On Fuzzy Soft Infra Semi open Set

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Abstract: *In this paper we introduce the concept of fuzzy soft generalized closed set (cl^*), fuzzy soft generalized open set (int^*), fuzzy soft Infra Semi open sets. We extensively apply these concepts and introduce fuzzy soft infra semi continuous, fuzzy soft infra irresolute, fuzzy soft infra semi connected. A detailed study on their properties and relation between these new concepts are discussed. Some of their inverse relations are illustrated with example.*

Keywords: *Fuzzy soft g-open set (int^*) (Fuzzy soft g-closed (cl^*)), Fuzzy soft infra semi open (FS infra semi closed), Fuzzy soft infra semi closure (FS infra semi interior), Fuzzy soft infra semi continuous, Fuzzy soft infra irresolute, Fuzzy soft infra semi connected space.*

I. INTRODUCTION

Fuzzy soft set is a combination of fuzzy sets and soft sets, in which soft set is defined over fuzzy set and are useful in solving the various uncertainties arising in the fields of engineering, social sciences, economics, environment, medical science etc. In 2011, Tanay et al[4] introduced fuzzy soft topological spaces. In fuzzy soft topology, fuzzy soft open sets and fuzzy soft continuity and different types of fuzzy soft sets and fuzzy soft functions which are stronger or weaker than fuzzy soft open sets and fuzzy soft continuity are found to be an important investigating area. So many investigators introduced various types of fuzzy soft open sets and fuzzy soft continuous function and many researchers extensively investigated such as fuzzy soft characterization, relation between various fuzzy soft open sets and fuzzy soft applications to the study of various fuzzy soft topological concepts.

In 2016, Hakeem A.Othman[8] introduced the concept of fuzzy Infra-semiopen sets and extensively applied this concept in fuzzy continuous functions. In this paper we introduce fuzzy soft infra semiopen set and related fuzzy soft sets in the fuzzy soft topological spaces. Further we investigate and characterize the relation between various types of fuzzy soft open sets and fuzzy soft continuity and also we introduce the concept of fuzzy soft infra-semi connected space and related results are investigated.

II. PRELIMINARIES

Definition 2.1[5]: Let $A \subset E$ and $\mathcal{F}(X)$ be the set of all fuzzy sets in X. Then a pair (f, A) is called a fuzzy soft set over X, denoted by f_A, where $f: A \to \mathcal{F}(X)$ is a function.

From the definition, it is clear that $f(a)$ is a fuzzy set in U, for each $a \in A$, and we will denote the membership function of $f(a)$ by $f_a: X \to [0,1]$.

Definition 2.2[5]: The complement of a fuzzy soft set (f, A) is the fuzzy soft set (f^c, A), which is denoted by $(f, A)^c$ and where $f^c: A \to F(X)$ is a fuzzy set valued function i.e., for each $a \in A, f^{'}(a)$ is a fuzzy set in X, whose membership function $f_a^{'}(x) = 1 - f_a(x)$ for all $x \in X$. Here $f_a^{'}$ is the membership function of $f^{'}(a)$.

Definition 2.3[9]: Let τ be a collection of fuzzy soft sets over a universe X with a fixed parameter set E, then (f_E, τ) is called fuzzy soft topology if

i) $\tilde{0}_E, \tilde{1}_E \in \tau$
ii) Union of any members of τ is a member of τ.
iii) Intersection of any two members of τ is a member of τ.

Each member of τ is called fuzzy soft open set i.e. A fuzzy soft set f_A over X is fuzzy soft open if and only if $f_A \in \tau$. A fuzzy soft set f_A over X is called fuzzy soft closed set if the complement of f_A is fuzzy soft open set.

Definition 2.4[9]: Let g_C be a fuzzy soft set in a fuzzy soft topological space (f_E, τ). Then

i) The fuzzy soft closure of g_C is a fuzzy soft set defined as

$$Fs\ cl\ g_C = \cap\{h_A \backslash g_C \subseteq h_A\ and\ h_A\ is\ a\ fuzzy\ soft\ closed\ set\}$$

ii) The fuzzy soft interior of g_C is a fuzzy soft set defined as

$$Fs\ int\ g_C = \cup\{k_B \backslash k_B \subseteq g_C\ and\ k_B\ is\ a\ fuzzy\ soft\ open\ set\}$$

Definition 2.5[10]: Let $(X, \tilde{\tau}, E)$ and $(Y, \tilde{\sigma}, K)$ be fuzzy soft topological spaces. Let $\rho: X \to Y$ and $\psi: E \to K$ be the two mappings and $g = (\rho, \psi)$ is said to be fuzzy soft continuous if the inverse image of every fuzzy soft open set in $(Y, \tilde{\sigma}, K)$ is fuzzy soft open in $(X, \tilde{\tau}, E)$ that is $g^{-1}(\tilde{\mu}) \in \tilde{\tau}$ for all $\tilde{\mu} \in \tilde{\sigma}$

Definition 2.6[17]: A fuzzy soft set f_A in fuzzy soft topological space (X, τ, E) is called fuzzy soft semi open if $f_A \leq Fscl\ Fsint(f_A)$, fuzzy soft semi closed if $Fsint\ Fscl(f_A) \leq f_A$.

