Intuitionistic Step N- Fuzzy Soft Normal Subgroup Over Q-Fuzzy Soft Version

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***Abstract**: In this paper, we have introduced the concept of step N-fuzzy soft subgroup and step N-fuzzy soft cosset of a given group. We study the concept of step N-fuzzy soft normal subgroup and have discussed various related properties. We have also studied the effect on the image and inverse image of step N-fuzzy soft subgroup (normal) under group homomorphism.*

***Keywords:** soft set, N-intuitionistic fuzzy soft set, step N-intuitionistic fuzzy soft subgroup, homomorphism, cosset, quotient group, isomorphic, Q-fuzzy soft set.*

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I. INTRODUCTION

The fuzzy set theory becomes a strong area of making observations in different area's like medical science, social sciences, engineering, management sciences, artificial intelligence, robotics, computer networks, decision making and so on. A fuzzy set was first introduced by Zadeh [13] and then the fuzzy sets have been used in the reconsideration of classical mathematics. Yuan et.al [12] introduced the concept of fuzzy subgroup with thresholds. Through membership function, we obtain information which makes possible for us to reach the conclusion. Due to unassociated sorts of unpredictably's occurring in different area's of life like economics, engineering, medical sciences, management sciences, psychology, sociology, decision making and fuzzy set as noted and often effective mathematical instruments have been offered to make, be moving in and grip those unpredictably's. A fuzzy subgroup with thresholds λ and μ is also called a (λ, μ)-fuzzy subgroup. A.Solairaju and R.Nagarajan introduced the concept of structures of Q- fuzzy groups [10]. A.Solairaju and R.Nagarajan studied some structure properties of upper Q-fuzzy index order with upper Q- fuzzy subgroups[11].A.Rosenfeld [9] defined fuzzy groups.Such inaccuracies are associated with the membership function that belongs to [0,1]. Since the establishment of fuzzy set, several extensions have been made such as Atanassov's([1], [2], [3], [4])work on intuitionistic fuzzy set (IFS) was quite remarkable as he extended the concept of FSs by assigning non-membership degree say "N(x)" along with membership degree say "P(x)" with condition that $0 \leq P(x)+N(x) \leq 1$. Form last few decades, the IFS has been explored by many researchers and successfully applied to many practical fields like medical diagnosis, clustering analysis, decision making pattern recognition ([1], [2], [3],[4]). Strengthening the concept of IFS suggest pythagorean fuzzy sets which somehow enlarge the space of positive membership and negative membership by introducing some new condition that $0 \leq P^2(x) + N^2(x) \leq 1$. Molodtsov [8] introduced the concept of soft sets that can be seen as a new mathematical theory for dealing with uncertainty. The soft set theory has been applied to many different fields with great success. Maji et.al ([5],[6],[7]) worked on theoretical study of soft sets in detail, and presented an application of soft set in the decision making problem using the reduction of rough sets. In this paper, we introduce the notion of step N-intuitionistic fuzzy soft cossets and establish their algebraic properties .We define the notion of step N-intuitionistic fuzzy soft subgroup and investigate the condition under which a fuzzy soft subgroup is step N-intuitionistic fuzzy soft subgroup. We also initiate the study of step N-intuitionistic fuzzy soft normal subgroup and quotient group with respect to step N-intuitionistic fuzzy soft normal subgroup and prove some of their various group theoretic properties.

II. PRELIMINARIES AND BASIC CONCEPTS

In this section, we study some fundamental characterization of step N-intuitionistic fuzzy soft subgroup which play a key role in obtaining the basic group theoretic results in terms of their respective fuzzy versions. Some details of these concepts are given below which are very essential for our further discussion.

Definition 2.1: Let U be any universal set, E set of parameters and A$\subseteq$ E. Then a pair (K,A) is called soft set over U, where K is a mapping from A to 2^U , the power set of U.

