Bipolar vague finite switchboard state Machines

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Abstract

The notion of bipolar vague finite switchboard state machine is introduced and investigate the related properties.

Keywords: *bipolar vague finite state machine, bipolar switching , bipolar retrievable, bipolar homomorphism, bipolar strong homomorphism.*

I. Introduction

In fuzzy set theory, the membership degrees of elements range over the interval [0, 1]. The membership degree expresses the degree of belongingness of elements to a fuzzy set. The membership degree 1 indicates that an element completely belongs to its corresponding fuzzy set, and the membership degree 0 indicates an element does not belong to the fuzzy set. The membership degrees on the interval (0, 1) indicate the partial membership to the fuzzy set . Sometimes, the membership degree means the satisfaction degree of elements to some property or constraint corresponding to a fuzzy set[2, 9]. In the viewpoint of satisfaction degree , the membership degree 0 is assigned to elements which do not satisfy some property. The elements with membership degree 0 are usually regarded as having the same characteristics in the fuzzy set representation. By the way, among such elements, some have irrelevant characteristics to the property corresponding to a fuzzy set and the others have contrary contrary characteristics to the property. The traditional fuzzy set representation cannot tell apart contrary elements from irrelevant elements. Only with membership degrees ranged on the interval [0, 1], it is difficult to express the difference of the irrelevant elements from the contrary elements in fuzzy sets. If a set representation could express this kind of difference , it would be more informative than the traditional fuzzy set representation. Based on these observations, Lee[6] introduced an extension of fuzzy sets named bipolar-valued fuzzy sets. He gave two kinds of representations of the notion of bipolar-valued fuzzy sets. Malik.et.al [8] introduced the notions of submachine of a fuzzy finite state machine, retrievable, separated and connected fuzzy finite state machines and discussed their basic properties. They also initiated a decomposition theorem for fuzzy finite state machines in terms of primary submachines. On the otherhand, Kumbhojkar and Chaudhari [5] provided several ways of constructing products of fuzzy finite state machines and their mutual relationship, through isomorphism and coverings. Li and Pedrycz [7] indicated that fuzzy finite state automata can be viewed as a mathematical model of computation in fuzzy systems. Recently, a higher order set with imprecision has been extended to automata. Based on Atanassov intuitionistic fuzzy sets [1] Jun proposed intuitionistic fuzzy finite state machines in [3] and also intuitionistic fuzzy finite switchboard machines in [4]. In this paper, using the notion of bipolar vague valued sets concepts of bipolar submachines, bipolar connected, bipolar retrievable, bipolar homomorphism, bipolar strong homomorphism and bipolar vague finite switchboard state machines (bvfssm) are introduced and related properties are investigated.

II. Preliminaries

Definition 2.7:[6]

Let X be the universe of discourse. A bipolar-valued fuzzy set A in X is an object having the form A = {(x, t^{+}_{A}(x), t^{-}_{A}(x)) / x $\in$ X} where t^{+}_{A} : X $\rightarrow$ [0, 1] and t^{-}_{A} : X $\rightarrow$ [-1, 0] are mappings. The positive membership degree t^{+}_{A}(x) denotes the satisfaction degree of an element x to the property corresponding to a bipolar-valued fuzzy set A = {(x, t^{+}_{A}(x), t^{-}_{A}(x)) / x $\in$ X} , and the negative membership degree t^{-}_{A}(x) denotes the satisfaction degree of x to some implicit counter-property of A = {(x, t^{+}_{A}(x), t^{-}_{A}(x)) / x $\in$ X}.

Remark 2.8:[6]

If t^{+}_{A}(x) $\neq$ 0 and t^{-}_{A}(x) = 0, it is the situation that x is regarded as having only positive satisfaction for A = {(x, t^{+}_{A}(x), t^{-}_{A}(x)) / x $\in$ X}. If t^{+}_{A}(x) = 0 and t^{-}_{A}(x) $\neq$ 0 , it is the situation that x does not satisfy the property of A = {(x, t^{+}_{A}(x), t^{-}_{A}(x)) / x $\in$ X} but somewhat satisfies the counter-property of A = {(x, t^{+}_{A}(x), t^{-}_{A}(x)) / x $\in$ X}. It is possible for an element x to be t^{+}_{A}(x) $\neq$ 0 and t^{-}_{A}(x) $\neq$ 0 when the membership function of the property overlaps that of its counter-property over some portion of the domain.

III. Bipolar vague finite state machines

Definition 3.1:
Let X be the universe of discourse. A bipolar-valued vague set A in X is an object having the form A ={(x, $[t^+_A$(x), $1-f^+_A$(x)], $[t^-_A$(x), $1-f^-_A$(x)]) / x∈X} where t^+_A, $1-f^+_A$:X →[0, 1] and t^-_A, $1-f^-_A$: X →[-1, 0] are mappings and $t^+_A+1-f^+_A \le 1$, $t^-_A+1-f^-_A \le -1$. The positive membership degree $[t^+_A$(x), $1-f^+_A$(x)] denotes the satisfaction region of an element x to the property corresponding to a bipolar-valued vague set A ={(x, $[t^+_A$(x), $1-f^+_A$(x)], [t^-_A(x), $1-f^-_A$(x)]) / x∈X}, and the negative membership degree $[t^-_A$(x), $1-f^-_A$(x)] denotes the satisfaction region of x to some implicit counter-property of A ={(x, $[t^+_A$(x), $1-f^+_A$(x)], [t^-_A(x), $1-f^-_A$(x)]) / x∈X}.

Remark 3.2:
If t^+_A(x) ≠ 0 and $1-f^+_A$(x) ≠ 0, t^-_A(x) and $1-f^-_A$(x) = 0, it is the situation that x is regarded as having only positive satisfaction for A ={(x, $[t^+_A$(x), $1-f^+_A$(x)], [t^-_A(x), $1-f^-_A$(x)]) / x∈X}. If t^+_A(x) = 0 and $1-f^+_A$(x) = 0, t^-_A(x) ≠ 0 and $1-f^-_A$(x) ≠ 0 , it is the situation that x does not satisfy the property of A ={(x, $[t^+_A$(x), $1-f^+_A$(x)], [t^-_A(x), $1-f^-_A$(x)]) / x∈X}but somewhat satisfies the counter-property of A ={(x, $[t^+_A$(x), $1-f^+_A$(x)], [t^-_A(x), $1-f^-_A$(x)]) / x∈X}. It is possible for an element x to be t^+_A(x) ≠ 0, $1-f^+_A$(x) ≠ 0 and t^-_A(x) ≠ 0, $1-f^-_A$(x) ≠ 0 when the membership function of the property overlaps that of its counter-property over some portion of the domain.