Definition 2.7[18]: Let $(X, \tilde{\tau}, E)$ and $(Y, \tilde{\sigma}, K)$ be two fuzzy soft topological spaces. A fuzzy soft mapping $g: (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ is said to be fuzzy soft semi continuous if for each fuzzy soft open set $\tilde{\mu}$ in $(Y, \tilde{\sigma}, K)$, the inverse image $g^{-1}(\tilde{\mu})$ is fuzzy soft semi open set in $(X, \tilde{\tau}, E)$.

Definition 2.8[18]: Let $(X, \tilde{\tau}, E)$ and $(Y, \tilde{\sigma}, K)$ be two fuzzy soft topological spaces. A fuzzy soft mapping $g: (X, \tilde{\tau}, E) \to (Y, \tilde{\sigma}, K)$ is said to be fuzzy soft semi open if for each fuzzy soft open set $\tilde{\lambda}$ in $(X, \tilde{\tau}, E)$, the image $g(\tilde{\lambda})$ is fuzzy soft semi open set in $(Y, \tilde{\sigma}, K)$.

Lemma 2.9[14]: Let η be any fuzzy set. Then

i) $\eta \leq cl^*(\eta) \leq cl(\eta)$.
ii) $int(\eta) \leq int^*(\eta) \leq (\eta)$.

Definition 2.10[17]: A fuzzy soft set f_A in fuzzy soft topological space (X, τ, E) is called fuzzy soft generalized closed set (Fs g-closed) if $Fscl\ (f_A) \leq H$, $\forall f_A \leq H$ and H is fuzzy soft open in X.

III. FUZZY SOFT INFRA SEMI OPEN SET

Definition 3.1: Let (f, A) be any fuzzy soft set. The Fuzzy soft Generalized closure of (f, A) is defined as the intersection of all Fuzzy soft g-closed sets containing (f, A) and it is denoted by $cl^*(f, A)$.

$$cl^*(f, A) = \barwedge \{(f, B): (f, B) \geq (f, A), \quad (f, B) is\ a\ fuzzy\ soft\ generalized - closed\ set\ of\ (X, \tau, E)\}.$$

Definition 3.2: Let (f, A) be any fuzzy soft set. The Fuzzy soft Generalized interior of (f, A) is defined as the union of all Fuzzy soft g-open sets contained in (f, A) and it is denoted by $int^*(f, A)$.

$$int^*(f, A) = \veebar \{(f, B): (f, B) \leq (f, A), \quad (f, B) is\ a\ fuzzy\ soft\ generalized - open\ set\ of\ (X, \tau, E)\}.$$

Lemma 3.3: Let (f, A) be any fuzzy soft set. Then,

i) $(f, A) \leq cl^*(f, A) \leq cl(f, A)$
ii) $(f, A) \geq int^*(f, A) \geq int\ (f, A)$

Definition 3.4: A fuzzy soft set (f, A) in a fuzzy soft topological space (X, τ, E) is called fuzzy soft infra-semiopen if $(f, B) \leq (f, A) \leq cl^*(f, B)$, where (f, B) is a fuzzy soft open set. The family of all fuzzy soft infra-semi open sets in (X, τ, E) will be denoted by $FSIso(X, \tau, E)$.

Definition 3.5: A fuzzy soft set (f, A) in a fuzzy soft topological space (X, τ, E) is called fuzzy soft infra-semiclosed if $int^*(f, B) \leq (f, A) \leq (f, B)$, where (f, B) is a fuzzy soft closed set. The family of all fuzzy soft infra-semiclosed sets in (X, τ, E) will be denoted by $FSIsc(X, \tau, E)$.

Theorem 3.6: Let (f,A) be any fuzzy soft subset of a fuzzy soft topological space(X,τ,E), then the following properties are equivalent:

i) $(f,A) \in FSIso(X,\tau,E)$.
ii) $(f,A) \leq cl^{*}int(f,A)$.
iii) $cl^{*}(f,A) = cl^{*}\big(int\,(f,A)\big)$.
iv) $(f,A)^{C}$ is a fuzzy soft infra-semiclosed set.
v) $int^{*}cl\,(f,A)^{C} \leq (f,A)^{C}$.
vi) $int^{*}(cl\,(f,A)^{C}) = int^{*}(f,A)^{C}$.

Proof:

$(i) \Rightarrow (ii)$: Assume that $(f,A) \in FSIso(X,\tau,E)$, then by definition there exists $(f,B) \in FSo(X,\tau,E)$ such that $(f,B) \leq (f,A) \leq cl^{*}(f,B)$. Since, $(f,B) \leq (f,A)$ and by the lemma 3.7, $(f,B) \leq int(f,A) \Rightarrow cl^{*}(f,B) \leq cl^{*}\big(int(f,A)\big)$ but $(f,A) \leq cl^{*}(f,B)$ therefore $(f,A) \leq cl^{*}\big(int\,(f,A)\big)$.