Example 2.2: Let X={c_1,c_2,c_3} be the set of three cars and E={costly(e_1), metallic colour(e_2), cheap(e_3)}be the set of parameters, where A={e_1,e_2} $\subset$ E. Then (K,A)={K(e_1)={c_1,c_2,c_3},K(e_2)={c_1,c_2}} is the crisp soft set over X.

Definition 2.3: Let U be an initial universe. Let P (U) be the power set of U, E be the set of all parameters and A$\subseteq$ E. A soft set (f_A, E) on the universe U is defined by the set of order pairs (f_A, E) = {(e, f_A (e)): e$\in$ E, $f_A\in$ P (U)} where f_A : E $\rightarrow$ P (U) such that f_A (e) = ϕ, if e $\notin$A. Here 'f_A' is called an approximate function of the soft set.

Example 2.4: Let U = {u_1,u_2, u_3, u_4} be a set of four shirts and E = {white(e_1),red(e_2), blue(e_3)} be a set of parameters. If A = {e_1, e_2} $\subseteq$ E. Let $f_A(e_1)$ = {u_1, u_2 ,u_3, u_4} and $f_A(e_2)$= {u_1,u_2,u_3}. Then we write the soft set (f_A, E)= {(e_1, {u_1, u_2 ,u_3, u_4}), (e_2,{u_1,u_2,u_3})} over U which describe the "colour of the shirts" which Mr. X is going to buy. We may represent the soft set in the following form: U={(e_1,u_1), (e_2,u_1), (e_1,u_2), (e_2,u_2), (e_1,u_3), (e_2,u_3), (e_1, u_4)}

Definition 2.5:Let U be the universal set, E set of parameters and A$\subset$ E. Let K(X) denote the set of all fuzzy subsets of U. Then a pair (K,A) is called fuzzy soft set over U, where K is a mapping from A to K(U).

Example 2.6:Let U={c_1,c_2,c_3} be the set of three cars and E={costly(e_1),metallic colour(e_2), cheap(e_3)} be the set of parameters, where A={e_1,e_2}$\subset$E. Then (K,A)={K(e_1)={c_1/0.8,c_2/0.6,c_3/0.1}, K(e_2)={c_1/0.2,c_2/0.5,c_3/0.9}} is the fuzzy soft set over U denoted by F_A.

Definition 2.7: Let X be a non-empty set. A mapping $A\colon X\times Q\rightarrow[-1,\,0]$ is called a negative fuzzy soft subset (abbreviated a N-fuzzy soft subset) of X with Q-fuzzy version. Let A and B be two step N-intuitionistic fuzzy soft subsets of a set X with Q-fuzzy version. Then the following characterizations of these fuzzy sets have been discussed in [J.N.Mordeson].

1. $A\leq B$ if and only if $A(x,e)\leq B(x,q),\ \textit{for all}\ \ x\in X\ \textit{and}\ \ q\in Q.$
2. $A=B$ if and only if $A\leq B\ \textit{and}\ B\leq A.$
3. The complement of the N-intuitionistic fuzzy soft set A is A^C and is defined is $A^C(x,q)=1-A(x,q)$.
4. $(A\cap B)(x,q)=\min\{A(x,q),\,B(x,q)\},\ \textit{for all}\ \ x\in X\ \textit{and}\ \ q\in Q.$
5. $(A\cup B)(x,q)=\max\{A(x,q),\,B(x,q)\},\ \textit{for all}\ \ x\in X\ \textit{and}\ \ q\in Q.$

Definition 2.8: Let A be N-intuitionistic fuzzy soft subset with Q-fuzzy version of a set X and $\alpha\in[-1,\,0]$. The set $A_\alpha=\{x\in X\,/\,A(x,q)\geq\alpha\}$ is called a level subset of N-intuitionistic fuzzy soft subset of A.

