Nano contra semi c(s) generalized continuity in Nano Topological Spaces

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Abstract: *The purpose of this paper is to introduce and study the stronger form of continuity called Nano contra semi c(s) generalized continuity in Nano topological spaces. Some of the properties of Nano contra semi c(s) - generalized continuous function are analyzed. Also its relation with other nano contra continuous functions are investigated.*

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I. Introduction

Continuous function is one of the main function in topology. Different types of continuous functions were studied by various authors in the recent development of topology. M.Lellis Thivagar and Carmel Richard [7] introduced nano topological space with respect to a subset X of a universe which is defined in terms of lower and upper approximations of X. The elements of Nano topological space are called Nano open sets. He has defined nano closed sets, nano interior and nano closure of a set in nano topological space. He [6] has also introduced a nano continuous function, nano open mappings, nano closed mappings and nano homeomorphisms in nano topological space.

In this paper we have introduced a new class of Continuous functions called Nano contra semi c(s) generalized continuous functions and obtain some characterizations in nano topological spaces.

Throughout this paper (U, $\tau_R(X)$), $(V,\tau_{R^1}(Y))$ and $(W,\tau_{R^{11}}(z))$ are nano topological spaces with respect to X, where $X \subseteq U, Y \subseteq V, Z \subseteq W$. R, R^1 and R^{11} are an equivalence relations on U, V and W. U/R, V/R^1 , W/R^{11} denotes the the family of equivalence classes by the equivalence relations R, R^1 and R^{11}respectively on U, V and W.

II. Preliminaries

In this section we recall some definitions and properties which are useful in this study.

Definition 2.1 :

Let U be a non- empty finite set of objects called the universe and R be an equivalence relation on U named as the indicernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U , R) is said to be the approximation space. Let X $\subseteq$ U.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by L_R (X). That is
$L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by U_R (X). That is
$U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not- X with respect to R and it is denoted by B_R (X). That is, $B_R(X) = U_R(X) – L_R(X)$.

Property 2.2: If (U , R) is an approximation space and X ,Y $\subseteq$U, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$;

(ii) $L_R(\phi) = U_R(\phi)$ *and* $L_R(U) \subseteq U_R(U) = U$;

(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;
(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;
(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;
(vi) $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;
(vii) $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$;
(viii) $U_R(X^C) = [L_R(X)]^C$ *and* $L_R(X^C) = [U_R(X)]^C$;
(ix) $U_R U_R(X) = L_R L_R(X) = U_R(X)$;
(x) $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3: Let U be the universe, R be an equivalence relation on U $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where X $\subseteq$ U. Then by property 2.3, $\tau_R(X)$ satisfies the following axioms:

(i) U and ϕ are in $\tau_R(X)$

(ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$ That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. The space (U, $\tau_R(X)$) is the nano topological space. The elements of $\tau_R(X)$ are called as nano – open sets of (U, $\tau_R(X)$).

Remark 2.4: If $\tau_R(X)$ is the nano topology on U with respect to X, then the set B = $\{U, L_R(x), B_R(x)\}$ is the basis for $\tau_R(X)$.

Definition 2.5: If (U , $\tau_R(X)$) is a nano topological space with respect to X where X $\subseteq$ U and if A $\subseteq$ U, then the nano interior of A is defined as the union of all nano – open subsets of A and it is denoted by Nint (A). That is Nint(A) is the largest nano- open subset of A. The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by Ncl(A). That is, Ncl(A) is the smallest nano closed set containing A.

Definition 2.6: A subset of a topological space (X, τ) is called

a) Ng - closed [2] if Ncl(A)$\subseteq$ U, whenever A $\subseteq$ U, and U is nano open in X.
b) Nsg - closed [1] if Nscl(A) $\subseteq$ U, whenever A $\subseteq$ U, and U is nano semi open in X.
c) Nt - set [5] if Nint(A) = Nint(Ncl(A)).
d) Nc(s) - set [10] if A = G$\cap$ F, where G is Ng–open and F is Nt-set.
e) Nsc(s)g - closed [10] if Nscl(A)$\subseteq$ U, whenever A $\subseteq$ U and U is Nc(s)-set in X.
f) Ns - open [7] if A $\subseteq$ Ncl (Nint (A)).
g) Np - open [7] if A $\subseteq$Nint (Ncl (A)).
h) Nα - open [7] if A $\subseteq$Nint (Ncl (Nint (A))).
i) Nr - open [7] if A = Nint (Ncl (A)).

Definition 2.7[10]: A subset A of $(U, \tau_R(X))$ is called an Nano semi c(s) generalized closed set if Nscl(A) $\subseteq$ U, whenever A $\subseteq$ U and U is Nc(s)- set in $(U, \tau_R(X))$. The complement of nano semi c(s) generalized closed set is nano semi c(s) generalized open set in $(U, \tau_R(X))$.

Definition 2.8[6]: Let $(U, \tau_R(X))$ and $(V, \tau_{R^1}(Y))$ be a nano topological spaces. Then the function $f : (U, \tau_R(X)) \to (V, \tau_{R^1}(Y))$ is said to be nano continuous on U if the inverse image of every nano open set in V is nano open set in U.

