Nano semi c* - generalized closure in nano topological space

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Abstract: *The aim of this paper is to introduce and study the concept of Nano semi c* generalized closure in nano topological spaces. Some of its basic properties are analyzed.*

AMS Classification : *54B05, 54C05*

Key words:*Nc* - set, Nano semi c* - generalizes closed set, Nano semi c* generalized closure*

1. Introduction

The concept of Nano topology was introduced by Lellis Thivagar, which was defined in terms of approximations and boundary region of a subset of an universe using an equivalence relation on it. He has also defined nano closed sets, nano interior, nano closure and nano continuity. A. Pushpalatha and S.Sridevi were introduced a concept of Nano semi c* generalized closed set in Nano topological space. In this paper, we have introduced a Nano semi c* generalized closure in nano topological spaces and also discuss some of its properties.

2. Preliminaries

This section is to recall some definitions and properties which are useful in this study.

Throughout this paper $(U,\tau_R(X))$ is a Nano Topological Space with respect to X where $X \subseteq U$, R is an equivalence relation on U, U/R denotes the family of equivalence relation of U by R and $(V,\tau_{R'}(Y))$ is is a Nano Topological Space with respect to Y where $Y \subseteq V$, R' is an equivalence relation on V, V/R' denotes the family of equivalence relation of V by R' and $(W,\tau_{R''}(Z))$ is is a Nano Topological Space with respect to W where $Z \subseteq W$, R'' is an equivalence relation on W, W/R'' denotes the family of equivalence relation of W by R''.

Definition 2.1 : Let U be a non- empty finite set of objects is called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U , R) is said to be the approximation space. Let X $\subseteq$ U.

(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by L_R (X). That is $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$, where R(x) denotes the equivalence class determined by x.

(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by U_R (X). That is $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \phi\}$

(iii) The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not- X with respect to R and it is denoted by B_R (X). That is, $B_R(X) = U_R(X) – L_R (X)$.

Property 2.2: If (U , R) is an approximation space and X ,Y $\subseteq$ U, then

(i) $L_R(X) \subseteq X \subseteq U_R(X)$;

(ii) $L_R(\phi) = U_R(\phi)$ *and* $L_R(U) \subseteq U_R(U) = U$;

(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;

(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;

(v) $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;

(vi) $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$;

(vii) $U_R(X^C) = [L_R(X)]^C$ *and* $L_R(X^C) = [U_R(X)]^C$;

(viii) $U_R U_R(X) = L_R L_R(X) = U_R(X)$;

(ix) $L_R L_R(X) = U_R L_R(X) = L_R(X)$.

Definition 2.3: Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where X $\subseteq$ U. Then by property 2.2, $\tau_R(X)$ satisfies the following axioms:

(i) U and ϕ are in $\tau_R(X)$

(ii) The union of the elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. The space $(U, \tau_R(X))$ is called as nano topological space. The elements of $\tau_R(X)$ are called nano – open sets of $(U, \tau_R(X))$.

Remark 2.4: If $\tau_R(X)$ is the nano topology on U with respect to X, then the set B = $\{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.5 : If (U , $\tau_R(X)$) is a nano topological space with respect to X where X $\subseteq$ U and if A $\subseteq$ U, then the nano interior of A is defined as the union of all nano – open subsets of A and it is denoted by N*int*(A). That is N*int*(A) is the largest nano- open subset of A. The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by N*cl*(A). That is, N*cl*(A) is the smallest nano closed set containing A.

***Definition - 2.6*:** If $(U, \tau_R(X))$ is a nano topological space and A⊆U. Then A is said to be

i. **nano semi-open[2]** if A⊆N*cl*(N*int*(A))
ii. **nano pre-open[2]** if A⊆N*int*(N*cl*(A))
iii. **nano regular open[2]** if A= N*int*(N*cl*(A))
iv. **nano α - open[2]**if A N*int*(N*cl*(N*int*(A)))
v. **nano c*-set [4]** if if A= G$\cap$F where G is Ng- open and F is Nα* - set in $(U, \tau_R(X))$
vi. **nano sc*g closed set[4]** if N*scl*(A) $\subseteq$ G, whenever A $\subseteq$ G, and G is Nc* - set in $(U, \tau_R(X))$.

3.Nano semi c* - generalized closure in nano topological space

In this section, we define the function called Nano semi c* generalized closure and study their some of its properties.

Definition :3.1

The nano semi c* - generalized closure of A is defined as the intersection of all Nsc*g – closed sets containing A, denoted by Nsc*gC*l*(A) which is the smallest Nsc*g – closed set containing A.

Example : 3.2

Let $U=\{a,b,c,d\}$ with $U/R=\{\{a\},\{c\},\{b,d\}\}$ and let X= {a,b}. Then $\tau_R(X)=\{\phi,U,\{a\},\{b,d\},\{a,b,d\}\}$ is nano topology on U with respect to X and $\tau_R^{\,c}(X)=\{\phi,U,\{c\},\{a,c\},\{b,c,d\}\}$. Nsc*g –closed set is $\{\phi,U,\{a\},\{c\},\{a,c\},\{b,d\}\{b,c,d\}\}$.

Remark: 3.3

For any set $A\subseteq U,\ A\subseteq Nsc^*gCl(A)\subseteq Nscl(A)\subseteq Ncl(A)$. In Ex -2, let $A=\{b\}\subseteq U$. Now $Nsc^*gCl(A)=\{b,d\}\,and\ Nscl(A)=\{b,d\}\,and\ Ncl(A)=\{b,c,d\}$. Thus, it follow that $A\subseteq Nsc^*gCl(A)\subseteq Nscl(A)\subseteq Ncl(A)$.

