b) be a binary topological space. Then the map $f:Z \to X\times Y$ is called a

(i) totally binary continuous function if $f^{-1}(A,B)$ is clopen in (Z,η) for every binary open set (A,B) in (X, Y, μ_b).

(ii) strongly binary continuous function if $f^{-1}(A,B)$ is clopen in (Z,η) for every binary set (A,B) in (X, Y, μ_b).

Definition 2.13[8]: A binary topological space (X,Y,μ_b) is said to be a $\mu T_{1/2}$ space if every $\mu_b$closed set is $\mu_b$gclosed.

Definition 2.14[3]: Let (Z,η) be a topological space and (X,Y, μ_b) be a binary topological space. Then the map $f:Z \to X\times Y$ is called

(i) a binary g-continuous function if $f^{-1}(A,B)$ is gclosed in (Z,η) for every binary closed set (A,B) in (X, Y, μ_b).

(ii) a binary g*-continuous function if $f^{-1}(A,B)$ is g*closed in (Z,η) for every binary closed set (A,B) in (X, Y, μ_b).

(iii) a binary semicontinuous function if $f^{-1}(A,B)$ is semiclosed in (Z,η) for every binary closed set (A,B) in (X, Y, μ_b).

(iv) a binary semiprecontinuous function if $f^{-1}(A,B)$ is semipreclosed in (Z,η) for every binary closed set (A,B) in (X, Y, μ_b).

(v) a binary α-continuous function if $f^{-1}(A,B)$ is αclosed in (Z,η) for every binary closed set (A,B) in (X, Y, μ_b).

3. BINARY REGULAR ^ GENERALIZED CONTINUOUS FUNCTIONS

In this section we introduce binary regular^generalized continuous function and study its relationship with other binary continuous functions.

Definition 3.1: Let (Z,η) be a topological space and (X,Y, μ_b) be a binary topological space. Then the map f: Z → X×Y is called a **binary regular^generalized continuous (shortly r^g-continuous) function** at a point z∈Z, if for any binary generalized open set (U,V) in (X, Y, μ_b) with f(z) ∈ (U,V) there exists a generalized open set G in (Z,η) such that z∈G and f(G) ⊆(U,V). f is called binary r^g-continuous if it is r^g continuous at each z ∈Z.

Definition 3.2: Let (Z,η) be a topological space and (X,Y, μ_b) be a binary topological space. Then the map f: Z → X×Y is called a **binary regular^generalized continuous (shortly r^g-continuous) function** if $f^{-1}(A,B)$ is r^g closed in (Z,η) for every closed set (A,B) in (X,Y, μ_b).

Theorem 3.3: Every (i) binary continuous

(ii) binary g-continuous

(iii) binary g*-continuous function is binary r^g-continuous function.

Proof: Straight forward [9].

Remark 3.4: The converse of the above theorem need not be true as shown in the following example.

Example 3.5: Consider Z = {a,b,c}, X = {x_1,x_2} and Y = {y_1,y_2}. Let η = {ϕ,{b},{a,b},Z} and μ_b = {(ϕ,ϕ),({x_2},{y_2}),(X,Y)}. Clearly η is a topology on Z and μ_b is a binary topology from X to Y. The closed sets of (Z,η) are {ϕ,{c},{a,c},Z}. Define f: Z → X×Y by f(a) = ({x_1},{y_1}) and f(b) = ({x_2},{y_2}) = f(c). Now f is a binary r^g-continuous function but it is not a binary continuous, binary g-continuous and binary g*-continuous function since f^{-1}({x_1},{y_1}) = {a} is not a closed, gclosed and g*closed set in (Z,η).

Remark 3.6: The concepts of binary semicontinuous, binary semiprecontinuous are independent to the concept of binary r^g-continuous function as shown in the following example.

Example 3.7: * Let Z = {1,2,3}, X ={x_1,x_2} and Y = {y_1,y_2}. Let η = {ϕ,{1},{3}, {1,2},{1,3},{2,3},Z} and μ_b = {(ϕ,ϕ), ({x_1},{y_1}),({x_2},{y_2}),(X,Y)}. Clearly η is a topology on Z and μ_b is a binary topology from X to Y. The binary closed sets are {(ϕ,ϕ), ({x_2},{y_2}), ({x_1},{y_1}),(X,Y)}.Define f: Z → X×Y as f(1) = ({x_1},{y_1}) = f(3) and f(2) = ({x_2},{y_2})Then f is a binary r^g-continuous function but it is not a binary semicontinuous and binary semiprecontinuous since the inverse image of ({x_1},{y_1}) is {1,3} is not a semiclosed and semipreclosed sets in (Z,η).