$(ii) \Rightarrow (iii)$: We have $(f,A) \leq cl^{*}int(f,A) \Rightarrow cl^{*}(f,A) \leq cl^{*}\big(int(f,A)\big)$. And we know that $int(f,A) \leq (f,A) \Rightarrow cl^{*}int(f,A) \leq cl^{*}(f,A)$, thus we have $cl^{*}(f,A) = cl^{*}\big(int\,(f,A)\big)$.

$(iii) \Rightarrow (iv)$: By (ii) we have $(f,A) \leq cl^{*}int(f,A)$, then $int\,(f,A) \leq (f,A) \leq cl^{*}int(f,A)$. Put $(f,B) = int(f,A)$, then $(f,B) \leq (f,A) \leq cl^{*}(f,B)$, where (f,A) is a fuzzy soft infra semi open set. Therefore $int^{*}\,(f,B)^{C} \leq (f,A)^{C} \leq (f,B)^{C}$, where $(f,B)^{C} \in FSc(X,\tau,E)$. Thus, $(f,A)^{C} \in FSIsc(X,\tau,E)$.

$(iv) \Rightarrow (v)$: We have $int^{*}\,(f,B)^{C} \leq (f,A)^{C} \leq (f,B)^{C}$, where $(f,B)^{C} \in FSc(X,\tau,E)$. Therefore, $cl\,(f,A)^{C} \leq (f,B)^{C}$ and $int^{*}(cl\,(f,A)^{C}) \leq int^{*}(f,B)^{C} \leq (f,A)^{C}$. Hence, $int^{*}cl\,(f,A)^{C} \leq (f,A)^{C}$.

$(v) \Rightarrow (vi)$: Assume that $int^{*}(cl\,(f,A)^{C}) \leq (f,A)^{C}$, which implies $int^{*}(cl\,(f,A)^{C}) \leq int^{*}(f,A)^{C}$ and we also know that $int^{*}\,(f,A)^{C} \leq int^{*}(cl\,(f,A)^{C})$. Thus, we get $int^{*}(cl\,(f,A)) = int^{*}(f,A)$.

$(vi) \Rightarrow (i)$: Let $int^{*}(cl\,(f,A)^{C}) \leq (f,A)^{C}$, then $\big(int^{*}(cl\,(f,A)^{C})\big)^{C} \leq ((f,A)^{c})^{c}$. Implies that $(f,A) \leq cl^{*}\big(int(f,A)\big)$. Hence, $int\,(f,A) \leq (f,A) \leq cl^{*}\big(int\,(f,A)\big)$. Put, $(f,B) = int\,(f,A)$. Then, $(f,B) \leq (f,A) \leq cl^{*}(f,B)$ and $(f,A) \in FSIso(X,\tau,E)$.

Definition 3.7: Let (f,A) be any fuzzy soft set in a fuzzy soft topological space (X,τ,E). The Fuzzy soft infra-semi closure is defined as the intersection of all Fuzzy soft infra semi closed sets containing(f,A).

$$Iscl\,(f,A) = \wedge\{(f,B) : (f,B) \geq (f,A),\ (f,B) \in FSIsc(X,\tau,E)\}.$$

Definition 3.8: Let (f,A) be any fuzzy soft set in a fuzzy soft topological space (X,τ,E). The Fuzzy soft infra-semi interior is defined as the union of all Fuzzy soft infra semi open sets contained in (f,A).

$$Isint\,(f,A) = \vee\{(f,B) : (f,B) \leq (f,A),\ (f,B) \in FSIso(X,\tau,E)\}.$$

Proposition 3.9: Let (f,A) and (f,B) be any fuzzy soft subsets in (X,τ,E) and $(f,A) \leq (f,B)$, then the following are true.

(i) $Is\,int\,(f,A)$ is the largest fuzzy soft infra semiopen set contained in (f,A).
(ii) $Is\,int\,(f,A) \leq (f,A)$.
(iii) $Is\,int\,(f,A) \leq Is\,int\,(f,B)$.
(iv) $Is\,int\,(Is\,int\,(f,A)) = Is\,int\,(f,A)$.
(v) $(f,A) \in FSIso(X,\tau,E) \Leftrightarrow Is\,int\,(f,A) = (f,A)$.
(vi) $Is\,int\,(f,A) \vee Is\,int\,(f,B) \leq Is\,int\,\big((f,A)\vee(f,B)\big)$.
(vii) $Is\,int\,(f,A) \wedge Is\,int\,(f,B) \geq Is\,int\,\big((f,A)\wedge(f,B)\big)$.

Proposition 3.10: Let (f,A) and (f,B) be any fuzzy soft subsets in (X,τ,E) and $(f,A) \leq (f,B)$, then the following are true.