Definition 2.9: Let A be N-intuitionistic fuzzy soft subset under Q-fuzzy version of a group G. Then 'A' is called N-intuitionistic fuzzy soft subgroup if

(NIFG–1) : $A(xy,q)\geq\min\{A(x,q),\,B(x,q)\},\ \textit{for all}\ \ x\in X\ \textit{and}\ \ q\in Q.$

(NIFG–2) : $A(x^{-1},q)\geq A(x,q),\ \textit{for all}\ \ x\in X\ \textit{and}\ \ q\in Q.$

It is easy to show that an N-intuitionistic fuzzy soft subset under Q-fuzzy version of a group G satisfies $A(x,q)\leq A(e,q)\ \textit{and}\ \ A(x^{-1},q)=A(x,q),\ \textit{for all}\ \ x\in X\ \textit{and}\ \ q\in Q$, where 'e' is the identity element of G.

Proposition 2.10: A function $A:G\times Q\rightarrow[-1,\,0]$ is an N-intuitionistic fuzzy soft subset under Q-fuzzy version of a group G. Then

$$A(xy^{-1},q) \geq \min\{A(x,q),\ A(y,q)\},\ \textit{for all}\ \ x,y \in G\ \textit{and}\ \ q \in Q.$$

Proposition 2.11: A function $A : G \times Q \to [-1,\ 0]$ is an N-intuitionistic fuzzy soft subset under Q-fuzzy version of a group G. Then

(i) $A(x,q) \leq A(e,q),\ \textit{for all}\ \ x \in G\ \textit{and}\ \ q \in Q,\ \textit{where}\ \ 'e'\ \textit{is identityel ement of}\ G.$

(ii) $A(xy^{-1},q) = A(e,q), \textit{which implies that}\ A(x,q) = A(y,q),\ \textit{for all}\ x,y \in G\ \textit{and}\ \ q \in Q.$

Theorem 2.12: Let G be a group and 'A' be N-intuitionistic fuzzy soft subset under Q-fuzzy version of a group G. Then 'A' is N-intuitionistic fuzzy soft subgroup if and only if the level subset A_α for $\alpha \in [-1,\ 0]$, $A(e,q) \geq \alpha$, is a subgroup of G, where 'e' is the identity of G.

Definition 2.13: Let $A : G \times Q \to [-1,\ 0]$ be an N-intuitionistic fuzzy soft subgroup of G. 'A' is called N-intuitionistic fuzzy soft subgroup if $A(xy,q) = A(yx,q),\ \textit{for all}\ \ x,y \in G\ \textit{and}\ \ q \in Q.$

III. PROPERTIES OF STEP N- INTUITIONISTIC FUZZY SOFT SUBSETS

Definition 3.1: Let $A : G \times Q \to [-1,\ 0]$ be an N-intuitionistic fuzzy soft normal subgroup of G. For any $x \in G$, the N-intuitionistic fuzzy soft set $xA : G \times Q \to [-1,\ 0]$ defined by $(xA)(y,q) = A(x^{-1}y,q),\ \textit{for all}\ \ y \in G\ \textit{and}\ \ q \in Q$ is called left N-intuitionistic fuzzy soft cosset of A. The right N-intuitionistic fuzzy soft cosset of A may be defined in a similar way.

Definition 3.2: Let $f : G_1 \times Q \to G_2 \times Q$ be a homomorphism from a group G_1 into a group G_2. Let A and B be N-intuitionistic fuzzy soft subsets of G_1 and G_2 respectively. Then $f(A)\ \textit{and}\ \ f^{-1}(B)$ are respectively the image of N-intuitionistic fuzzy soft set A and the inverse image of N-intuitionistic fuzzy soft set B, for every $y \in G_2$ defined as

$$f(A)(y,q) = \begin{cases} \sup A(x,q)/\, x \in f^{-1}(y,q),\ \textit{if}\ \ f^{-1}(y,q) \neq \phi \\ 1, \qquad\qquad\qquad\qquad\qquad \textit{if}\ \ f^{-1}(y,q) \neq \phi \end{cases}$$

For every $x \in G,\ \ f^{-1}(B)(x,q) = B\, f(x,q).$

Remark 3.3: It is quite evident that a group homomorphism 'f' admits the following characterizations.