Note 3.3:
For the sake of simplicity, we shall use the symbol A = (X, V^+_A, V^-_A) for the bipolar-valued vague set A = A ={(x, $[t^+_A$(x), $1-f^+_A$(x)], [t^-_A(x), $1-f^-_A$(x)]) / x∈X}, and use the notion of bipolar vague sets instead of the notion of bipolar-valued vague sets, where $V^+_A = [t^+_A, 1-f^+_A]$ and $V^-_A = [t^-_A, 1-f^-_A]$.

Definition 3.4:
A bipolar vague finite state machine (bvfsm, for short) is a triple M = (Q, X , A), where Q and X are finite nonempty sets, called the set of states and set of input symbols, respectively, and A = (X, V^-_A, V^+_A) is a bipolar vague set in Q x X x Q.

Remark 3.5:

Let X^* denote the set of all words of elements of X of finite length. Let λ denote the empty word in X^* and $|x|$ denote the length of x for every x ∈ X^*.

Definition 3.6:

Let M = (Q, X , A) be a bvfsm. Define a bipolar vague set A_* = (X, $V^-_{A_*}$, $V^+_{A_*}$) in Q x X^* x Q by $V^-_{A_*}(q, \lambda, p) = \begin{cases} -1 \text{ if } q = p \\ 0 \text{ if } q \neq p \end{cases}$, $V^+_{A_*}(q, \lambda, p) = \begin{cases} 1 \text{ if } q = p \\ 0 \text{ if } q \neq p \end{cases}$

$V^-_{A_*}(q, xa, p) = \inf_{r\in Q}[V^-_{A_*}(q, x, r) \vee V^-_A(r, a, p)]$,

$V^+_{A_*}(q, xa, p) = \sup_{r\in Q}[V^+_{A_*}(q, x, r) \wedge V^+_A(r, a, p)]$ for all p, q ∈ Q, x ∈ X^* and a ∈ X.

Remark 3.7:

Let M = (Q, X , A) be a bvfsm. Then $V^-_{A_*}(q, xy, p) = \inf_{r\in Q}[V^-_{A_*}(q, x, r) \vee V^-_{A_*}(r, y, p)]$,

$V^+_{A_*}(q, xy, p) = \sup_{r\in Q}[V^+_{A_*}(q, x, r) \wedge V^+_{A_*}(r, y, p)]$ for all p, q ∈ Q, x, y ∈ X^*.

Definition 3.8:

Let M = (Q, X , A) be a bvfsm and let p, q ∈ Q. Then p is called a bipolar immediate successor of q if the following conditions holds:

(∃a ∈ X)($V^-_A(q, a, p) < 0$, $V^+_A(q, a, p) > 0$). We say that p is a bipolar successor of q if the following holds:

$(\exists x \in X^*)(V^-_{A_*}(q, x, p) < 0, V^+_{A_*}(q, x, p) > 0)$. We denote by S(q) the set of all bipolar successors of q. For any subset T of Q, the set of all bipolar successors of T, denoted by S(T), is defined to be the set S(T) = $\cup\{S(q) / q \in T\}$.

Definition 3.9:

Let M = (Q, X , A) be a bvfsm. Let $(\forall T \subseteq Q)$. Let $A_Q = (V^-_{A_Q}, V^+_{A_Q})$ be bipolar vague set in T x X x T and N = (T, X, A_Q) be a bvfsm. Then N is called a bipolar submachine of M, if

1. A / T x X x T = A_Q, i.e., A^- / T x X x T = A^-_Q and A^+ / T x X x T = A^+_Q.
2. $S(T) \subseteq T$.

We assume that B = (B, X , A) is a bvfsm M. Obviously if K is a bipolar submachine of N and N is a bipolar submachine of M, then K is a bipolar submachine of M.

Definition 3.10:

A bvfsm M = (Q, X , A) is said to be strongly bipolar connected if $p \in S(q)$ for every p, q $\in$Q. A bipolar submachine N = (T, X, A_Q) of a bvfsm M = (Q, X , A) is said to be proper if $T \neq B$ and $T \neq Q$.

Theorem 3.11:

A bvfsm M = (Q, X , A) is strongly bipolar connected if and only if M = (Q, X , A) has a proper bipolar submachines.

Proof:

Suppose that M = (Q, X , A) is strongly bipolar connected. Let N = (T, X, A_Q) be a bipolar submachine of M such that $T \neq \Phi$. Then there exists $q \in T$. If $p \in Q$ then $p \in S(q)$ since M is strongly bipolar connected. It follows that $p \in S(q) \subseteq S(T) \subseteq T$ so that T = Q. Hence M = N, i.e., M has no proper bipolar submachines. Let p, q $\in$Q and let N = (S(q), X , A_Q) where $A_Q = (V^-_{A_Q}, V^+_{A_Q})$ is given by

$V^-_{A_Q} = V^-_A$ / S(q) x X x S(q) and $V^+_{A_Q} = V^+_A$ / S(q) x X x S(q) Then N is a bipolar submachine of M and $S(q) \neq \Phi$, and so S(q) = Q. Thus $p \in S(q)$, and therefore M is strongly bipolar connected.

Definition 3.12:

For a bvfsm A = (V^-_A, V^+_A) in a set X, the bipolar support of A is defined to be the set Supp(A) = $\{x \in X / V^-_A(x) < 0, V^+_A(x) > 0\}$.

Definition 3.13:

For a bipolar vague set A = $\{(x, (V^-_A(x), V^+_A(x)) / x \in X\}$ and $(\beta, \alpha) \in [-1,0] \times [0, 1]$, we define $V^-_{A_\beta} = \{x \in X / V^-_A(x) \leq \beta\}$, $V^+_{A_\alpha} = \{x \in X / V^+_A(x) \geq \alpha\}$ which are called the negative β- cut of A and the positive α-cut of A, respectively. The set V(A, (α, β)) = $\{x \in X / V^+_A(x) \geq \alpha$ and $V^-_A(x) \leq \beta\}$ is called a (β, α)-level subset of A.