(i) $Is\ cl\ (f,A)$ is the smallest fuzzy soft infra-semiclosed set containing (f,A).
(ii) $(f,A) \leq Is\ cl\ (f,A)$.
(iii) $Is\ cl\ (f,A) \leq Is\ cl\ (f,B)$.
(iv) $Is\ cl\ (Is\ cl\ (f,A)) = Is\ cl\ (f,A)$.
(v) $(f,A) \in FSIsc(X,\tau,E) \Leftrightarrow Is\ cl\ (f,A) = (f,A)$.
(vi) $Is\ cl\ (f,A) \vee Is\ cl\ (f,B) \leq Is\ cl\ \big((f,A) \vee (f,B)\big)$.
(vii) $Is\ cl\ (f,A) \wedge Is\ cl\ (f,B) \geq Is\ cl\ \big((f,A) \wedge (f,B)\big)$.

Theorem 3.11: Let (f,A) be any fuzzy soft subset of a fuzzy soft topological space (X,τ,E). Then the following are true

i) $\big(Is\ int\ (f,A)\big)^C = Is\ cl\ (f,A)$.
ii) $\big(Is\ cl\ (f,A)\big)^C = Is\ int\ (f,A)$.
iii) $Is\ int\ (f,A) = (f,A) \wedge cl^*\ \big(int\ (f,A)\big)$.
iv) $Is\ cl\ (f,A) = (f,A) \vee int^*\big(cl\ (f,A)\big)$.

Proof:

i)
$$\begin{aligned}\big(Is\ int\ (f,A)\big)^C &= (\vee\{(f,B):(f,B) \leq (f,A), (f,B) \in FSIso(X,\tau,E)\})^C \\ &= \wedge\{(f,B)^C:(f,B)^C \geq (f,A)^c, (f,B)^C \in FSIsc(X,\tau,E)\} \\ &= Is\ cl\ (f,A).\end{aligned}$$

ii) Similar to (i).

iii) $Is\ int$ is fuzzy soft infra semi open, then we have

$$Is\ int\ (f,A) \leq cl^*\Big(int\big(Is\ int\ (f,A)\big)\Big) \leq cl^*\big(int\ (f,A)\big)$$

So, $Is\ int\ (f,A) \leq (f,A) \wedge cl^*\big(int\ (f,A)\big)$ (1)

We have $int\ (f,A) \leq (f,A) \wedge cl^*\big(int\ (f,A)\big) \leq cl^*\big(int\ (f,A)\big)$.

By definition 3.4 $\Big((f,A) \wedge cl^*\big(int\ (f,A)\big)\Big) \in FSIso(X,\tau,E)$ and $(f,A) \wedge cl^*\big(int\ (f,A)\big) \leq (f,A)$, then $(f,A) \wedge cl^*\big(int\ (f,A)\big) \leq Is\ int\ (f,A)$ (2)

From (1) and (2), we have, $Is\ int\ (f,A) = (f,A) \wedge cl^*\ \big(int\ (f,A)\big)$.

iv) Similar to (iii).

Theorem 3.12:

i) The union of any number of a fuzzy soft infra-semi open set is a fuzzy soft infra-semi open set.
ii) The intersection of any number of a fuzzy soft infra-semi closed set is a fuzzy soft infra-semi closed set.

Proof:

i) Let $\{(f,A)_\rho\}$ be a family of fuzzy soft infra-semi open set. Then by theorem 3.6, for each ρ, $(f,A)_\rho \leq cl^*\big(int\ (f,A)_\rho\big)$ and $\vee(f,A)_\rho \leq \vee\Big(cl^*\big(int\ (f,A)_\rho\big)\Big) \leq cl^*\big(int(\vee(f,A)_\rho)\big)$. Hence, $(f,A)_\rho$ is a fuzzy soft infra-semi open set.
ii) It is obvious.

Theorem 3.13: If $(f,A) \in FSIso(X,\tau,E)$, then $Is\ cl(f,A) \in FSIso(X,\tau,E)$.

Proof:

If $(f,A) \in FSIso(X,\tau,E)$, then by theorem 3.6 $(f,A) \leq cl^*\big(int\ (f,A)\big)$.In general, $(f,A) \leq Is\ cl(f,A) \Rightarrow cl^*\big(int\ (f,A)\big) \leq cl^*(int\big(Is\ cl\ (f,A)\big)$. Thus, $(f,A) \leq cl^*\big(int\ (f,A)\big) \leq cl^*(int\big(Is\ cl\ (f,A)\big)$.This implies that

$(f,A) \le cl^*(int(Is\ cl\ (f,A)) \Rightarrow Is\ cl(f,A) \le Is\ cl\ \left(cl^*\left(int(Is\ cl\ (f,A))\right)\right)$.
$\Rightarrow Is\ cl(f,A) \le cl^*\left(int(Is\ cl\ (f,A))\right)$. Hence, $Is\ cl(f,A) \in FSIso(X,\tau,E)$.