(i) $f(A)f(x,q) \geq A(x,q),\ \textit{for every element}\ \ x \in G\ \textit{and}\ \ q \in Q.$

(ii) When 'f' is bijective map, $f(A)f(x,q) = A(x,q),\ \textit{for all}\ \ x \in G\ \textit{and}\ \ q \in Q.$

Definition 3.4: A function $\delta : [-1,\ 0] \times [-1,\ 0] \to [-1,\ 0]$ is said to be a δ-norm if and only if 'δ' admits following properties for all a, b, c, d in $[-1,\ 0]$

(i) $\delta(a,b) = \delta(b,a)$

(ii) $\delta(a,\delta(b,c)) = \delta(\delta(a,b),c)$

(iii) $\delta(a,1) = \delta(1,a) = 1$

(iv) If $a \leq c\ \textit{and}\ b \leq d$ then $\delta(a,b) = \delta(c,d).$

Definition 3.5: Let $\delta_b : [-1,\ 0] \times [-1,\ 0] \to [-1,\ 0]$ be the bounded difference norm defined by $\delta_b(a,b) = \max\,(a+b+1,\ 0),\ -1 \leq a \leq 0,\ -1 \leq b \leq 0$. Clearly the bounded difference norm satisfies all the axioms of δ-norm.

Definition 3.6: Let A be N-intuitionistic fuzzy soft subset of a set X and $\alpha \in [-1,\ 0]$. The N-intuitionistic fuzzy soft set A_* of X is called step N-intuitionistic fuzzy soft subset of X (with respect to intuitionistic fuzzy set A) and is defined as

$$A_*(x,q) = \delta_b(A(x,q),\alpha), \ \ for \ all \ \ x \in X, \ q \in Q.$$

Remark 3.7: It is important to note that one can obtain the classical fuzzy soft subset $A(x,q)$ by choosing the value of α = - 1 in the above definition. Whereas the case become crisp for the choice α = 0. These algebraic facts lead to more that the case illustrates the step N-intuitionistic fuzzy soft version with respect to any intuitionistic fuzzy soft subset for the value of α, when $\alpha \in [-1, \ 0]$.

Theorem 3.8: Let A and B be any two intuitionistic fuzzy soft sets of X. Then $(A \cap B)_* = A_* \cap B_*$

***Proof*:** In view of definition 3.6, we have

$$\begin{aligned}(A \cap B)_*(x,q) &= \delta_b((A \cap B)(x,q),\alpha) \\ &= \delta_b(\min(A(x,q), B(x,q)), \ \alpha) \\ &= \min(\delta_b(A(x,q), B(x,q)), \ \alpha) \\ &= \min(\delta_b(A(x,q),\alpha), \ \delta_b(A(y,q),\alpha), \ \alpha) \\ &= \min(A_*(x,q), A_*(y,q))\end{aligned}$$

This implies that $(A \cap B)_* = A_* \cap B_*$.

Definition 3.9: Let A be an intuitionistic fuzzy soft subset of a group G and $\alpha \in [-1, \ 0]$. Then A is called step N-intuitionistic fuzzy soft subgroup of G. In other words, A is step N-intuitionistic fuzzy soft subgroup under intuitionistic Q-fuzzy version if A* satisfies the following:

(SNIFG–1) : $A_*(xy,q) \geq \min\{A_*(x,q), \ A_*(y,q)\}, \ for\ all \ \ x, y \in G \ and \ \ q \in Q.$

(SNIFG–2) : $A_*(x^{-1},q) \geq \ \{A_*(x,q)\}, \ for\ all \ \ x, y \in G \ and \ \ q \in Q.$

Proposition 3.10: If $A : G \times Q \to [-1, \ 0]$ is a step N-intuitionistic fuzzy soft subgroup of a group G, then

(i) $A_*(x,q) \leq A_*(e,q), \ for\ all \ x \in G \ and \ \ q \in Q, \ where \ 'e' \ is\ identity\ element\ of\ G.$