Definition 3.14:

Let $M_1 = (Q_1, X_1, A_1)$ and $M_2 = (Q_2, X_2, A_2)$ bvfsms. A pair (α, β) of mappings α: $Q_1 \rightarrow Q_2$ and β: $X_1 \rightarrow X_2$ is called a homomorphism written (α, β) : $M_1 \rightarrow M_2$ if

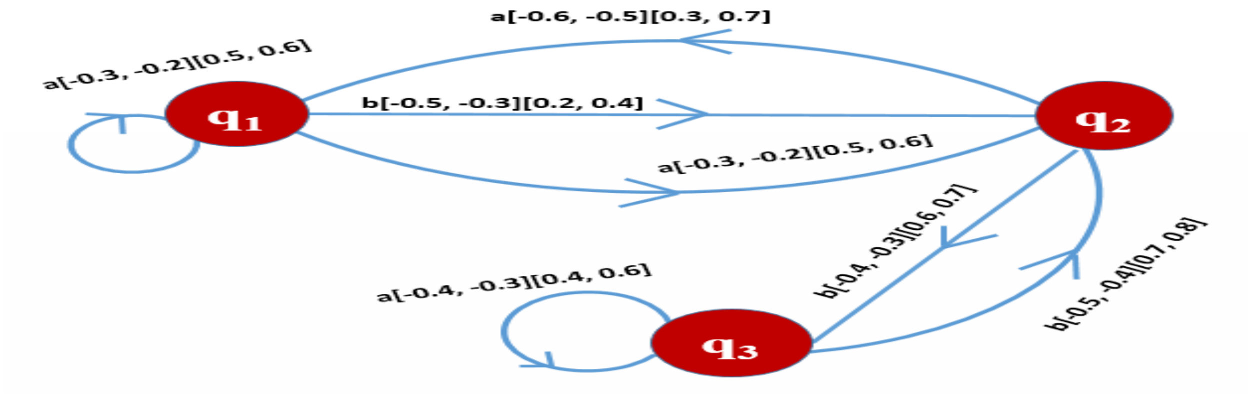
$V_{A_1}{}^-(q, x, p) \leq V_{A_2}{}^-(\alpha(q), \beta(x), \alpha(p))$, $V_{A_1}{}^+(q, x, p) \leq V_{A_2}{}^+(\alpha(q), \beta(x), \alpha(p))$ $\forall$q, p $\in Q_1$ and $\forall x \in X_1$.

Example 3.15:

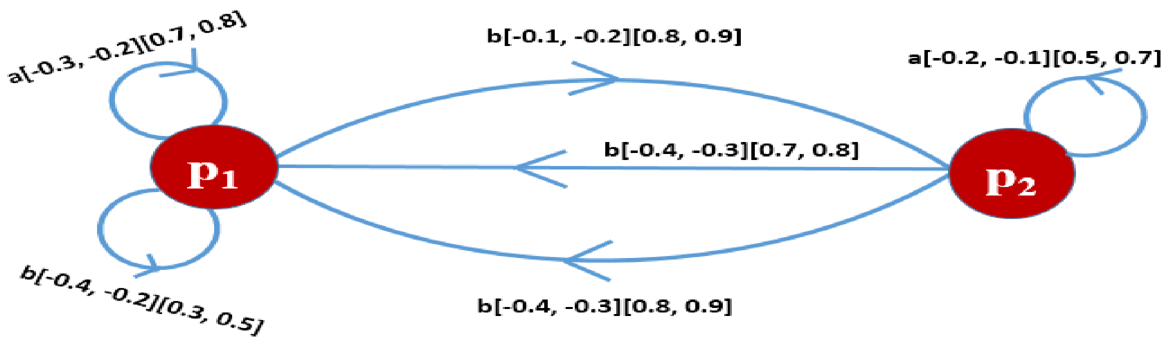
Let $M_1 = (Q_1, X_1, A_1)$ and $M_2 = (Q_2, X_2, A_2)$ be bvfsms, where $Q_1 = \{q_1, q_2, q_3\}$, X_1 = {a, b}, $Q_2 = \{p_1, p_2\}$, X_2 = {a, b} and A_1, A_2 are defined as follows.

Define α : $Q_1 \to Q_2$ and β : $X_1 \to X_2$ as follows $\alpha(q_1) = \alpha(q_2) = p_1$, $\alpha(q_3) = p_2$, β(a) = a and β(b) = b

M



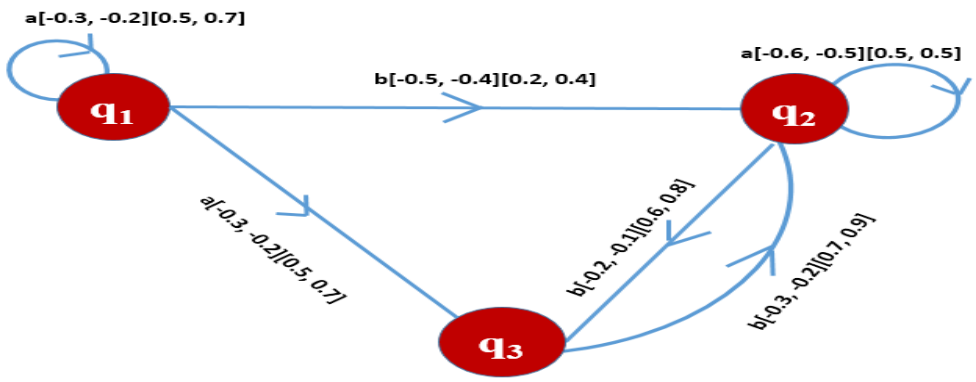
M_1



M_2(A homomorphic image of M_1)