* Let Z = {a,b,c}, X ={x_1,x_2} and Y = {y_1,y_2}. Let η = {ϕ,{a},{b},{a,b},Z} and μ_b = {(ϕ,ϕ), ({x_1},{y_1}),({x_2},{y_2}),(X,Y)}. Clearly η is a topology on Z and μ_b is a binary topology from X to Y. Define f: Z → X×Y as f(a)= ({x_1},{y_1}), f(b)= ({x_2},{y_2}) and f(c) = ϕ then f is a binary semicontinuous and binary semiprecontinuous but it is not a binary r^g-continuous function since f^{-1}({x_2},{y_2}) = {b} is both binary semiclosed and binary semipreclosed sets but it is not a binary r^g closed set in (Z,η).

Remark 3.8: The concept of r^g-continuous function is independent to the concept of α-continuous function.

Example 3.9: * Let Z = {a,b,c}, X ={x_1,x_2} and Y = {y_1,y_2}. Let η = {ϕ,{a},{b},{a,b},Z} and μ_b = {(ϕ,ϕ), ({x_1},{y_2}),(X,Y)}. Clearly η is a topology on Z and μ_b is a binary topology from X to Y. Define f: Z → X×Y as f(a)= ({x_2},{y_1}) = f(b), f(c) = (ϕ,ϕ) then f is a binary r^g-continuous function but it is not a binary α-continuous function since the inverse image of ({x_2},{y_1}) = {a,b} is r^g closed set but it is not an αclosed set in (Z,η).

*****Let Z = {a,b,c,d}, X ={x_1,x_2} and Y = {y_1,y_2}. Let η = {ϕ,{a},{c},{a,c},{c,d}, {a,c,d},Z} and μ_b = {(ϕ,ϕ), ({x_2},{y_2}),(X,Y)}. Clearly η is a topology on Z and μ_b is a binary topology from X to Y. Define f: Z → X×Y as f(a) = (ϕ,ϕ) = f(b), f(c) = (ϕ,{y_2}) f(d) = ({x_1},{y_1}), then f is a binary α-continuous but it is not a binary r^g-continuous since the inverse image of $(\{x_1\},\{y_1\}) = \{d\}$ is an αclosed set but it is not a r^g closed set in (Z,η).

Definition 3.10: Let (Z,η) be a topological space and (X,Y, μ_b) be a binary topological space. Then the map f: Z → X×Y is called a

(i) totally binary r^g-continuous function if $f^{-1}(A,B)$ is r^g-clopen in (Z,η) for every binary closed set (A,B) in (X, Y, μ_b).

(ii) strongly binary r^g-continuous function if $f^{-1}(A,B)$ is r^g-clopen in (Z,η) for every binary set (A,B) in (X, Y, μ_b).

Example 3.11: Let Z = {1,2,3}, $X = \{x_1,x_2\}$ and $Y = \{y_1,y_2\}$. Let η = {ϕ,{1},{3}, {1,2},{1,3},{2,3},Z} and $\mu_b = \{(\phi,\phi), (\{x_1\},\{y_1\}),(\{x_2\},\{y_2\}),(X,\{y_1\}),(X,Y)\}$. Clearly η is a topology on Z and μ_b is a binary topology from X to Y. The binary closed sets are $\{(\phi,\phi), (\{x_2\},\{y_2\}), (\{x_1\},\{y_1\}),(X,Y)\}$.Define f: Z → X×Y as f(1) = (X,ϕ) = f(3) and $f(2) = (\phi,\{y_2\})$. In (Z,η) all the subsets of Z are r^g closed sets. Hence all the sets are both r^g closed and r^g open sets, i.e.,r^g clopen sets. Thus f is both totally binary r^g continuous function and strongly binary r^g continuous function.

Theorem 3.12: Every strongly binary r^g-continuous function is totally binary r^g-continuous function.

Proof: Straight forward from the definition 3.10.

Remark 3.13: The converse of the above theorem need not be true as seen in the following example.

Example 3.14: Let Z = {a,b,c}, $X = \{x_1,x_2\}$ and $Y = \{y_1,y_2\}$. Let η = {ϕ,{a},{b},{a,b},Z} and $\mu_b = \{(\phi,\phi), (\{x_1\},\{y_1\}),(X,Y)\}$. Clearly η is a topology on Z and μ_b is a binary topology from X to Y. Define f: Z → X×Y as $f(a)= (\{x_1\},\{y_1\})$, f(b) = (ϕ,ϕ) ,$f(c)= (\{x_2\},\{y_2\})$. Then f is totally binary r^g-continuous function but it is not strongly binary r^g continuous since the inverse image of $(\{x_1\},\{y_1\}) = \{a\}$ is not r^g clopen in (Z,η).