Theorem 3.14: Let (f,A) and (f,B) be fuzzy soft subsets of a fuzzy soft topological space (X,τ,E) and

i) If $(f,B) \in FSIso(X,\tau,E)$ with $(f,B) \le (f,A) \le cl^*(f,B)$, then $(f,A) \in FSIso(X,\tau,E)$.
ii) If $(f,B) \in FSIsc(X,\tau,E)$ with $int^*(f,B) \le (f,A) \le (f,B)$, then $(f,A) \in FSIsc(X,\tau,E)$.

Proof:

i) Let (f,A) and (f,B) be fuzzy soft subsets of a fuzzy soft topological space (X,τ,E). Consider $(f,B) \in FSIso(X,\tau,E)$ with $(f,B) \le (f,A) \le cl^*(f,B)$. Since $(f,B) \in FSIso(X,\tau,E)$ by definition 3.4 we have $(f,C) \le (f,B) \le cl^*(f,C)$. Here, $(f,C) \le (f,B) \le (f,A)$ thus $(f,C) \le (f,A)$ and $(f,B) \le cl^*(f,C)$ implies that $cl^*(f,B) \le cl^*(f,C)$. Therefore, $(f,C) \le (f,A) \le cl^*(f,C)$. Thus, $(f,A) \in FSIso(X,\tau,E)$.
ii) Similar to (i)

Theorem 3.15: A fuzzy soft set $(f,A) \in FSIso(X,\tau,E)$ if and only if for every fuzzy soft singleton $(f,p) \le (f,A)$, there exists a fuzzy soft set $(f,D) \in FSIso(X,\tau,E)$ such that $(f,p) \le (f,D) \le (f,A)$.

Theorem 3.16: Let (f,A) be any fuzzy soft subset of (X,τ,E), then the following holds,

i) If (f,A) is a fuzzy soft infra-semi open (fuzzy soft infra-semi closed) set, then (f,A) is a fuzzy soft semi open (fuzzy soft semi closed) set.
ii) If (f,A) is a fuzzy soft open (fuzzy soft closed) set, then (f,A) is a fuzzy soft infra-semi open (fuzzy soft infra-semi closed) set.

Example 3.17:

Let $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $(F_1,A), (F_2,A) \& (F_3,A)$ be the fuzzy soft sets defined as follows

$(F_1,A) = \left[\left\{e_1, \left(\frac{x_1}{0.3}, \frac{x_2}{0.5}\right)\right\}, \left\{e_2, \left(\frac{x_1}{0.2}, \frac{x_2}{0.4}\right)\right\}\right]$

$(F_2,A) = \left[\left\{e_1, \left(\frac{x_1}{0.6}, \frac{x_2}{0.7}\right)\right\}, \left\{e_2, \left(\frac{x_1}{0.5}, \frac{x_2}{0.6}\right)\right\}\right]$

$(F_3,A) = \left[\left\{e_1, \left(\frac{x_1}{0.7}, \frac{x_2}{0.5}\right)\right\}, \left\{e_2, \left(\frac{x_1}{0.6}, \frac{x_2}{0.4}\right)\right\}\right]$

If $\tau = (0_x, (F_1,A), 1_x)$. Then, (F_2,A) is not a fuzzy soft infra-semi open (fuzzy soft $open$) set.

Example 3.18:

Let $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $(F_1,A), (F_2,A) \& (F_3,A)$ be the fuzzy soft sets defined as follows

$(F_1,A) = \left[\left\{e_1, \left(\frac{x_1}{0.4}, \frac{x_2}{0.7}\right)\right\}, \left\{e_2, \left(\frac{x_1}{0.3}, \frac{x_2}{0.8}\right)\right\}\right]$

$(F_2,A) = \left[\left\{e_1, \left(\frac{x_1}{0.4}, \frac{x_2}{0.5}\right)\right\}, \left\{e_2, \left(\frac{x_1}{0.3}, \frac{x_2}{0.4}\right)\right\}\right]$

$(F_3,A) = \left[\left\{e_1, \left(\frac{x_1}{0.5}, \frac{x_2}{0.5}\right)\right\}, \left\{e_2, \left(\frac{x_1}{0.5}, \frac{x_2}{0.6}\right)\right\}\right]$

If $\tau = (0_x,\ (F_1,A), (F_2,A), 1_x)$. Then, (F_3,A) is a fuzzy soft infra-semi open set and it is also fuzzy soft semi open set.

Example 3.19:

Let $X = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $(F_1, A), (F_2, A) \& (F_3, A)$ be the fuzzy soft sets defined as follows

$$(F_1, A) = \left[\left\{e_1, \left(\frac{x_1}{0.4}, \frac{x_2}{0.5}\right)\right\}, \left\{e_2, \left(\frac{x_1}{0.5}, \frac{x_2}{0.4}\right)\right\}\right]$$

$$(F_2, A) = \left[\left\{e_1, \left(\frac{x_1}{0.5}, \frac{x_2}{0.7}\right)\right\}, \left\{e_2, \left(\frac{x_1}{0.4}, \frac{x_2}{0.6}\right)\right\}\right]$$

$$(F_3, A) = \left[\left\{e_1, \left(\frac{x_1}{0.5}, \frac{x_2}{0.5}\right)\right\}, \left\{e_2, \left(\frac{x_1}{0.4}, \frac{x_2}{0.4}\right)\right\}\right]$$

If $\tau = (0_x,\ (F_1, A), (F_2, A), 1_x)$. Then, (F_3, A) is a fuzzy soft infra-semi open set.