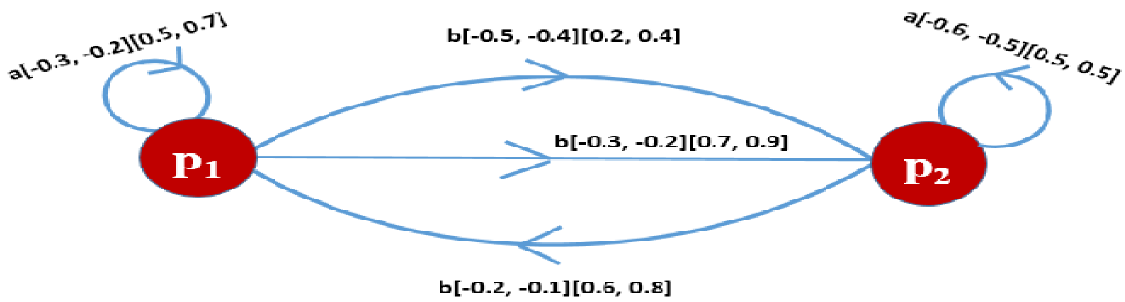
Definition 3.16:
Let $M_1 = (Q_1, X_1, A_1)$ and $M_2 = (Q_2, X_2, A_2)$ be bvfsms. A pair (α, β) of mappings α: $Q_1 \to Q_2$ and β: $X_1 \to X_2$ is called a strong homomorphism written if
$V_{A_2}{}^{-}(\alpha(q), \beta(x), \alpha(p)) = \vee\{V_{A_1}{}^{-}(q, x, t) / t \in Q_1, \alpha(t) = \alpha(p)\}$ and
$V_{A_2}{}^{+}(\alpha(q), \beta(x), \alpha(p)) = \vee\{V_{A_1}{}^{+}(q, x, t) / t \in Q_1, \alpha(t) = \alpha(p)\}$ $\forall q, p \in Q_1$ and $\forall x \in X_1$.

Example 3.17:
Let $M_1 = (Q_1, X_1, A_1)$ and $M_2 = (Q_2, X_2, A_2)$ be bvfsms, Where $Q_1 = \{q_1, q_2, q_3\}$, X_1 = {a, b}, $Q_2 = \{p_1, p_2\}$, X_2 = {a, b} and A_1, A_2 are defined as follows.



Define α : $Q_1 \to Q_2$ and β : $X_1 \to X_2$ as follows $\alpha(q_1) = \alpha(q_3) = p_1$, $\alpha(q_2) = p_2$, β(a) = a and β(b) = b.

M_1



M_2(A strong homomorphic image of M_1)

Definition 3.18:
Let $M_1 = (Q_1, X_1, A_1)$ and $M_2 = (Q_2, X_2, A_2)$ be two bvfsms. Let (α, β) : $M_1 \to M_2$ be bipolar homomorphism. Define β^* : $X_1^* \to X_2^*$ by $\beta^*(\lambda) = \lambda$ and $\beta^*(ua) = \beta^*(u)\beta(a)$ $\forall u \in X_1^*$, $a \in X_1$.

Proposition 3.19:
Let $M_1 = (Q_1, X_1, A_1)$ and $M_2 = (Q_2, X_2, A_2)$ be two bvfsms. Let (α, β) : $M_1 \to M_2$ be a strong homomorphism . Then $\forall q, r \in Q_1$ and $\forall x \in X_1$ if $V_{A_2}{}^-(\alpha(q), \beta(x), \alpha(r)) < 0$ and $V_{A_2}{}^+(\alpha(q), \beta(x), \alpha(r)) > 0$ then there exist $t \in Q_1$ such that $V_{A_1}{}^-(q, x, t) < 0$, $V_{A_1}{}^+(q, x, t) > 0$ and $\alpha(t) = \alpha(r)$.Furthermore $\forall p \in Q_1$ if $\alpha(p) = \alpha(q)$ then $V_{A_1}{}^-(q, x, t) \geq V_{A_1}{}^-(p, x, r)$ and $V_{A_1}{}^+(q, x, t) \geq V_{A_1}{}^+(p, x, r)$.
Proof:
Let $p, q, r \in Q_1$ and $x \in X_1$, $V_{A_2}{}^-(\alpha(q), \beta(x), \alpha(r)) = \vee\{V_{A_1}{}^-(q, x, s) / s \in Q_1, \alpha(s) = \alpha(r)\} < 0$ and $V_{A_2}{}^+(\alpha(q), \beta(x), \alpha(r)) = \vee\{V_{A_1}{}^+(q, x, s) / s \in Q_1, \alpha(s) = \alpha(r)\} > 0$. Since Q_1 is finite there exist $t \in Q_1$ such that $\alpha(t) = \alpha(r)$ and $V_{A_1}{}^-(q, x, t) = \vee\{V_{A_1}{}^-(q, x, s) / s \in Q_1, \alpha(s) = \alpha(r)\} < 0$ and $V_{A_1}{}^+(q, x, t) = \vee\{V_{A_1}{}^+(q, x, s) / s \in Q_1, \alpha(s) = \alpha(r)\} > 0$. Suppose $\alpha(p) = \alpha(q)$ then $V_{A_1}{}^-(q, x, t) = V_{A_2}{}^-(\alpha(q), \beta(x), \alpha(r)) = V_{A_2}{}^-(\alpha(p), \beta(x), \alpha(r)) \geq V_{A_1}{}^-(p, x, r)$
$V_{A_1}{}^+(q, x, t) = V_{A_2}{}^+(\alpha(q), \beta(x), \alpha(r)) = V_{A_2}{}^+(\alpha(p), \beta(x), \alpha(r)) \geq V_{A_1}{}^+(p, x, r)$.

Proposition 3.20:
Let $M_1 = (Q_1, X_1, A_1)$ and $M_2 = (Q_2, X_2, A_2)$ be two bvfsms. Let (α, β) : $M_1 \to M_2$ be a homomorphism. Define β^* ; $X_1^* \to X_2^*$. Then $\beta^*(uv) = \beta^*(u)\beta^*(v)$ $\forall u, v \in X_1^*$.
Proof:
Let $u, v \in X_1^*$ and $|v| = n$. If n = 0 then v = λ and hence $\beta^*(uv) = \beta^*(u) = \beta^*(u)\ \beta^*(v)$. Suppose now the result is true $\forall y \in X_1^*$ such that $|y| = n - 1$, n > 0. Let v = ya where $y \in X_1^*$, $a \in X_1$ and $|y| = n-1$.

Then $\beta^*(uv) = \beta^*(uya) = \beta^*(uy)\beta^*(a) = \beta^*(u)\beta^*(y)\ \beta^*(a) = \beta^*(u)\beta^*(ya) = \beta^*(u)\beta^*(v)$. Therefore $\beta^*(uv) = \beta^*(u)\ \beta^*(v)$, $\forall u, v \in X_1^*$.