Definition 2.9[9]: Let $(U, \tau_R(X))$ and $(V, \tau_{R^1}(Y))$ be a nano topological spaces. Then the function $f : (U, \tau_R(X)) \to (V, \tau_{R^1}(Y))$ is said to be nano semi c(s) generalized continuous (briefly Nsc(s)g – continuous) on U if the inverse image of every nano open set in V is nano semi c(s) g – open set in U.

III. Nano contra semi c(s) generalized continuous function in Nano topological spaces

In this section we define and study the new class of function, namely nano contra semi c(s) generalized continuous functions in nano topological spaces.

Definition 3.1: Let $(U,\tau_R(X))$ and $(V,\tau_{R^1}(Y))$ be a nano topological spaces. Then the function $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ is said to be nano contra semi c(s) generalized continuous (briefly Ncsc(s)g – continuous) on U if the inverse image of every nano open set in V is nano semi c(s) g – closed set in U.

Example 3.2: Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X= {a, b}. Then the nano topology is $\tau_R(X)=\{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$. Let V = {x, y, z, w} with V/R[1] = {{x, z},{y},{w}} and Y = {x, w}. Then $\tau_{R^1}(Y)=\{\phi, V, \{w\}, \{x, z, w\}, \{x, z\}\}$. $\tau_R{}^c(X)=\{\phi, U, \{c\}, \{b, c, d\}, \{a, c\}\}$, $\tau_{R^1}{}^c(Y)=\{\phi, V, \{y\}, \{x, y, z\}, \{y, w\}\}$ are the complements of $\tau_R(X)$ and $\tau_{R^1}(Y)$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ by f(a) = y, f(b) = x, f(c) = w and f(d) = z. Then $f^{-1}(\{w\})=\{c\}, f^{-1}(\{x,z,w\})=\{b,c,d\}, f^{-1}(\{x,z\})=\{b,d\}\, and\, f^{-1}(V)=U$. That is the inverse image of every in nano open set in V is Nsc(s)g – closed set in U. Therefore f Ncsc(s)g – continuous function.

Theorem 3.3: A function $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ is said to be Ncsc(s)g - continuous iff the inverse image of every nano closed set in V is Nsc(s)g – open set in U.

Proof: Let f be Ncsc(s)g – continuous and F be nano closed set in V. That is V – F is nano open set in V. Since f is Ncsc(s)g – continuous, $f^{-1}(V-F)$ is Nsc(s)g – closed set in U. That is $f^{-1}(V)-f^{-1}(F)=U-f^{-1}(F)$ is Nsc(s)g – closed set in U. Hence $f^{-1}(F)$ is Nsc(s)g – open set in U, if f is Ncsc(s)g – continuous on U. Conversely, let us assume that the inverse image of every nano closed set in V is Nsc(s)g – open set in U. Let G be nano open set in V. Then V – G is nano closed set in V. By our assumption $f^{-1}(V-G)$ is Nsc(s)g – open set in U. That is $f^{-1}(V)-f^{-1}(G)=U-f^{-1}(G)$ is Nsc(s)g – open set in U. Hence $f^{-1}(G)$ is Nsc(s)g – close set in U. That is the inverse image of every nano open set in V is Nsc(s)g – closed set in U. That is f is Ncsc(s)g – continuous on U.

Theorem 3.4: Every nano contra continuous function is Ncsc(s)g – continuous function.

Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ be a nano contra continuous function and A be nano closed set in $(V,\tau_{R^1}(Y))$ and therefore A^c is nano open in $(V,\tau_{R^1}(Y))$. Then the inverse image of A under the map f is nano closed in $(U,\tau_R(X))$. Since every nano closed is Nsc(s)g – closed set, $f^{-1}(A^c)$ is Nsc(s)g – closed set in $(U,\tau_R(X))$. But $f^{-1}(A^c)=(f^{-1}(A))^c$, so we have $f^{-1}(A)$ is Nsc(s)g – open set in U whenever A is Nano closed set in V. By theorem 3.3, f is Ncsc(s)g – continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 3.5: Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X= {a, b}. Then the nano topology is $\tau_R(X)=\{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$. Let V = {x, y, z, w} with V/R[1] = {{x, z},{y},{w}} and Y = {x, w}. Then $\tau_{R^1}(Y)=\{\phi, V, \{w\}, \{x, z, w\}, \{x, z\}\}$. $\tau_R{}^c(X)=\{\phi, U, \{c\}, \{b, c, d\}, \{a, c\}\}$, $\tau_{R^1}{}^c(Y)=\{\phi, V, \{y\}, \{x, y, z\}, \{y, w\}\}$ are the complements of $\tau_R(X)$ and $\tau_{R^1}(Y)$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ by f(a) = y, f(b) = x, f(c) = w and f(d) = z. Then $f^{-1}(\{w\})=\{c\}, f^{-1}(\{x,z,w\})=\{b,c,d\}, f^{-1}(\{x,z\})=\{b,d\}\, and\, f^{-1}(V)=U$. Therefore f is

Ncsc(s)g – continuous function but not Nano contra – continuous function, since $f^{-1}(\{x,z\})=\{b,d\}$ is not Nano closed set in U.

Remark 3.6: The concept of Nsc(s)g – continuity and Ncsc(s)g – continuity are independent as shown in the following example.