IV. FUZZY SOFT INFRA SEMI CNTINUOUS FUNCTION

Definition 4.1: A function $h: (X, \tau, E) \to (Y, \mu, E)$ is said to be fuzzy soft infra-semi continuous if $h^{-1}(f, A) \in FSIso(X, \tau, E)$, $[FSIsc(X, \tau, E)]$ for every fuzzy soft open [fuzzy soft closed]set $(f, A) \in (Y, \mu, E)$.

Definition 4.2: A function $h: (X, \tau, E) \to (Y, \mu, E)$ is said to be fuzzy soft infra-irresolute if $h^{-1}(f, A) \in FSIso(X, \tau, E)$, $[FSIsc(X, \tau, E)]$ for every fuzzy soft infra-semi open [fuzzy soft infra-semi closed] set $(f, A) \in (Y, \mu, E)$.

Theorem 4.3: Let (X, τ, E) and (Y, μ, E) be a fuzzy soft topological space and $h: (X, \tau, E) \to (Y, \mu, E)$, then the following properties are equivalent

- i) h is fuzzy soft infra-semi continuous.
- ii) For every fuzzy soft singleton set $(f, t) \in (X, \tau, E)$ and every fuzzy soft open set $(f, B) \in (Y, \mu, E)$ with $h(f, t) \leq (f, B)$, then there exists a fuzzy soft infra-semi open set, $(f, A) \in (X, \tau, E)$ such that $(f, t) \leq (f, A)$ and $(f, A) \leq h^{-1}(f, B)$.
- iii) For each fuzzy soft singleton set $(f, t) \in (X, \tau, E)$ and each fuzzy soft open set $(f, B) \in (Y, \mu, E)$ with $h(f, t) \leq (f, B)$, then there exists a fuzzy soft infra-semi open set, $(f, A) \in (X, \tau, E)$ such that $(f, t) \leq (f, A)$ and $h(f, A) \leq (f, B)$.
- iv) $h^{-1}(f, B) \in FSIsc(X, \tau, E), \forall (f, B) \in FSc(Y, \mu, E)$.
- v) $h\big(Is\ cl\ (f, A)\big) \leq cl\big(h(f, A)\big), \forall\ (f, A) \in (X, \tau, E)$.
- vi) $Is\ cl\ \big(h^{-1}(f, B)\big) \leq h^{-1}\big(cl\ (f, B)\big), \forall\ (f, B) \in (Y, \mu, E)$.
- vii) $h^{-1}\big(int\ (f, B)\big) \leq Is\ int\ \big(h^{-1}(f, B)\big), \forall\ (f, B) \in (Y, \mu, E)$.

Proof:

$(i) \Rightarrow (ii)$: Let $(f, B) \in FSo(Y, \mu, E)$ and a fuzzy soft singleton set $(f, t) \in (X, \tau, E)$ such that $h(f, t) \subseteq (f, B)$, then there exists $(f, A) \in FSo(Y, \mu, E)$ such that $h(f, t) \leq (f, A) \leq (f, B)$. Since, h is a fuzzy soft infra-semi continuous, $(f, A) = h^{-1}(f, A)$ is a fuzzy soft infra-semi open set and $(f, t) \leq (f, A) = h^{-1}(f, A) \leq h^{-1}(f, B)$.

$(ii) \Rightarrow (iii)$: Let $(f, B) \in FSo(Y, \mu, E)$ and (f, t) be a fuzzy soft singleton in (X, τ, E) such that $h(f, t) \subseteq (f, B)$, then there exists a fuzzy soft infra-semi open set, (f, A) such that $(f, t) \leq (f, A)$ and $(f, A) \leq h^{-1}(f, B)$. Hence, $(f, t) \leq (f, A)$ and $h(f, A) \leq h^{-1}\big(h(f, B)\big) \leq (f, B)$.

$(iii) \Rightarrow (i)$: Consider, $(f, t) \leq h^{-1}(f, B)$ and $(f, B) \in FSo(Y, \mu, E)$. Thus, $h(f, t) \leq h\big(h^{-1}(f, B)\big) \leq (f, B)$. Then there exists a fuzzy soft infra-semi open set (f, A) such that $(f, t) \leq (f, A)$ and $h(f, A) \leq (f, B)$. Thus, $(f, t) \leq (f, A) \leq h^{-1}\big(k(f, A)\big) \leq h^{-1}(f, B)$. Therefore by theorem 3.19, $h^{-1}(f, B) \in (X, \tau, E)$ is a fuzzy soft infra-semi open set. Hence, h is a fuzzy soft infra-semi continuous.