Proposition 3.21:
Let $M_1 = (Q_1, X_1, A_1)$ and $M_2 = (Q_2, X_2, A_2)$ be two bvfsms. Let $(\alpha, \beta) : M_1 \to M_2$ be a homomorphism. Then $V_{A_{1*}}^-(q, x, p) \le V_{A_2}^-(\alpha(q), \beta^*(x), \alpha(p))$ and
$V_{A_{1*}}^+(q, x, p) \le V_{A_2}^+(\alpha(q), \beta^*(x), \alpha(p))$ $\forall$ q, p $\in Q_1$ and x $\in X_1^*$.
Proof:
Let q, p $\in Q_1$ and x $\in X_1^*$. We prove the result by induction on $|x|$= n. If n = 0 then x = λ and $\beta^*(x) = \beta^*(\lambda) = \lambda$
$V_{A_{1*}}^-(q, \lambda, p) = -1 = V_{A_2}^-(\alpha(q), \beta^*(\lambda), \alpha(p))$ if q = p
$V_{A_{1*}}^-(q, \lambda, p) = 0 = V_{A_2}^-(\alpha(q), \beta^*(\lambda), \alpha(p))$ if $q \neq p$
$V_{A_{1*}}^+(q, \lambda, p) = 1 = V_{A_2}^+(\alpha(q), \beta^*(\lambda), \alpha(p))$ if q = p
$V_{A_{1*}}^+(q, \lambda, p) = 0 = V_{A_2}^+(\alpha(q), \beta^*(\lambda), \alpha(p))$ if $q \neq p$. Suppose now the result is true $\forall$ y $\in X^*$ such that $|y|$= n – 1, n > 0. Let x = ya where y $\in X_1^*$, a $\in X_1$ and $|y|$ = n-1.
$V_{A_{1*}}^-(q, x, p) = V_{A_{1*}}^-(q, ya, p) = \wedge_{r\in Q_1}\{V_{A_{1*}}^-(q, y, r) \vee V_{A_{1*}}^-(r, a, p)\} \le \wedge_{r\in Q_1}\{V_{A_{2*}}^-(\alpha(q), \beta^*(y), \alpha(r)) \vee V_{A_2}^-(\alpha(r), \beta(a), \alpha(p))$ (by homomorphism) $\le \wedge_{r'\in Q_2}\{V_{A_{2*}}^-(\alpha(q), \beta^*(y), r') \vee V_{A_2}^-(r', \beta(a), \alpha(p))\} = V_{A_{2*}}^-(\alpha(q), \beta^*(y)\beta(a), \alpha(p)) = V_{A_{2*}}^-(\alpha(q), \beta^*(ya), \alpha(p)) = V_{A_{2*}}^-(\alpha(q), \beta^*(x), \alpha(p))$. Hence $V_{A_{1*}}^-(q, x, p) \le V_{A_{2*}}^-(\alpha(q), \beta^*(x), \alpha(p))$. Now, $V_{A_{1*}}^+(q, x, p) = V_{A_{1*}}^+(q, ya, p) = \vee_{r\in Q_1}\{V_{A_{1*}}^+(q, y, r) \wedge V_{A_{1*}}^+(r, a, p)\} \le \vee_{r\in Q_1}\{V_{A_{2*}}^+(\alpha(q), \beta^*(y), \alpha(r)) \wedge V_{A_2}^+(\alpha(r), \beta(a), \alpha(p))$ (by homomorphism) $\le \vee_{r'\in Q_2}\{V_{A_{2*}}^+(\alpha(q), \beta^*(y), r') \wedge V_{A_2}^+(r', \beta(a), \alpha(p))\} = V_{A_{2*}}^+(\alpha(q), \beta^*(y)\beta(a), \alpha(p)) = V_{A_{2*}}^+(\alpha(q), \beta^*(ya), \alpha(p)) = V_{A_{2*}}^+(\alpha(q), \beta^*(x), \alpha(p))$. Hence $V_{A_{1*}}^+(q, x, p) \le V_{A_{2*}}^+(\alpha(q), \beta^*(x), \alpha(p))$.

Proposition 3.22:
Let $M_1 = (Q_1, X_1, A_1)$ and $M_2 = (Q_2, X_2, A_2)$ be two bvfsms. Let $(\alpha, \beta) : M_1 \to M_2$ be a homomorphism. Then α is one-one if and only if $V_{A_{1*}}^-(q, x, p) = V_{A_{2*}}^-(\alpha(q), \beta^*(x), \alpha(p))$, $V_{A_{1*}}^+(q, x, p) = V_{A_{2*}}^+(\alpha(q), \beta^*(x), \alpha(p))$ $\forall$ q, p $\in Q_1$ and x $\in X_1^*$.
Proof:
Suppose α is one – one. Let p, q $\in Q_1$ and $\in X_1^*$. Let $|x|$ = n. We prove the result by induction on n. Let n = 0 then x = λ and $\beta^*(\lambda) = \lambda$ Now α(q) = α(p) if and only if q = p. Hence $V_{A_{1*}}^-(q, x, p) = -1$ if and only if $V_{A_{2*}}^-(\alpha(q), \beta(\lambda), \alpha(p)) = -1$ and $V_{A_{1*}}^+(q, x, p) = 1$ if and only if $V_{A_{2*}}^+(\alpha(q), \beta(\lambda), \alpha(p)) = 1$ (By Strong homomorphism). Suppose the result is true $\forall y \in X_1^*$, $|y|$ = n-1, n > 0. Let x = ya, $|y|$ = n-1, y $\in X_1^*$, a $\in X_1$. Then $V_{A_{2*}}^-(\alpha(q), \beta^*(x), \alpha(p)) = V_{A_{2*}}^-(\alpha(q), \beta^*(ya), \alpha(p)) = V_{A_{2*}}^-(\alpha(q), \beta^*(y)\beta(a), \alpha(p)) = \wedge_{r\in Q_1}\{V_{A_{2*}}^-(\alpha(q), \beta^*(y), \alpha(r)) \vee V_{A_2}^-(\alpha(r), \beta(a), \alpha(p))\} = \wedge_{r\in Q_1}\{V_{A_{1*}}^-(q, y, r) \vee V_{A_1}^-(r,a, p)\} = V_{A_{1*}}^-(q, ya, p) = V_{A_{1*}}^-(q, x, p)$. Now $V_{A_{2*}}^+(\alpha(q), \beta^*(x), \alpha(p)) = V_{A_{2*}}^+(\alpha(q), \beta^*(ya), \alpha(p)) = V_{A_{2*}}^+(\alpha(q), \beta^*(y)\beta(a), \alpha(p)) = \vee_{r\in Q_1}\{V_{A_{2*}}^+(\alpha(q), \beta^*(y), \alpha(r)) \wedge V_{A_2}^+(\alpha(r), \beta(a), \alpha(p))\} = \vee_{r\in Q_1}\{V_{A_{1*}}^+(q, y, r) \wedge V_{A_1}^+(r,a, p)\} = V_{A_{1*}}^+(q, ya, p) = V_{A_{1*}}^+(q, x, p)$.
Conversely, Let q, p $\in Q_1$ and let α(q) = α(p). Then $V_{A_{2*}}^-(\alpha(q), \lambda, \alpha(p)) = V_{A_{1*}}^-(q, \lambda, p)$ and $V_{A_{2*}}^+(\alpha(q), \lambda, \alpha(p)) = V_{A_{1*}}^+(q, \lambda, p)$. Hence q = p. Hence α is one-one.