Example 3.7: Let U = { a, b, c, d } with U/R = { { a }, { c }, { b, d } } and X= { a, b } then $\tau_R(X)=\{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$ and $\tau_R{}^c(X)=\{\phi, U, \{c\}, \{b, c, d\}, \{a, c\}\}$. Then Nsc(s)g – closed sets are { ϕ, U, { a },{ c },{ a, c },{ b, c },{ b, d },{ c, d },{ a, b, c }, { a, c, d }, { b, c, d }} and Nsc(s)g – open sets are { ϕ, U, { a },{ b },{ d},{ a, b },{ a, c }, { a, d },{ b, d },{ a, b, d }, { b, c, d }}. Let V = {x, y, z, w} with V/R[1] = {{x, z},{y},{w}} and Y = {x, w}. Then $\tau_{R^1}(Y)=\{\phi, V, \{w\}, \{x, z, w\}, \{x, z\}\}$.

(i) Define $f:(U,\tau_R(X))\rightarrow(V,\tau_{R^1}(Y))$ by f(a) = w, f(b) = x, f(c) = y and f(d)=z. Then $f^{-1}(\{w\})=\{a\}, f^{-1}(\{x,z,w\})=\{a,b,d\}, f^{-1}(\{x,z\})=\{b,d\}\, and\, f^{-1}(V)=U$. Therefore f is Nsc(s)g – continuous function but not Ncsc(s)g - continuous function, since $f^{-1}(\{x,z,w\})=\{a,b,d\}$ is not Nsc(s)g - closed set in U where $\{x,z,w\}$ is Nano open in V. Therefore f is Nsc(s)g – continuous but not Ncsc(s)g – continuous function.

(ii) Define $f:(U,\tau_R(X))\rightarrow(V,\tau_{R^1}(Y))$ by f(a) = y, f(b) = x, f(c) = w and f(d)=z. Then $f^{-1}(\{w\})=\{c\}, f^{-1}(\{x,z,w\})=\{b,c,d\}, f^{-1}(\{x,z\})=\{b,d\}\, and\, f^{-1}(V)=U$. Therefore f is Ncsc(s)g – continuous function but not Nsc(s)g - continuous function, since $f^{-1}(\{w)=\{c\}$ is not Nsc(s)g - open set in U where $\{w\}$ is Nano open in V. Therefore f is Ncsc(s)g – continuous but not Nsc(s)g – continuous function.

Theorem 3.8: Every nano contra semi continuous function is Ncsc(s)g – continuous function.

Proof: Let $f:(U,\tau_R(X))\rightarrow(V,\tau_{R^1}(Y))$ be a nano contra semi continuous function and A be nano semi closed set in $(V,\tau_{R^1}(Y))$ and therefore A^c is nano semi open in $(V,\tau_{R^1}(Y))$. Then the inverse image of A under the map f is nano semi closed in $(U,\tau_R(X))$. Since every nano semi closed is Nsc(s)g – closed set, $f^{-1}(A^c)$ is Nsc(s)g – closed set in $(U,\tau_R(X))$. But $f^{-1}(A^c)=(f^{-1}(A))^c$, so we have $f^{-1}(A)$ is Nsc(s)g – open set in U whenever A is Nano semi closed set in V. By theorem 3.3, f is Ncsc(s)g – continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 3.9: Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X= {a, b}. Then the nano topology is $\tau_R(X)=\{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$. Let V = {x, y, z, w} with V/R[1] = {{x, z},{y},{w}} and Y = {x, w}. Then $\tau_{R^1}(Y)=\{\phi, V, \{w\}, \{x, z, w\}, \{x, z\}\}$. $\tau_R{}^c(X)=\{\phi, U, \{c\}, \{b, c, d\}, \{a, c\}\}$, $\tau_{R^1}{}^c(Y)=\{\phi, V, \{y\}, \{x, y, z\}, \{y, w\}\}$ are the complements of $\tau_R(X)$ and $\tau_{R^1}(Y)$ respectively. Define $f:(U,\tau_R(X))\rightarrow(V,\tau_{R^1}(Y))$ by f(a) = w, f(b) = y, f(c) = x and f(d) = z. Then $f^{-1}(\{w\})=\{a\}, f^{-1}(\{x,z,w\})=\{a,c,d\}, f^{-1}(\{x,z\})=\{c,d\}\, and\, f^{-1}(V)=U$. Therefore f is Ncsc(s)g – continuous function but not Nano contra semi – continuous function, since $f^{-1}(\{x,z\})=\{c,d\}$ is not Nano semi closed set in U.

Theorem 3.10: Every nano contra generalized continuous function is Ncsc(s)g – continuous function.

Proof: Let $f:(U,\tau_R(X))\rightarrow(V,\tau_{R^1}(Y))$ be a nano contra generalized continuous function and A be nano generalized closed set in $(V,\tau_{R^1}(Y))$ and therefore A^c is Ng - open in $(V,\tau_{R^1}(Y))$. Then the inverse image

of A under the map f is nano generalized closed in $(U,\tau_R(X))$. Since every nano generalized closed is Nsc(s)g – closed set, $f^{-1}(A^c)$ is Nsc(s)g – closed set in $(U,\tau_R(X))$. But $f^{-1}(A^c)=(f^{-1}(A))^c$, so we have $f^{-1}(A)$ is Nsc(s)g – open set in U whenever A is Ng - closed set in V. By theorem 3.3, f is Ncsc(s)g – continuous function.
The converse of the above theorem need not be true as seen from the following example.