(ii) $A_*(xy^{-1},q) = A_*(e,q), \ which\ implies\ that\ A_*(x,q) = A_*(y,q), \ for\ all \ \ x, y \in G \ and \ \ q \in Q.$

Proof: (i) $A_*(e,q) = A_*(xx^{-1},q) \geq \min\{A_*(x,q), \ A_*(x^{-1},q)\}$

$$= \min\{A_*(x,q), \ A_*(y,q)\}$$

$$= A_*(x,q)$$

Hence, $A_*(e,q) \geq A_*(x,q), \ \ for\ all \ \ x \in G \ and \ q \in Q.$

(ii) $A_*(x,q) = A_*(xy^{-1}y,q)$

$$\begin{aligned}&\geq \min\{A_*(xy^{-1},q), \ A_*(y,q)\} \\ &= \min\{A_*(e,q), \ A_*(y,q)\} \\ &= A_*(y,q)\end{aligned}$$

Hence, $A_*(x,q) \geq A_*(y,q)$.

Similarly, $A_*(y,q) \geq A_*(x,q)$.

This implies that $A_*(x,q) = A_*(y,q). \ \ for \ \ all \ \ x, y \in G.$

In the following result, we establish a condition under which step N-intuitionistic fuzzy soft subset of a group G is step N-intuitionistic fuzzy soft subgroup.

Theorem 3.11: Let A* be step N-intuitionistic fuzzy soft subset of a group G.Then A*is step N-intuitionistic fuzzy soft subgroup under Q-fuzzy version of G if and only if A_*^{δ} is a subgroup of G for all $\delta \leq A(e, \ q)$.

Proof: It is quite obvious that A* is non-empty, since A* is step N-intuitionistic fuzzy soft subset of a group G, $A_0(x,q) \leq A_0(e,q), \ \ \ for\ all \ \ x \in G \ and \ q \in Q.$

But $x, y \in A_*^{\ \delta}, \ then \ \ \ A_*(x,q) \geq \delta \ \ \ and \ \ A_*(y,q) \geq \delta$

Now, $A_*(xy^{-1},q) \geq \min(A_*(x,q), \ A_*(y^{-1},q))$

$$\begin{aligned}&= \min(A_*(x,q), \ A_*(y,q)) \\ &\geq \min\{\delta, \ \delta\} \ = \ \delta\end{aligned}$$

Therefore, $xy^{-1} \in A_*^{\delta}$.

Hence, A_*^{δ} is a subgroup of G.

Conversely, suppose A_*^{δ} is subgroup of G, for all $\delta \leq A_*(e,q)$

$Let\ x, y \in G\ and\ Let\ A_*(x,q) = a$

$A_*(y,q) = b,\ where\ a, b \in [0, 1]$.

Let $C = \min(a,b). Then\ x, y \in A_*^{\delta},\ where\ C \leq A_*(e,q)$.

So, by the assumption A_*^{C} is a subgroup of G.

This implies that $xy^{-1} \in A_*^{C}$ and hence $A_*(xy^{-1}, q) \geq \min(A_*(x,q), A_*(y,q))$

Consequently, A*is step N-intuitionistic fuzzy soft subgroup of G.

Proposition 3.12**:** Every intuitionistic fuzzy soft subgroup of a group G is step N-intuitionistic fuzzy soft subgroup of G under Q-fuzzy version.

Proof: Let A be an intuitionistic fuzzy soft subgroup of a group G and let $x, y \in G$.

Then,
$$\begin{aligned} A_*(xy,q) &= \delta_b(A(xy,q), \alpha) \\ &\geq \delta_b(\min(A(x,q), A(y,q), \alpha)) \\ &= \min(\delta_b(A(x,q), A(y,q), \alpha)) \\ &= \min(\delta_b(A(x,q), \alpha), \delta_b(A(y,q), \alpha)) \text{ and} \end{aligned}$$

$$A_*(xy,q) \geq \min(A_*(x,q), A_*(y,q))$$

Therefore, $A_*(x^{-1},q) = \delta_b(A(x^{-1},q), \alpha) = \delta_b(A(x,q), \alpha) = A_*(x,q)$

Consequently, A is step N-intuitionistic fuzzy soft subgroup of G.