Theorem : 3.4

If A and B are subsets of a space U. Then

(i) Nsc*gC*l*(U) = U and Nsc*gC$l(\phi)=\phi$.

(ii) $A\subseteq Nsc^*gCl(A)$

(iii) If B is any Nsc*g- closed set containing A, then $Nsc^*gCl(A)\subseteq B$

(iv) If $A\subseteq B\,then\ Nsc^*gCl(A)\subseteq Nsc^*gCl(B)$

Proof:

(i) By the definition of Nsc*g – closure, U is the only Nsc*g –closed set containing U. Therefore

Nsc*gC*l*(U) = Intersection of all the Nsc*g- closed set containing U

$= \cap\{U\}=U$

Nsc*gC(U) = U. Similarly $Nsc^*gCl(\phi)=\phi$.

(ii) By the definition of Nsc*g- closure of A, it is obvious that $A\subset Nsc^*gCl(A)$.

(iii) Let B be any Nsc*g – closed set containing A. Since NSc*gC*l*(A) is the intersection of all Nsc*g- closed set containing A, Nsc*gCl(A) is containing in every Nsc*g –closed set containing A. Hence $Nsc^*gCl(A)\subseteq B$.

(iv) Let A and B be subsets of U such that $A\subset B$. By the definition,

Nsc*gC*l*(B)= $\cap\{F: B\subset F\in Nsc^*g-closed(U)\}$.

If $B\subset F\in Nsc^*g-closed\,(U)$, then $Nsc^*gCl(B)\subset F$. Since $A\subset B\subset F\in Nsc^*g-closed\,(U)$, we have $Nsc^*g-closed\,(U)\subset F$. Therefore, $Nsc^*gCl(A)\subset\cap\{F: B\subset F\in Nsc^*g-closed(U)\}=Nsc^*gCl(B)$

(i.e..) $Nsc^*gCl(A)\subset Nsc^*gCl(B)$.

Theorem : 3.5

If $A\subset(U,\tau_R(X))$ is Nsc*g – closed then Nsc*gC*l*(A) = A.

Proof:

Let A be Nsc*g- closed subset of U. We know that $A\subset Nsc^*gCl(A)$. Also $A\subset A$, and A is Nsc*g- closed. By theorem 4(iii), $Nsc^*gCl(A)\subset A$. Hence Nsc*gC*l*(A) = A.

Remark: 3.6

The converse of the above theorem need not be true as seen from the following example.

Example : 3.7

Let $U=\{a,b,c,d,e\}, U/R=\{\{a,c\},\{b\},\{d\},\{e\}\}$ and X= $\{a,b\}\subseteq U$ then $\tau_R(X)=\{\phi,U,\{b\},\{a,c\},\{a,b,c\}\}$. Let A= {a} $\subset U$. Nsc*gC*l*(A) = Nsc*gCl({a})= {a} =A. But A= {a} is not a Nsc*g – closed set.

Theorem: 3. 8

If A and B are subsets of a space $(U,\tau_R(X))$, then $Nsc^*gCl(A\cap B)\subset Nsc^*gCl(A)\cap Nsc^*gCl(B)$.

Proof:

Let A and B be subsets of U. Clearly $A \cap B \subset A$ *and* $A \cap B \subset B.$ By Theorem 4(iv),

$Nsc*gCl(A \cap B) \subset Nsc*gCl(A)$ *and* $Nsc*gCl(A \cap B) \subset Nsc*gCl(B).$

Hence $Nsc*gCl(A \cap B) \subset Nsc*gCl(A) \cap Nsc*gCl(B).$

Theorem: 3.9

If A and B are subsets of a space $(U, \tau_R(X))$ then $Nsc*gCl(A \cup B) = Nsc*gCl(A) \cup Nsc*gCl(B).$

Proof:

Let A and B be subsets of $(U, \tau_R(X))$. Clearly $A \subset A \cup B$ *and* $B \subset A \cup B.$ We have $Nsc*gCl(A) \cup Nsc*gCl(B) \subset Nsc*gCl(A \cup B).$

Let $x \in Nsc*gCl(A \cup B).$ *Suppose* $x \notin Nsc*gCl(A) \cup Nsc*gCl(B).$ Then their exists Nsc*g –closed set A_1 and B_1 with $A \subset A_1, B \subset B_1$ *and* $x \notin A_1 \cup B_1.$ *We have* $A \cup B \subset A_1 \cup B_1$ *and* $A_1 \cup B_1$ is Nsc*g – closed set such that $x \notin A_1 \cup B_1$.*Thus* $x \notin Nsc*gCl(A \cup B)$ which is a contradiction to our assumption. Therefore $Nsc*gCl(A \cup B) \subset Nsc*gCl(A) \cup Nsc*gCl(B).$

Hence $Nsc*gCl(A \cup B) = Nsc*gCl(A) \cup Nsc*gCl(B).$

Theorem: 3. 10

If A is a subset of a space $(U, \tau_R(X))$, then $Nsc*gCl(A) \subset Ncl(A).$

Proof:

Let A be a subset of a space $(U, \tau_R(X))$, By definition, $NCl(A) = \cap\{F : U \subset F \in NC(U)\}.$

If $A \subset F \in NC(U)$. Then $A \subset F \in Nsc*g-closed(U)$ because every nano closed set is Nsc*g-closed set. That is $Nsc*gCl(A) \subset F.$ Therefore $Nsc*gCl(A) \subset \cap\{f \subset X; F \in NC(U)\} = Ncl(A).$ Hence $Nsc*gCl(a) \subset Ncl(A).$

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