Example 3.11: Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X= {a, b}. Then the nano topology is $\tau_R(X)=\{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$. Let V = {x, y, z, w} with V/R[1] = {{x, z},{y},{w}} and Y = {x, w}. Then $\tau_{R^1}(Y)=\{\phi, V, \{w\}, \{x, z, w\}, \{x, z\}\}$. $\tau_R{}^c(X)=\{\phi, U, \{c\}, \{b, c, d\}, \{a, c\}\}$, $\tau_{R^1}{}^c(Y)=\{\phi, V, \{y\}, \{x, y, z\}, \{y, w\}\}$ are the complements of $\tau_R(X)$ and $\tau_{R^1}(Y)$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ by f(a) = w, f(b) = y, f(c) = x and f(d) = z. Then $f^{-1}(\{w\})=\{a\}, f^{-1}(\{x,z,w\})=\{a,c,d\}, f^{-1}(\{x,z\})=\{c,d\}\, and\, f^{-1}(V)=U$. Therefore f is Ncsc(s)g – continuous function but not Nano contra generalized – continuous function, since $f^{-1}(\{w\})=\{a\}$ is not Ng - closed set in U.

Theorem 3.12: Every nano contra αg - continuous function is Nsc(s)g – continuous function.
Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ be a $Nano\, contra\, \alpha g$ - continuous function and A be $N\alpha g$ - closed set in $(V,\tau_{R^1}(Y))$ and therefore A^c is N αg - open in $(V,\tau_{R^1}(Y))$. Then the inverse image of A under the map f is nano αg closed in $(U,\tau_R(X))$. Since every nano αg closed set is Nsc(s)g – closed set, $f^{-1}(A^c)$ is Nsc(s)g – closed set in $(U,\tau_R(X))$. But $f^{-1}(A^c)=(f^{-1}(A))^c$, so we have $f^{-1}(A)$ is Nsc(s)g – open set in U whenever A is N αg - closed set in V. By theorem 3.3, f is Ncsc(s)g – continuous function.
The converse of the above theorem need not be true as seen from the following example.

Example 3.13: Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X= {a, b}. Then the nano topology is $\tau_R(X)=\{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$. Let V = {x, y, z, w} with V/R[1] = {{x, z},{y},{w}} and Y = {x, w}. Then $\tau_{R^1}(Y)=\{\phi, V, \{w\}, \{x, z, w\}, \{x, z\}\}$. $\tau_R{}^c(X)=\{\phi, U, \{c\}, \{b, c, d\}, \{a, c\}\}$, $\tau_{R^1}{}^c(Y)=\{\phi, V, \{y\}, \{x, y, z\}, \{y, w\}\}$ are the complements of $\tau_R(X)$ and $\tau_{R^1}(Y)$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ by f(a) = w, f(b) = y, f(c) = x and f(d) = z. Then $f^{-1}(\{w\})=\{a\}, f^{-1}(\{x,z,w\})=\{a,c,d\}, f^{-1}(\{x,z\})=\{c,d\}\, and\, f^{-1}(V)=U$. Therefore f is Ncsc(s)g – continuous function but not Nano contra αg – continuous function, since $f^{-1}(\{w\})=\{a\}$ is not N αg - closed set in U.

Theorem 3.14: Every $Ncg\alpha$ - function is Ncsc(s)g – continuous function.
Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ be a $Nano\, contra\, g\alpha$ - continuous function and A be $Ng\alpha$ - closed set in $(V,\tau_{R^1}(Y))$ and therefore A^c is $Ng\alpha$ - open in $(V,\tau_{R^1}(Y))$. Then the inverse image of A under the map f is nano $g\alpha$ closed in $(U,\tau_R(X))$. Since every nano $g\alpha$ closed set is Nsc(s)g – closed set, $f^{-1}(A^c)$ is Nsc(s)g – closed set in $(U,\tau_R(X))$. But $f^{-1}(A^c)=(f^{-1}(A))^c$, so we have $f^{-1}(A)$ is Nsc(s)g – open set in U whenever A is N $g\alpha$ - closed set in V. By theorem 3.3, f is Ncsc(s)g – continuous function.
The converse of the above theorem need not be true as seen from the following example.

Example 3.15: Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X= {a, b}. Then the nano topology is $\tau_R(X)=\{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$. Let V = {x, y, z, w} with V/R^1 = {{x, z},{y},{w}} and Y = {x, w}. Then $\tau_{R^1}(Y)=\{\phi, V, \{w\}, \{x, z, w\}, \{x, z\}\}$. $\tau_R{}^c(X)=\{\phi, U, \{c\}, \{b, c, d\}, \{a, c\}\}$, $\tau_{R^1}{}^c(Y)=\{\phi, V, \{y\}, \{x, y, z\}, \{y, w\}\}$ are the complements of $\tau_R(X)$ and $\tau_{R^1}(Y)$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ by f(a) = y, f(b) = x, f(c) = w and f(d) = z. Then $f^{-1}(\{w\})=\{c\}, f^{-1}(\{x,z,w\})=\{b,c,d\}, f^{-1}(\{x,z\})=\{b,d\}$ *and* $f^{-1}(V)=U$. Therefore f is Ncsc(s)g – continuous function but not Nano contra $g\alpha$ – continuous function, since $f^{-1}(\{x,z\})=\{b,d\}$ is not $Ng\alpha$ - closed set in U.