Remark 3.13: The converse of the above proposition 3.12 need not be true.

Example 3.14: Let $G = \{e, a, b, ab\},\ where\ a^2 = b^2 = e\ and\ ab = ba$ be the Klein 4-group. Let the fuzzy set A of G be defined as

$$A = \{\langle e, -0.2\rangle, \langle a, -0.4\rangle, \langle b, -0.1\rangle, \langle ab, -0.7\rangle\}$$

Taking $\alpha = -0.3$,

$$\begin{aligned} A_*(x,q) &= \delta_b(A(x,q), \alpha) \\ &= \max(A(x,q) + \alpha + 1, 0) \\ &= \max(A(x,q) - 0.3 + 1, 0) \end{aligned}$$

$$A_*(x,q) = 0,\ for\ all\ x \in G, q \in G.$$

Moreover, we have $a^{-1} = a,\ b^{-1} = b\ \ and\ \ (ab)^{-1} = ab$.

This implies that A is step N-intuitionistic fuzzy soft subgroup of G. But clearly A is not intuitionistic fuzzy soft subgroup of G.

Proposition 3.15**:** Intersection of two step N-intuitionistic fuzzy soft subgroups of a group G is also step N-intuitionistic fuzzy soft subgroup.

Corollary 3.16: The intersection of any finite number of step N-intuitionistic fuzzy soft subgroups of a group G is also step N-intuitionistic fuzzy soft subgroup of a group G.

Remark 3.17: The union of any finite number of step N-intuitionistic fuzzy soft subgroups of a group G need not be step N-intuitionistic fuzzy soft subgroup of a group G.

Example 3.18: Consider the group of integers Z. Define, the two intuitionistic fuzzy soft subsets A and B of Z as follows

$$A(x,q) = \begin{cases} -0.4, & if \;\; x = 2z \\ 0, & otherwise \end{cases}$$

$$B(x,q) = \begin{cases} -0.23, & if \;\; x = 3z \\ -0.07, & otherwise \end{cases}$$

It can be easily verified that A and B are step N-intuitionistic fuzzy soft subgroups of Z.

Now, $(A \cap B)(x,q) = \max\{A(x,q),\, B(x,q)\}$

Therefore, $(A \cap B)(x,q) = \begin{cases} -0.4, & if \;\; x \in 2z \\ -0.23, & if \;\; x \in 3z - 2z \\ -0.07, & otherwise \end{cases}$

Take $x = 15 \;\; and \;\; y = 4, \; then$

$(A \cup B)(x,q) = -0.4 \;\; and \;\; (A \cup B)(y,q) = -0.23$

But, $(A \cup B)(x - y, q) = (A \cup B)(15 - 4, q)$

$= (A \cup B)(11, q) = -0.07 \;\; and$

$\min((A \cup B)(x,q),\, (A \cup B)(y,q)) = \min(-0.4,\, -0.23) = -0.4$

Clearly, $(A \cup B)(x - y, q) < \min((A \cup B)(x,q),\, (A \cup B)(y,q))$

Consequently, we see that, the union of two step N-intuitionistic fuzzy soft subgroups of a group G need not be step N-intuitionistic fuzzy soft subgroup of a group G.

***Definition 3.19*:** Let A be step N-intuitionistic fuzzy soft subgroup of a group G and $\alpha \in [-1, 0]$. Then A is called step N-intuitionistic fuzzy soft normal subgroup of G if and only if $x\mathrm{A}_* = \mathrm{A}_* x, \;\; for \; all \;\; x \in G.$

The following result leads to note that every intuitionistic fuzzy soft normal subgroup of a group G is step N-intuitionistic fuzzy soft normal subgroup of G.

Proposition 3.20: Every intuitionistic fuzzy soft normal subgroup of a group G is step N-intuitionistic fuzzy soft normal subgroup of G.