Definition 3.23:

Let M = (Q, X, A) be a bvfsm. M is said to be retrievable if $\forall q\in Q$, $\forall y\in X^*$ if $\exists u \in Q$ such that $V_{A_*}(q, y, u) = [V^-_{A_*}(q, y, u) < 0, V^+_{A_*}(q, y, u) > 0]$, then $\exists x\in X^*$ such that $V_{A_*}(q, yx, q) = [V^-_{A_*}(q, yx, q) < 0, V^+_{A_*}(q, yx, q) > 0]$.

Definition 3.24:

Let M = (Q, X, A) be a bvfsm. M is said to be quasi-retrievable if $\forall q \in Q$, $\forall y \in X^*$ if $\exists$ t $\in$ Q such that V_{A_*}(q, y, t) = [$V^-_{A_*}$(q, y, t) < 0, $V^+_{A_*}$(q, y, t) > 0], then $\exists x \in X^*$ such that V_{A_*}(q, yx, q) = [$V^-_{A_*}$(q, x, q) < 0, $V^+_{A_*}$(t, x, q) > 0].

Where $V^-_{A_*}(q, yx, p) = \inf_{u\in Q}[V^-_{A_*}(q, y, u) \vee V^-_{A_*}(u, x, q)] < 0$,

$V^+_{A_*}(q, yx, p) = \sup_{u\in Q}[V^+_{A_*}(q, y, u) \wedge V^+_{A_*}(u, x, q)] > 0$.

Definition 3.25:

Let M = (Q, X , A) be a bvfsm and let q, r, s $\in$ Q. Then r and s are said to be bipolar q-related if there exists y $\in X^*$ such that V_{A_*}(q, y, r) = [$V^-_{A_*}$(q, y, r) < 0, $V^+_{A_*}$(q, y, r) > 0] and V_{A_*}(q, y, s) = [$V^-_{A_*}$(q, y, s) < 0, $V^+_{A_*}$(q, y, s) > 0] . We say that r and s are bipolar q-twins if

1. r and s are q-related.
2. S(r) = S(s).

Definition 3.26:

Let M = (Q, X, A) be a bvfsm. We say that M satisfies the exchange property if the following condition holds:

Let p, q $\in$ Q and let T $\subseteq$ Q. Suppose that if p $\in$ S(T $\cup$ {q}), p does not belong to S(T), then q $\in$ S(T $\cup$ {p}).

IV. Bipolar Retrievable

Definition 4.1:

A bvfsm M = (Q, X , A) is said to be bipolar retrievable if $(\forall q \in Q)\ (\forall y \in X^*)(\exists u \in Q)\ (V^-_{A_{Q^*}}(q, y, u) < 0,\ V^+_{A_{Q^*}}(q, y, u) > 0)$

$\Rightarrow (\exists x \in X^*)\ (V^-_{A_{Q^*}}(u, x, q) < 0,\ V^+_{A_{Q^*}}(u, x, q) > 0)$.

Definition 4.2:

Let M = (Q, X , A) be a bvfsm and let q, r, s $\in$ Q. Then r and s are said to be bipolar q-related if there exists y $\in X^*$ such that $V^-_{A_{Q^*}}$(q, y, r) < 0, $V^-_{A_{Q^*}}$(q, y, s) < 0, $V^+_{A_{Q^*}}$(q, y, r) > 0 and $V^+_{A_{Q^*}}$(q, y, s) > 0. We say that r and s are bipolar q-twins if

3. r and s are bipolar q-related.
4. S(r) = S(s).

Proposition 4.3:

Let M = (Q, X , A) be a bvfsm. Then the following assertions are equivalent.

1. $\forall$q, r, s $\in$ Q, if r and s are bipolar q-related, then r and s are bipolar q-twins.
2. $(\forall p, q, r \in Q)\ (\forall x, y \in X^*)\ (V^-_{A_{Q^*}}(q, y, r) < 0,\ V^-_{A_{Q^*}}(q, yx, p) < 0,\ V^+_{A_{Q^*}}(q, y,r) > 0,\ V^+_{A_{Q^*}}(q, yx, p) > 0 \Rightarrow p \in S(r))$.

Proof:

(1)$\Rightarrow$(2) Since r and s are bipolar q-twins so that p$\in$S(s) = S(r).

(2)$\Rightarrow$(1) Suppose that (2) is valid. Let q, r, s $\in$ Q be such that r and s are bipolar q-related . Then there exists y $\in X^*$ such that $V^-_{A_{Q^*}}$(q, y, r) < 0, $V^-_{A_{Q^*}}$(q, y, s) < 0, $V^+_{A_{Q^*}}$(q, y, r) > 0 and $V^+_{A_{Q^*}}$(q, y, s) > 0 . If p $\in$ S(s), then there exists x $\in X^*$ such that $V^-_{A_{Q^*}}$(s, x, p) < 0 and $V^+_{A_{Q^*}}$(s, x, p) > 0. Then $V^-_{A_{Q^*}}(q, xy, p) = \inf_{u\in Q}[V^-_{A_{Q^*}}(q, y, u) \vee V^-_{A_{Q^*}}(u, x, p)] < 0$ and

${V^+}_{A_{Q^*}}(q, xy, p) = \sup_{u\in Q}[{V^+}_{A_{Q^*}}(q, y, u) \wedge {V^+}_{A_{Q^*}}(u, x, p)] > 0$. Thus $p \in S(r)$ by hypothesis. Similarly if $p \in S(r)$ then $p \in S(s)$. Therefore r and s are bipolar q-twins.