Theorem 3.16: Every Ncg* - continuous function is Ncsc(s)g – continuous function.

Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ be a $Ncg*$ - continuous function and A be $Ng*$ - closed set in $(V,\tau_{R^1}(Y))$ and therefore A^c is $Ng*$ - open in $(V,\tau_{R^1}(Y))$. Then the inverse image of A under the map f is $Ng*$ closed in $(U,\tau_R(X))$. Since every $Ng*$ closed set is Nsc(s)g – closed set, $f^{-1}(A^c)$ is Nsc(s)g – closed set in $(U,\tau_R(X))$. But $f^{-1}(A^c)=(f^{-1}(A))^c$, so we have $f^{-1}(A)$ is Nsc(s)g – open set in U whenever A is $Ng*$ - closed set in V. By theorem 3.3, f is Ncsc(s)g – continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 3.17: Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X= {a, b}. Then the nano topology is $\tau_R(X)=\{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$. Let V = {x, y, z, w} with V/R^1 = {{x, z},{y},{w}} and Y = {x, w}. Then $\tau_{R^1}(Y)=\{\phi, V, \{w\}, \{x, z, w\}, \{x, z\}\}$. $\tau_R{}^c(X)=\{\phi, U, \{c\}, \{b, c, d\}, \{a, c\}\}$, $\tau_{R^1}{}^c(Y)=\{\phi, V, \{y\}, \{x, y, z\}, \{y, w\}\}$ are the complements of $\tau_R(X)$ and $\tau_{R^1}(Y)$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ by f(a) = y, f(b) = x, f(c) = w and f(d) = z. Then $f^{-1}(\{w\})=\{c\}, f^{-1}(\{x,z,w\})=\{b,c,d\}, f^{-1}(\{x,z\})=\{b,d\}$ *and* $f^{-1}(V)=U$. Therefore f is Ncsc(s)g – continuous function but not $Ncg*$ – continuous function, since $f^{-1}(\{x,z\})=\{b,d\}$ is not $Ng*$ - closed set in U.

Theorem 3.18: Every nano contra regular continuous function is Nsc(s)g – continuous function.

Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ be a Ncr - continuous function and A be nano regular closed set in $(V,\tau_{R^1}(Y))$ and therefore A^c is Nr - open in $(V,\tau_{R^1}(Y))$. Then the inverse image of A under the map f is nano regular closed in $(U,\tau_R(X))$. Since every nano regular closed is Nsc(s)g – closed set, $f^{-1}(A^c)$ is Nsc(s)g – closed set in $(U,\tau_R(X))$. But $f^{-1}(A^c)=(f^{-1}(A))^c$, so we have $f^{-1}(A)$ is Nsc(s)g – open set in U whenever A is Nr - closed set in V. By theorem 3.3, f is Ncsc(s)g – continuous function.

The converse of the above theorem need not be true as seen from the following example.

Example 3.19: Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X= {a, b}. Then the nano topology is $\tau_R(X)=\{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$. Let V = {x, y, z, w} with V/R^1 = {{x, z},{y},{w}} and Y = {x, w}. Then $\tau_{R^1}(Y)=\{\phi, V, \{w\}, \{x, z, w\}, \{x, z\}\}$. $\tau_R{}^c(X)=\{\phi, U, \{c\}, \{b, c, d\}, \{a, c\}\}$, $\tau_{R^1}{}^c(Y)=\{\phi, V, \{y\}, \{x, y, z\}, \{y, w\}\}$ are the complements of $\tau_R(X)$ and $\tau_{R^1}(Y)$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ by f(a) = w, f(b) = y, f(c) = x and f(d) = z. Then $f^{-1}(\{w\})=\{a\}, f^{-1}(\{x,z,w\})=\{a,c,d\}, f^{-1}(\{x,z\})=\{c,d\}$ *and* $f^{-1}(V)=U$. Therefore f is

Ncsc(s)g – continuous function but not Nano contra regular continuous function, since $f^{-1}(\{w\})=\{a\}$ is not Nr - closed set in U.

Theorem 3.20: Every $Nc\alpha$ - function is Ncsc(s)g – continuous function.
Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ be a $Nc\alpha$ - continuous function and A be $N\alpha$ - closed set in $(V,\tau_{R^1}(Y))$ and therefore A^c is $N\alpha$ - open in $(V,\tau_{R^1}(Y))$. Then the inverse image of A under the map f is $N\alpha$ - closed in $(U,\tau_R(X))$. Since every $N\alpha$ - closed set is Nsc(s)g – closed set, $f^{-1}(A^c)$ is Nsc(s)g – closed set in $(U,\tau_R(X))$. But $f^{-1}(A^c)=(f^{-1}(A))^c$, so we have $f^{-1}(A)$ is Nsc(s)g – open set in U whenever A is $N\alpha$ - closed set in V. By theorem 3.3, f is Ncsc(s)g – continuous function.
The converse of the above theorem need not be true as seen from the following example.