Proof: Let A be an intuitionistic fuzzy soft normal subgroup of a group G. Then for any $x \in G$, we have $x\mathrm{A} = \mathrm{A}x$, which implies that $xA(g) = Ax(g), \;\; for \; any \; g \in G.$

Then, we have $A(x^{-1}g, q) = A(gx^{-1}, q)$

Which implies that $\delta_b(A(x^{-1}g, q),\, \alpha) = \delta_b(A(gx^{-1}, q),\, \alpha)$

Hence, $x\mathrm{A}_* = \mathrm{A}_* x, \;\; for \; all \;\; x \in G.$

Consequently, A is step N-intuitionistic fuzzy soft normal subgroup of a group G.

The converse of the above result need not be true.

Example 3.21: Consider the dihedral group of degree 3 with finite presentation

$$G = D_3 = \langle a, b \,;\, a^3 = b^2 = e,\, ba = a^2 b \rangle$$

Define the fuzzy soft subgroup of D_3 by

$$A(x,q) = \begin{cases} -0.7, & if \;\; x \in \langle b \rangle \\ -0.02, & otherwise \end{cases}$$

Taking $\alpha = -0.02$, we have

$$x A_*(g) = \delta_b(A(x^{-1}y,\, \alpha)) = \delta_b(A(x^{-1}y, -0.2)) = 0 = A_* x$$

This shows that A is step N-intuitionistic fuzzy soft normal subgroup of a group G.

$$A(a^2(ab)) = A(a^3 b) = A(b) = -0.7$$

$$A((ab)a^2) = A(a(ba)a) = A(a(a^2 b)a) = A(a^3 ba) = A(ba) = -0.02$$

This implies that A is not an intuitionistic fuzzy soft normal subgroup of G.

Proposition 3.22: Let A be step N-intuitionistic fuzzy soft normal subgroup with Q-fuzzy version of a group G .Then $A_*(y^{-1}xy,q)=A_*(x,q)$ or equivalently $A_*(xy,q)=A_*(yx,q)$ hold $\mathit{for\ all}\ x \in G\ \mathit{and}\ q \in Q.$

Proof: Since A is step N-intuitionistic fuzzy soft normal subgroup of a group G, $x\mathrm{A}_* = \mathrm{A}_* x$, holds $\mathit{for\ all}\ x \in G$.

This implies that $x\mathrm{A}_*(y^{-1},q) = \mathrm{A}_* x(y^{-1},q),\ \mathit{for\ all}\ \ y \in G\ \ \mathit{and}\ \ q \in G.$

In view of definition-3.19, the above relation becomes

$\delta_b(A(x^{-1}y^{-1},q),\ \alpha) = \delta_b(A(y^{-1}x^{-1},q),\ \alpha),$

Which implies that $A_*((yx)^{-1},q) = A_*((xy)^{-1},q)$

Consequently, $A_*(xy,q) = A_*(yx,q).$

Definition 3.23: Let A be step N-intuitionistic fuzzy soft normal subgroup of a group G. We define a set $G_{A_*} = \{x \in G / A_*(x,q) = A_*(e,q)\},$ where 'e' is the identity element of G.

The following result illustrates that the set G_{A_*} is in fact a normal subgroup of G.

Proposition 3.24**:** Let A be step N-intuitionistic fuzzy soft normal subgroup of a group G under Q-fuzzy version. Then G_{A_*} is a normal subgroup of G.

Proof: Obviously, $G_{A_*} \neq \phi,\ \mathit{for}\ \ e \in G_{A_*}$

Let $x, y \in G_{A_*}$ be any element. Then we have

$$A_*(xy^{-1},q) \geq \min(A_*(x,q),\ A_*(y,q))$$
$$= \min(A_*(e,q),\ A_*(e,q)) = A_*(e,q).$$

This implies that, $A_*(xy^{-1},q) \geq A_*(e,q)$

But, $A_*(xy^{-1},q) \leq A_*(e,q)$

Therefore, $A_*(xy^{-1},q) = A_*(e,q)$ which implies that $xy^{-1} \in G_{A_*}$

Hence, G_{A_*} is a subgroup of G. Further, let $x \in G_{A_*}$ and $y \in G,\ \mathit{we\ have}$

$A_*(y^{-1}xy^{-1},q) = A_*(x,q) = A_*(e,q).$

This implies that $y^{-1}xy^{-1} \in G_{A_*}$

Consequently, G_{A_*} is a normal subgroup of G.