Theorem 3.15: Let (X,Y, μ_b) be a binary generalized topological space and (Z,η) be a generalised topological space. Let f: Z → X×Y be a function such that $Z - f^{-1}(A,B) = f^{-1}(X-A,Y-B)$ for all $A \subseteq X$ and $B \subseteq Y$. Then f is binary regular^generalized continuous (r^g-continuous) if and only if $f^{-1}(A,B)$ is r^g closed in (Z,η) for every binary closed set (A,B) in (X,Y, μ_b).

4. BINARY REGULAR ^ GENERALIZED IRRESOLUTE FUNCTIONS

Definition 4.1: A function f: Z → X×Y is said to be a binary regular^generalized-irresolute **(shortly μ_br^g-irresolute)** function if $f^{-1}(A,B)$ is r^g closed in (Z,η) for every binary r^g closed set (A,B) in (X,Y, μ_b).

Example 4.2: Let Z = {1,2,3}, $X = \{x_1,x_2\}$ and $Y = \{y_1,y_2\}$. Let η = {ϕ,{1,2},{2,3},Z} and $\mu_b = \{(\phi,\phi), (\{x_1\},\{y_1\}),(\{x_2\},\{y_2\}) ,(X,Y)\}$. Clearly η is a topology on Z and μ_b is a binary topology from X to Y. Define f: - Z → X×Y by $f(1) = (\{x_1\},\{y_1\}) = f(2)$ and f(3) = (ϕ,ϕ). Then f is a binary r^g-irresolute function.

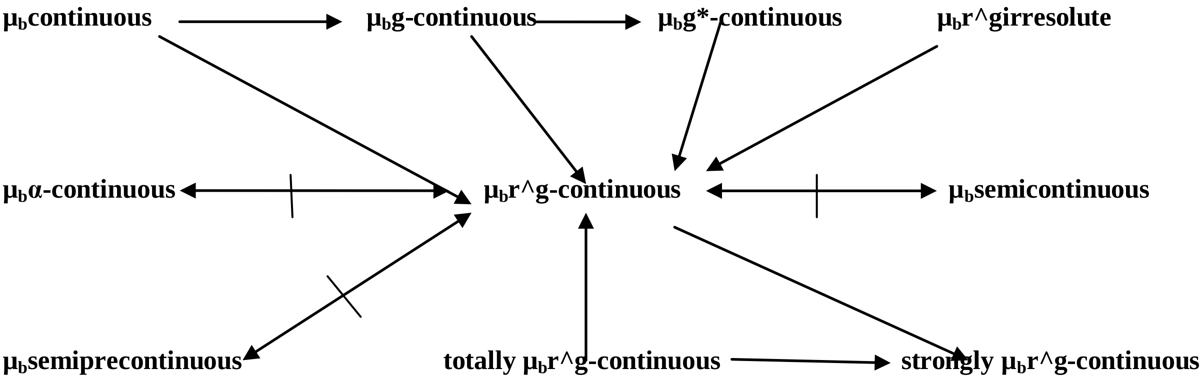
Theorem 4.3: Every binary r^g-irresolute function is binary r^g-continuous function.

Proof: Straight forward from the fact that every binary closed set is binary r^g closed set.

Remark 4.4: The converse of the above theorem need not be true as seen in the following example.

Example 4.5: Let Z = {a,b,c}, $X = \{x_1,x_2\}$ and $Y = \{y_1,y_2\}$. Let η = {ϕ,{a},{b},{a,b},Z} and $\mu_b = \{(\phi,\phi), (\{x_1\},\{y_1\}), (\{x_2\},\{y_2\}) ,(X,Y)\}$. Clearly η is a topology on Z and μ_b is a binary topology from X to Y. Define f: Z → X×Y by $f(a) = (\{x_2\},\{y_1\})$, $f(b) = (\{x_1\},\{y_2\}) =$ f(c) , then f is binary r^g-continuous function since $f^{-1}(\{x_1\},\{y_1\}) = f^{-1} (\{x_2\},\{y_2\}) = \phi$ which is r^g closed in Z but it is not a binary r^g-irresolute function since the inverse image of a binary r^g closed set $(\{x_2\},\{y_2\}) = \{a\}$ which is not an r^g closed set in (Z,η).

The above discussions are implemented in the following diagram.



where A ⟶ B represents A implies B but not conversely and A ⟷ B represents A and B are independent.

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