Example 3.21: Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X= {a, b}. Then the nano topology is $\tau_R(X)=\{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$. Let V = {x, y, z, w} with V/R[1] = {{x, z},{y},{w}} and Y = {x, w}. Then $\tau_{R^1}(Y)=\{\phi, V, \{w\}, \{x, z, w\}, \{x, z\}\}$. $\tau_R{}^c(X)=\{\phi, U, \{c\}, \{b, c, d\}, \{a, c\}\}$, $\tau_{R^1}{}^c(Y)=\{\phi, V, \{y\}, \{x, y, z\}, \{y, w\}\}$ are the complements of $\tau_R(X)$ and $\tau_{R^1}(Y)$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ by f(a) = y, f(b) = x, f(c) = w and f(d) = z. Then $f^{-1}(\{w\})=\{c\}, f^{-1}(\{x,z,w\})=\{b,c,d\}, f^{-1}(\{x,z\})=\{b,d\}\ and\ f^{-1}(V)=U$. Therefore f is Ncsc(s)g – continuous function but not $Nc\alpha$ – continuous function, since $f^{-1}(\{x,z\})=\{b,d\}$ is not $N\alpha$ - closed set in U.

Theorem 3.22: Every Ncsc(s)g – continuous function is Ncsg – continuous function.
Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ be a Ncsc(s)g – continuous function and A be Nsc(s)g – closed set in $(V,\tau_{R^1}(Y))$ and therefore A^c is Nsc(s)g - open in $(V,\tau_{R^1}(Y))$. Then the inverse image of A under the map f is Nsc(s)g - closed in $(U,\tau_R(X))$. Since every Nsc(s)g - closed set is Nsg – closed set, $f^{-1}(A^c)$ is Nsg – closed set in $(U,\tau_R(X))$. But $f^{-1}(A^c)=(f^{-1}(A))^c$, so we have $f^{-1}(A)$ is Nsg – open set in U whenever A is Nsc(s)g - closed set in V. By theorem 3.3, f is Ncsg – continuous function.
The converse of the above theorem need not be true as seen from the following example.

Example 3.23: Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X= {a, b}. Then the nano topology is $\tau_R(X)=\{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$. Let V = {x, y, z, w} with V/R[1] = {{x, z},{y},{w}} and Y = {x, w}. Then $\tau_{R^1}(Y)=\{\phi, V, \{w\}, \{x, z, w\}, \{x, z\}\}$. $\tau_R{}^c(X)=\{\phi, U, \{c\}, \{b, c, d\}, \{a, c\}\}$, $\tau_{R^1}{}^c(Y)=\{\phi, V, \{y\}, \{x, y, z\}, \{y, w\}\}$ are the complements of $\tau_R(X)$ and $\tau_{R^1}(Y)$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ by f(a) = y, f(b) = w, f(c) = x and f(d) = z. Then $f^{-1}(\{w\})=\{b\}, f^{-1}(\{x,z,w\})=\{b,c,d\}, f^{-1}(\{x,z\})=\{c,d\}\ and\ f^{-1}(V)=U$. Therefore f is Ncsg – continuous function but not Ncsc(s)g – continuous function, since $f^{-1}(\{w\})=\{b\}$ is not Nsc(s)g - closed set in U.

Theorem 3.24: Every Ncsc(s)g – continuous function is Ncrgb – continuous function.
Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ be a Ncsc(s)g – continuous function and A be Nsc(s)g – closed set in $(V,\tau_{R^1}(Y))$ and therefore A^c is Nsc(s)g - open in $(V,\tau_{R^1}(Y))$. Then the inverse image of A under the map f is Nsc(s)g - closed in $(U,\tau_R(X))$. Since every Nsc(s)g - closed set is Ncrgb – closed set, $f^{-1}(A^c)$ is

Ncrgb – closed set in $(U,\tau_R(X))$.But $f^{-1}(A^c)=(f^{-1}(A))^c$, so we have $f^{-1}(A)$ is Ncrgb – open set in U whenever A is Nsc(s)g - closed set in V. By theorem 3.3, f is Ncrgb – continuous function.
The converse of the above theorem need not be true as seen from the following example.

Example 3.25: Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X= {a, b}. Then the nano topology is $\tau_R(X)=\{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$. Let V = {x, y, z, w} with V/R[1] = {{x, z},{y},{w}} and Y = {x, w}. Then $\tau_{R^1}(Y)=\{\phi, V, \{w\}, \{x, z, w\}, \{x, z\}\}$ $\tau_{R^1}{}^c(Y)=\{\phi, V, \{y\}, \{x, y, z\}, \{y, w\}\}$ are the complements of $\tau_R(X)$ and $\tau_{R^1}(Y)$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ by f(a) = x, f(b) = z, f(c) = y and f(d) = w. Then $f^{-1}(\{w\})=\{d\}, f^{-1}(\{x,z,w\})=\{a,b,d\}, f^{-1}(\{x,z\})=\{a,b\} \, and \, f^{-1}(V)=U$. Therefore f is Ncrgb – continuous function but not Ncsc(s)g – continuous function, since $f^{-1}(\{w\})=\{d\}, f^{-1}(\{x,z,w\})=\{a,b,d\} \, and \, f^{-1}(\{x,z\})=\{a,b\}$ are not Nsc(s)g - closed sets in U.