Proposition 3.25: Let A be step N-intuitionistic fuzzy soft normal subgroup of a group G. Then

(i) $x\mathrm{A}_* = y\mathrm{A}_*\ \mathit{if\ \ and\ only\ if}\ \ x^{-1}y \in G.$

(ii) $\mathrm{A}_* x = \mathrm{A}_* y\ \ \mathit{if\ \ and\ only\ if}\ \ xy^{-1} \in G.$

Proof: (i) Suppose that $x\mathrm{A}_* = y\mathrm{A}_*,\ \ \mathit{for\ all}\ \ x, y \in G.$ In view definition-3.23, the above relation yields

$$\begin{aligned} A_*(x^{-1}y,q) &= \delta_b(A(x^{-1}g,q),\ \alpha) \\ &= (xA_*)(y,q) \\ &= (yA_*)(y,q) \\ &= \delta_b(A(y^{-1}g,q),\ \alpha) \\ &= \delta_b(A(e,q),\ \alpha) \\ &= A_*(e,\ q) \end{aligned}$$

This implies that $xy^{-1} \in G_{A_*}$

Conversely, let $xy^{-1} \in G_{A_*}$, then $A_*(x^{-1}y,q) = A_*(e,\ q)$

For any element $Z \in G_{A_*}$, $(xA_*)(z,q) = \delta_b(A(x^{-1}z,q),\ \alpha)$

$$
\begin{aligned}
&= A_*\left(x^{-1}z, q\right) \\
&= A_*\left(\left(x^{-1}y\right)\left(y^{-1}z\right), q\right) \\
&\geq \min\left(A_*\left(x^{-1}y, q\right),\ A_*\left(y^{-1}z, q\right)\right) \\
&= \min\left(A_*\left(e, q\right),\ A_*\left(y^{-1}z, q\right)\right) \\
&= A_*\left(y^{-1}z, q\right) = \left(yA_*\right)\left(z, q\right)
\end{aligned}
$$

Interchanging the roles of x and y, we get $\left(xA_*\right)(z,q) = \left(yA_*\right)(z,q),\ for\ all\ z \in G.$

Consequently, $\left(xA_*\right) = \left(yA_*\right)$.

(ii) One can prove this part analogous to (i).

***Proposition 3.26*:** Let A be step N-intuitionistic fuzzy soft normal subgroup of a group G and x, y, u, v be elements in G. If $x\mathrm{A}_* = u\mathrm{A}_*\ \ and\ \ y\mathrm{A}_* = v\mathrm{A}_*, then\ xy\mathrm{A}_* = uv\mathrm{A}_*$.

Proof: Given $x\mathrm{A}_* = u\mathrm{A}_*$ and $y\mathrm{A}_* = v\mathrm{A}_*$, we have $x^{-1}u\ and\ \ y^{-1}v \in G_{A_*}$

Consider, $(xy)^{-1}uv = y^{-1}\left(x^{-1}u\right)\left(yy^{-1}\right)v$

$$= \left[y^{-1}\left(x^{-1}u\right)y\right]\left(y^{-1}v\right)$$

This implies that, $(xy)^{-1}uv \in G_{A_*}$

Consequently, $xy\mathrm{A}_* = yx\mathrm{A}_*$

CONCLUSION

In this paper, we have introduced the concept of step N-intuitionistic fuzzy soft subgroup and step N-fuzzy soft cosset of a given group. The concept of step N-fuzzy soft normal subgroup and its various related properties have discussed.

FUTURE WORK

we shall extend this idea to intuitionistic fuzzy sets, vague soft sets and will investigate its various algebraic structures.

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