Proposition 4.4:

A bvfsm M = (Q, X, A) is a bipolar retrievable if and only if it satisfies

1. $(\forall q \in Q)\ (\forall y \in X^*)(\exists u \in Q)\ ({V^-}_{A_{Q^*}}(q, y, u) < 0,\ {V^+}_{A_{Q^*}}(q, y, u) > 0) \Rightarrow (\exists x \in X^*)\ ({V^-}_{A_{Q^*}}(q, yx, q) < 0,\ {V^+}_{A_{Q^*}}(q, yx, q) > 0)$.
2. q, r, s $\in$ Q, if r and s are bipolar q-related then r and s are bipolar q-twins.

Proof: Obvious.

V. Bipolar vague finite swithchboard state machines

Definition 5.1:

A bvfsm M = (Q, X, A) is said to be switching if it satisfies

${V^-}_{A_*}(q, a, p) = {V^-}_{A_*}(p, a, q)$, ${V^+}_{A_*}(q, a, p) = {V^+}_{A_*}(p, a, q)$ for all p, q $\in$ Q, and a $\in$ X.

Definition 5.2:

A bvfsm M = (Q, X, A) is said to be commutative if it satisfies

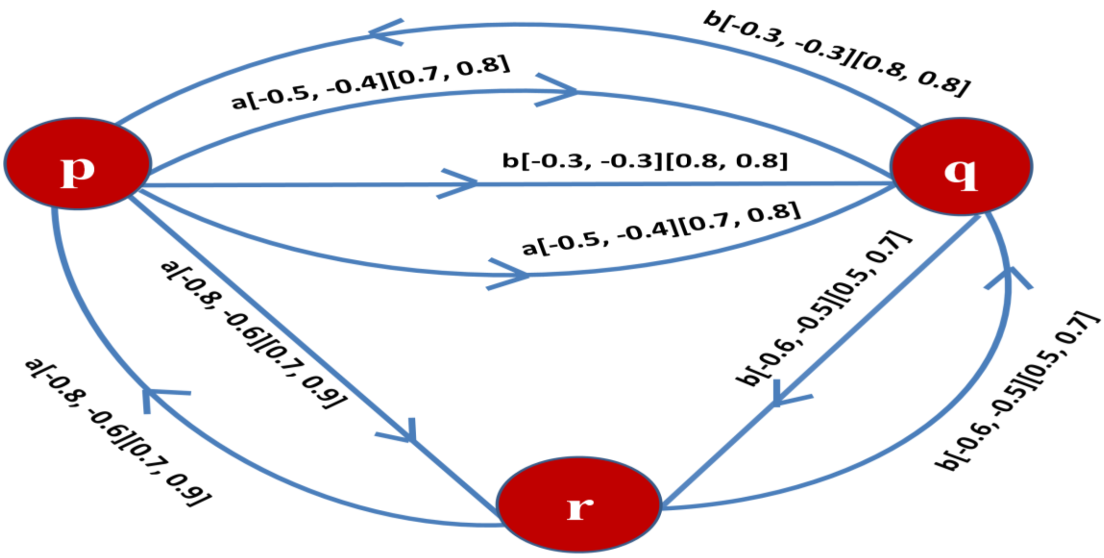
${V^-}_{A_*}(q, ab, p) = {V^-}_{A_*}(p, ba, q)$, ${V^+}_{A_*}(q, ab, p) = {V^+}_{A_*}(p, ba, q)$ for all p, q $\in$ Q, and a, b $\in$ X.

Definition 5.3:

f a bvfsm M = (Q, X, A) is both swiching and commutative, we say that M is a bipolar vague finite switchboard state machine (bvfssm, for short)

Example 5.4:

Let M = (Q, X, A) is a bvfsm, where Q = {p, q, r}, X = {a, b} and let $A_Q = ({V^-}_{A_Q}, {V^+}_{A_Q})$ is defined as follows:



It is easy to see that M = (Q, X, A) is bvfssm.

Proposition 5.5:

If M = (Q, X, A) is a commutative bvfsm, then ${V^-}_{A_*}(q, xa, p) = {V^-}_{A_*}(q, ax, p)$,

$V^+_{A_*}(q, xa, p) = V^+_{A_*}(q, ax, p)$ for all p, q ∈ Q, and a ∈ X and x ∈ X^*.

Proof:

Let p, q ∈ Q, a ∈ X and x ∈ X^*. Suppose $|x| = n$. If n = 0, then x = λ.

Thus $V^-_{A_*}(q, xa, p) = V^-_{A_*}(q, \lambda a, p) = V^-_{A_*}(q, a, p) = V^-_{A_*}(q, a\lambda, p) = V^-_{A_*}(q, xa, p)$ and $V^+_{A_*}(q, xa, p) = V^+_{A_*}(q, \lambda a, p) = V^+_{A_*}(q, a, p) = V^+_{A_*}(q, a\lambda, p) = V^+_{A_*}(q, ax, p)$. Suppose the result is true for all u ∈ X^* with $|u| = n-1$, n > 0. Let b ∈ X be such that x = ub. Then $V^-_{A_*}(q, xa, p) = V^-_{A_*}(q, uba, p) = \inf_{r\in Q}[V^-_{A_*}(q, u, r) \vee V^-_{A_*}(r, ba, p)] = \inf_{r\in Q}[V^-_{A_*}(q, u, r) \vee V^-_{A_*}(r, ab, p)] = V^-_{A_*}(q, uab, p) = \inf_{r\in Q}[V^-_{A_*}(q, ua, r) \vee V^-_{A}(r, b, p)] = \inf_{r\in Q}[V^-_{A_*}(q, au, r) \vee V^-_{A}(r, b, p)] = V^-_{A_*}(q, aub, p) = V^-_{A_*}(q, ax, p)$ and $V^+_{A_*}(q, xa, p) = V^+_{A_*}(q, uba, p) = \sup_{r\in Q}[V^+_{A_*}(q, u, r) \wedge V^+_{A_*}(r, ba, p)] = \sup_{r\in Q}[V^+_{A_*}(q, u, r) \wedge V^+_{A_*}(r, ab, p)] = V^+_{A_*}(q, uab, p) = \sup_{r\in Q}[V^+_{A_*}(q, ua, r) \wedge V^+_{A}(r, b, p)] = \sup_{r\in Q}[V^+_{A_*}(q, au, r) \wedge V^+_{A}(r, b, p)] = V^+_{A_*}(q, aub, p) = V^+_{A_*}(q, ax, p)$. This completes the proof.