Theorem 3.26: Every Ncsc(s)g – continuous function is $Nc\beta$ – continuous function.
Proof: Let $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ be a Ncsc(s)g – continuous function and A be Nsc(s)g – closed set in $(V,\tau_{R^1}(Y))$ and therefore A^c is Nsc(s)g - open in $(V,\tau_{R^1}(Y))$.Then the inverse image of A under the map f is Nsc(s)g - closed in $(U,\tau_R(X))$. Since every Nsc(s)g - closed set is $N\beta$ – closed set, $f^{-1}(A^c)$ is $N\beta$ – closed set in $(U,\tau_R(X))$.But $f^{-1}(A^c)=(f^{-1}(A))^c$, so we have $f^{-1}(A)$ is $N\beta$ – open set in U whenever A is Nsc(s)g - closed set in V. By theorem 3.3, f is $Nc\beta$ – continuous function.
The converse of the above theorem need not be true as seen from the following example.

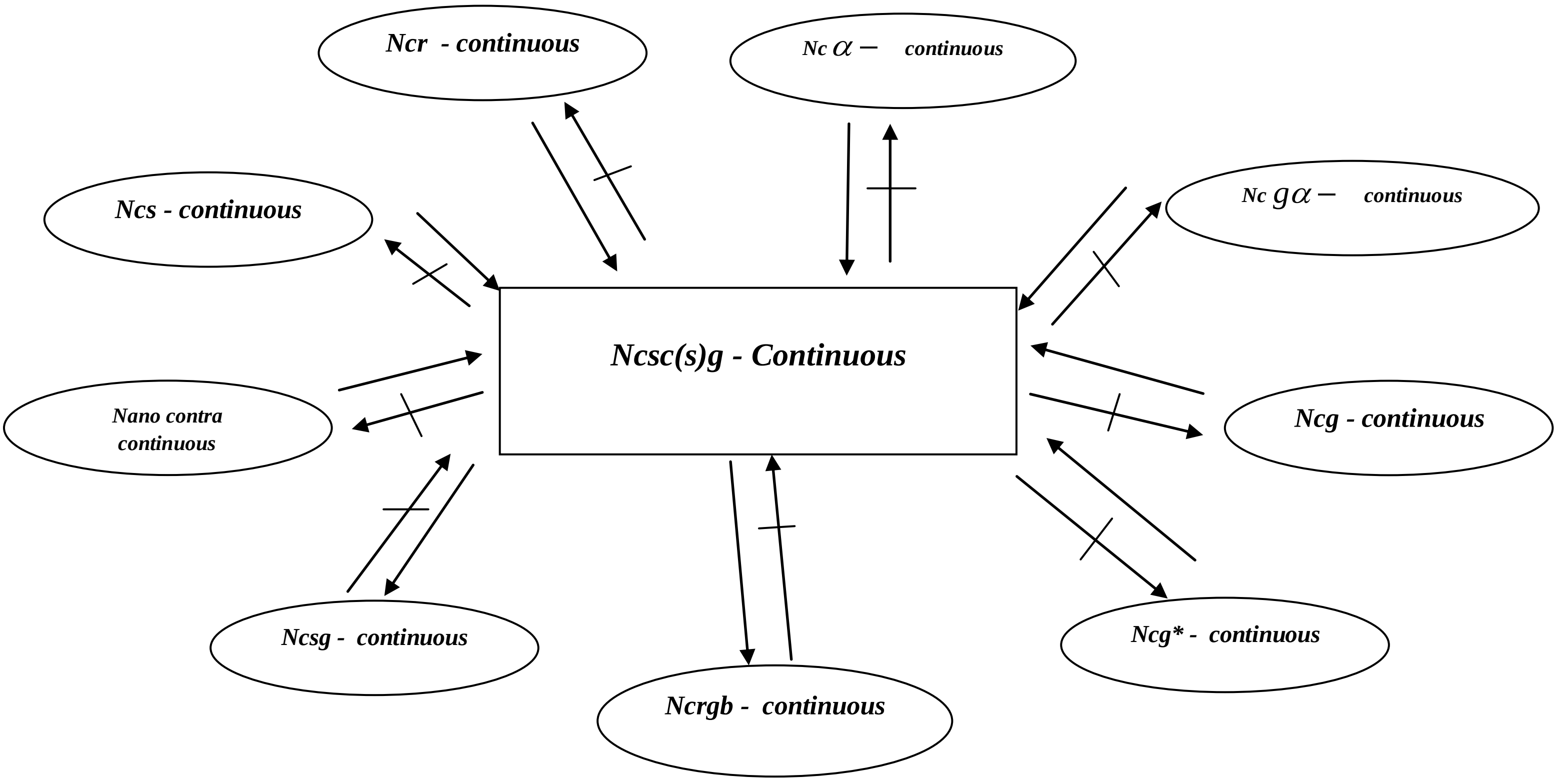
Example 3.27: Let U = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X= {a, b}. Then the nano topology is $\tau_R(X)=\{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$. Let V = {x, y, z, w} with V/R[1] = {{x, z},{y},{w}} and Y = {x, w}. Then $\tau_{R^1}(Y)=\{\phi, V, \{w\}, \{x, z, w\}, \{x, z\}\}$. $\tau_R{}^c(X)=\{\phi, U, \{c\}, \{b, c, d\}, \{a, c\}\}$, $\tau_{R^1}{}^c(Y)=\{\phi, V, \{y\}, \{x, y, z\}, \{y, w\}\}$ are the complements of $\tau_R(X)$ and $\tau_{R^1}(Y)$ respectively. Define $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ by f(a) = y, f(b) = z, f(c) = x and f(d) = w. Then $f^{-1}(\{w\})=\{d\}, f^{-1}(\{x,z,w\})=\{b,c,d\}, f^{-1}(\{x,z\})=\{b,c\} \, and \, f^{-1}(V)=U$. Therefore f is $Nc\beta$ – continuous function but not Ncsc(s)g – continuous function, since $f^{-1}(\{w\})=\{d\}$ is not Nsc(s)g - closed set in U.