$(i) \Rightarrow (iv)$: Let $(f, B) \in FSc(Y, \mu, E)$. Then, $(f, B)^c \in FSo(Y, \mu, E)$ that implies, $h^{-1}((f, B)^c) \in FSIso(X, \tau, E)$. Hence, $h^{-1}(f, B) \in FSIsc(X, \tau, E)$.

$(iv) \Rightarrow (v)$: Consider $(f,A) \in (X,\tau,E)$ and $(f,A) \leq h^{-1}\big(h(f,A)\big) \leq h^{-1}\big(cl\, h(f,B)\big)$. Then, we get $Is\, cl\ (f,A) \leq h^{-1}\big(cl\, \mathbb{G}(f,B)\big) = Is\, cl\ \Big(h^{-1}\big(cl\, h(f,B)\big)\Big)$. Therefore, $h\big(Is\, cl\ (f,A)\big) \leq cl\big(h(f,A)\big)$.

$(v) \Rightarrow (vi)$: Let $(f,B) \in (Y,\mu,E)$, then $h^{-1}(f,B) \in (X,\tau,E)$. By (iv), we get $h\Big(Is\, cl\big(h^{-1}(f,B)\big)\Big) \leq cl\ \Big(h\big(h^{-1}(f,B)\big)\Big) \leq cl\ (f,B)$, implies $Is\, cl\ \big(h^{-1}(f,B)\big) \leq h^{-1}\big(cl\ (f,B)\big)$.

$(vi) \Rightarrow (vii)$: Let $(f,B) \in (Y,\mu,E)$, then $(f,B)^c \in (Y,\mu,E)$. By (vi), we get $Is\, cl\big(h^{-1}(f,B)\big) \leq h^{-1}\big(cl\ (f,B)^c\big)$, which implies $h^{-1}\big(int\ (f,B)\big) \leq Is\ int\ \big(h^{-1}(f,B)\big)$.

$(vii) \Rightarrow (i)$: Consider, $(f,B) \in (Y,\mu,E)$. By (vi), we have $h^{-1}(f,B) = h^{-1}\big(int\ (f,B)\big) \leq Is\ int\ \big(h^{-1}(f,B)\big)$ which implies $h^{-1}(f,B) \in FSIso(X,\tau,E)$. Hence, h is a fuzzy soft infra-semi continuous function.

Theorem 4.4: Let $h: (X,\tau,E) \to (Y,\mu,E)$, then the following properties are equivalent

i) h is fuzzy soft infra-semi continuous
ii) $h^{-1}(f,B) \in FSIsc(X,\tau,E), \forall\ (f,B) \in FSc(Y,\mu,E)$.
iii) $int^{*}\ cl\ (f,B) \leq \Big(h^{-1}\big(cl\ (f,B)\big)\Big), \forall\ (f,B) \in (Y,\mu,E)$.
iv) $h\big(cl^{*}\ int\ (f,A)\big) \leq cl\ \big(h(f,A)\big), \forall (f,A) \in (X,\mu,E)$.

Remark: The composition of two fuzzy soft infra-semi continuous functions may not be a fuzzy soft infra-semi continuous.

Theorem 4.5: Let $k: (X,\tau,E) \to (Y,\mu,E)$ be a fuzzy soft infra-semi continuous and $h: (Y,\mu,E) \to (Z,\mathrm{v},E)$ be a fuzzy soft continuous function, then $h \circ k: (X,\tau,E) \to (Z,\mathrm{v},E)$ is a fuzzy soft infra-semi continuous function.

Proof: Let $(f,B) \in (Z,\nu,E)$ be a fuzzy soft open set. Thus, $\big((h \circ k)^{-1}(f,B)\big) = \Big(\ k^{-1}\big(h^{-1}(f,B)\big)\Big)$. Then, $h^{-1}\big(\ k^{-1}(f,C)\big) \in (X,\tau,E)$ is a fuzzy soft infra-semi open set.

Example 4.6:

Let $X = Y = Z = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $(F_1,A), (F_2,A) \& (F_3,A)$ be the fuzzy soft sets defined as follows

$$(F_1,A) = \left[\left\{e_1, \left(\frac{x_1}{0.4}, \frac{x_2}{0.5}\right)\right\},\ \left\{e_2, \left(\frac{x_1}{0.3}, \frac{x_2}{0.5}\right)\right\}\right]$$

$$(F_2,A) = \left[\left\{e_1, \left(\frac{x_1}{0.3}, \frac{x_2}{0.6}\right)\right\},\ \left\{e_2, \left(\frac{x_1}{0.4}, \frac{x_2}{0.7}\right)\right\}\right]$$

$$(F_3,A) = \left[\left\{e_1, \left(\frac{x_1}{0.5}, \frac{x_2}{0.5}\right)\right\},\ \left\{e_2, \left(\frac{x_1}{0.5}, \frac{x_2}{0.5}\right)\right\}\right]$$

If $\tau_x = (0_x,\ (F_1,A), (F_2,A), 1_x)$ and $\tau_y = \{0_x, h_3, 1_x\}$ and the function $g: (X,\tau_x) \to (Y,\tau_y)$ be the fuzzy soft identity functions. Then, g is not a fuzzy soft infra semi continuous function.