Proposition 5.6:

If M = (Q, X, A) is a bvfssm, then $V^-_{A_*}(q, x, p) = V^-_{A_*}(p, x, q)$, $V^+_{A_*}(q, x, p) = V^+_{A_*}(p, x, q)$ for all p, q ∈ Q and x ∈ X^*.

Proof:

Let p, q ∈ Q and x ∈ X^*. We prove the result by induction on $|x| = n$. Since x = λ whenever n = 0, we have $V^-_{A_*}(q, x, p) = V^-_{A_*}(q, \lambda, p) = V^-_{A_*}(p, \lambda, q) = V^-_{A_*}(p, x, q)$ and $V^+_{A_*}(q, x, p) = V^+_{A_*}(q, \lambda, p) = V^+_{A_*}(p, \lambda, q) = V^+_{A_*}(p, x, q)$. Hence the result is true for n = 0. Assume that the result is valid for all u ∈ X^* with $|u| = n-1$, n > 0, that is $V^-_{A_*}(q, u, p) = V^-_{A_*}(p, u, q)$, $V^+_{A_*}(q, u, p) = V^+_{A_*}(p, u, q)$. Let a ∈ X and x ∈ X^* be such that x = ua. Then $V^-_{A_*}(q, x, p) = V^-_{A_*}(q, ua, p) = \inf_{r\in Q}[V^-_{A_*}(q, u, r) \vee V^-_{A}(r, a, p)] = \inf_{r\in Q}[V^-_{A_*}(r, u, q) \vee V^-_{A}(p, a, r)] = \inf_{r\in Q}[V^-_{A_*}(r, u, q) \vee V^-_{A_*}(p, a, r)] = \inf_{r\in Q}[V^-_{A_*}(p, a, r) \vee V^-_{A_*}(r, u, q)] = V^-_{A_*}(p, au, q) = V^-_{A_*}(q, ua, p) = V^-_{A_*}(p, x, q)$ and $V^+_{A_*}(q, x, p) = V^+_{A_*}(q, ua, p) = \sup_{r\in Q}[V^+_{A_*}(q, u, r) \wedge V^+_{A}(r, a, p)] = \sup_{r\in Q}[V^+_{A_*}(r, u, q) \wedge V^+_{A}(p, a, r)] = \sup_{r\in Q}[V^+_{A_*}(r, u, q) \wedge V^+_{A_*}(p, a, r)] = \sup_{r\in Q}[V^+_{A_*}(p, a, r) \wedge V^+_{A_*}(r, u, q)] = V^+_{A_*}(p, au, q) = V^+_{A_*}(q, ua, p) = V^+_{A_*}(p, x, q)$. This shows that the result is true for $|u| = n$. This completes the proof.

Proposition 5.7:

If M = (Q, X, A) is a bvfssm, then $V^-_{A_*}(q, xy, p) = V^-_{A_*}(q, yx, p)$, $V^+_{A_*}(q, xy, p) = V^+_{A_*}(q, yx, p)$ for all p, q ∈Q and x, y ∈ X^*.

Proof:

Let p, q ∈ Q and x, y ∈X^*. Assume that $|y| = n$. If n = 0, then y = λ and so $V^-_{A_*}(q, xy, p) = V^-_{A_*}(q, x\lambda, p) = V^-_{A_*}(q, \lambda x, p) = V^-_{A_*}(q, yx, p)$ and $V^+_{A_*}(q, xy, p) = V^+_{A_*}(q, x\lambda, p) = V^+_{A_*}(q, \lambda x, p) = V^+_{A_*}(q, yx, p)$. Suppose that $V^-_{A_*}(q, xu, p) = V^-_{A_*}(q, ux, p)$ and $V^+_{A_*}(q, xu, p) = V^+_{A_*}(q, ux, p)$. Let $|u| = n$, n > 0. Let y = ua where a ∈ X and u ∈ X^* with $|u| = n-1$, n > 0. Then $V^-_{A_*}(q, xy, p) = V^-_{A_*}(q, xua, p) = \inf_{r\in Q}[V^-_{A_*}(q, xu, r) \vee V^-_{A}(r, a, p)] = \inf_{r\in Q}[V^-_{A_*}(q, ux, r) \vee V^-_{A}(r, a, p)] = \inf_{r\in Q}[V^-_{A_*}(r, ux, q) \vee V^-_{A}(p, a, r)] = \inf_{r\in Q}[V^-_{A}(p, a, r) \vee V^-_{A_*}(r, ux, q)] = V^-_{A_*}(p, aux, q) = \inf_{r\in Q}[V^-_{A_*}(p, au, r) \vee V^-_{A}(r, x, q)] = \inf_{r\in Q}[V^-_{A_*}(p, ua, r) \vee V^-_{A}(r, x, q)] = V^-_{A_*}(p, uax, q) = V^-_{A_*}(q, uax, p) = V^-_{A_*}(q, yx, p)$ and $V^+_{A_*}(q, xy, p) = V^+_{A_*}(q, xua, p) = \sup_{r\in Q}[V^+_{A_*}(q, xu, r) \wedge V^+_{A}(r, a, p)] = \sup_{r\in Q}[V^+_{A_*}(q, ux, r) \wedge V^+_{A}(r, a, p)] = \sup_{r\in Q}[V^+_{A_*}(r, ux, q) \wedge V^+_{A}(p, a, r)] = \sup_{r\in Q}[V^+_{A}(p, a, r) \wedge V^+_{A_*}(r, ux, q)] = V^+_{A_*}(p, aux, q) = \sup_{r\in Q}[V^+_{A_*}(p, au, r) \wedge V^+_{A}(r, x, q)] = \sup_{r\in Q}[V^+_{A_*}(p, ua, r) \wedge V^+_{A}(r, x, q)] = V^+_{A_*}(p, uax, q) = V^+_{A_*}(q, uax, p) = V^+_{A_*}(q, yx, p)$. This completes the proof.

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