Remark 3.28: Composition of two Ncsc(s)g – continuous function need not be a Ncsc(s)g – continuous function.

Example 3.29: Let U = {a, b, c, d}, V = {x, y, z, w} and W = {a, b, c, d} with U/R = {{a}, {c}, {b, d}} and X= {a, b}, V/R[1] = {{x, z},{y},{w}} and Y = {x, w}, W/R[11] = {{a}, {d}, {b, c}} and Z = {a, c}. Then the corresponding nano topologies of U,V and W are $\tau_R(X)=\{\phi, U, \{a\}, \{a, b, d\}, \{b, d\}\}$, $\tau_{R^1}(Y)=\{\phi, V, \{w\}, \{x,z,w\}, \{x,z\}\}$ and $\tau_{R^{11}}(Z)=\{\phi, W, \{a\}, \{a, b, c\}, \{b, c\}\}$ and its compliments are $\tau_R{}^c(X)=\{\phi, U, \{c\}, \{b, c, d\}, \{a, c\}\}$, $\tau_{R^1}{}^c(Y)=\{\phi, V, \{b\}, \{a, b, c\}, \{b, d\}\}$
and $\tau_{R^{11}}{}^c(Z)=\{\phi, W, \{d\}, \{b,c,d\}, \{a,d\}\}$ repectively. Define the function $f:(U,\tau_R(X))\to(V,\tau_{R^1}(Y))$ by f(a) = w, f(b) = y, f(c) = x and f(d) = z and $g:(V,\tau_{R^1}(Y))\to(W,\tau_{R^{11}}(z))$ as identity function by g(a) = b, g(b) = a, g(c) = c, g(d) = d. Then f and

g are Ncsc(s)g – continuous function. But their composition $g \circ f : (U, \tau_R(X)) \rightarrow (W, \tau_{R^{11}}(z))$ is not Ncsc(s)g – continuous function, since the inverse image of the nano closed set $\{a\}$ is $\{b\}$. But it is not a Nsc(s)g – closed set in $(U, \tau_R(X))$.

Remark 3.30: From the above discussion and result, the relationship between the Ncsc(s)g – continuous functions and existing contra continuous functions are as below in the diagrams.



REFERENCES

[1] K.Bhuvaneshwari and K.Ezhilarasi, On Nano semi generalized and Nano generalized semi closed sets, IJMCAR, 4(3)(2014), 117-124.
[2] K.Bhuvaneshwari and K.Mythili Gnanapriya, Nano generalized closed sets, International journal of scientific and Research Publications 4(5)(2014), 1-3.
[3] Chandrasekar.S, Rajesh Kannan.T, Suresh.M, "Contra – Nano sg – continuity in Nano topological spaces", International journal on Research Innovations in Engineering Science and Technology, (2017), 2 (4), 110-116.
[4] A.Dhanis Arul Mary and I.Arockiarani (2015), "Note on contra nano gb – closed maps", Journal of Global Research in Mathematical Archives, (2014), 2(10) 55 - 66.
[5] Jayalakshmi.A and Janaki.C, "A new form of nano locally closed sets in nano topological spaces", Global Journal of Pure and Applied Mathematics (19) 9 (2017), 5997 – 6006.
[6] M.Lellis Thivagar, Carmel Richard, "On Nano Continuity", Math.TheoryModel, 7 (2013) 32-37.
[7] M.Lellis Thivagar, Carmel Richard, "On Nano Forms of Weakly open sets", International Journal of Mathematics and Statistics Invention, 1 (2013), 31-37.
[8] Shalini.S.B, Indirani.K, "On Nano generalized β continuous functions and nano generalized β irresolute functions in Nano topological spaces", (2017) 13 (1) 79-86.
[9] S.Visalakshi and A.Pushpalatha, "On Nano Semi c(s) Generalized Continuous functions in Nano Topological Spaces", Malaya journal of Mathematik, (2019) 7 (1) 67-71 .
[10] S.Visalakshi and A.Pushpalatha, "On Nano Semi c(s) Generalized Closed Sets in Nano Topological Spaces", Malaya journal of Mathematik, Submitted.