Example 4.7:

Let $X = Y = Z = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $(F_1,A), (F_2,A) \& (F_3,A)$ be the fuzzy soft sets defined as follows

$$(F_1,A) = \left[\left\{e_1, \left(\frac{x_1}{0.4}, \frac{x_2}{0.5}\right)\right\},\ \left\{e_2, \left(\frac{x_1}{0.3}, \frac{x_2}{0.5}\right)\right\}\right]$$

$$(F_2,A) = \left[\left\{e_1, \left(\frac{x_1}{0.5}, \frac{x_2}{0.7}\right)\right\},\ \left\{e_2, \left(\frac{x_1}{0.5}, \frac{x_2}{0.7}\right)\right\}\right]$$

$$(F_3,A) = \left[\left\{e_1, \left(\frac{x_1}{0.5}, \frac{x_2}{0.5}\right)\right\},\ \left\{e_2, \left(\frac{x_1}{0.5}, \frac{x_2}{0.5}\right)\right\}\right]$$

If $\tau_x = (0_x, (F_1, A), (F_2, A), 1_x)$, $\tau_y = \{0_x, h_3, 1_x\}$ and the function $h: (X, \tau_x) \to (Y, \tau_y)$ be the fuzzy soft identity functions. Then, h is a fuzzy soft infra- semi continuous function.

Definition 4.8: A fuzzy soft function $h: (X, \tau, E) \to (Y, \mu, E)$ is called a fuzzy soft infra-semi open if $h(f, A) \in FSIso(Y, \mu, E), \forall\, (f, A) \in FSo(X, \tau, E)$.

Definition 4.9: A fuzzy soft function $h: (X, \tau, E) \to (Y, \mu, E)$ is called a fuzzy soft infra-semi closed if $h(f, A) \in FSIsc(Y, \mu, E), \forall\, (f, A) \in FSc(X, \tau, E)$.

Example 4.10:

Let $X = Y = Z = \{x_1, x_2\}$, $E = \{e_1, e_2\}$ and $(F_1, A), (F_2, A) \& (F_3, A)$ be the fuzzy soft sets defined as follows

$$(F_1, A) = \left[\left\{e_1, \left(\frac{x_1}{0.4}, \frac{x_2}{0.5}\right)\right\}, \left\{e_2, \left(\frac{x_1}{0.3}, \frac{x_2}{0.5}\right)\right\}\right]$$

$$(F_2, A) = \left[\left\{e_1, \left(\frac{x_1}{0.3}, \frac{x_2}{0.6}\right)\right\}, \left\{e_2, \left(\frac{x_1}{0.4}, \frac{x_2}{0.7}\right)\right\}\right]$$

$$(F_3, A) = \left[\left\{e_1, \left(\frac{x_1}{0.5}, \frac{x_2}{0.5}\right)\right\}, \left\{e_2, \left(\frac{x_1}{0.5}, \frac{x_2}{0.5}\right)\right\}\right]$$

If $\tau_x = \{0_x, h_3, 1_x\}$ and $\tau_y = (0_x, (F_1, A), (F_2, A), 1_x)$ and the function $g: (X, \tau_x) \to (Y, \tau_y)$ be the fuzzy soft identity functions. Then, g is not a fuzzy soft infra semi open (infra-semiclosed) function.

Theorem 4.11: Let (X, τ, E) and (Y, μ, E) be two fuzzy soft topological space and $h: (X, \tau, E) \to (Y, \mu, E)$, then the following properties are equivalent

i) h is a fuzzy soft infra-semi open.

ii) $h\big(int\,(f, A)\big) \le Is\ int\,\big(h(f, A)\big), \forall (f, A) \in (X, \tau, E)$.

iii) $int\,\big(h^{-1}(f, B)\big) \le h^{-1}\big(Is\ int(f, B)\big), \forall (f, B) \in (Y, \mu, E)$.

iv) $h^{-1}\big(Is\ cl\,(f, B)\big) \le cl\,\big(h^{-1}(f, B)\big), \forall (f, B) \in (Y, \mu, E)$.

v) $h\big(int\,(f, A)\big) \le int^{*}cl\,\big(h(f, A)\big), \forall (f, A) \in (X, \tau, E)$.

Proof:

$(i) \Rightarrow (ii)$: Let h be a fuzzy soft infra-semi open function and $(f, A) \in (X, \tau, E), h\big(int\,(f, A)\big) \le h(f, A)$. We know that $Is\ int\Big(h\big(int\,(f, A)\big)\Big) \le Is\ int\big(h(f, A)\big)$. Hence, $h\big(int\,(f, A)\big) \le Is\ int\,\big(h(f, A)\big)$.

$(ii) \Rightarrow (iii)$: Consider $(f, B) \in (Y, \mu, E)$, then $h^{-1}(f, B) \in (X, \tau, E)$. Put $h^{-1}(f, B) = (f, A)$ in (ii), we get $h\Big(int\,\big(h^{-1}(f, B)\big)\Big) \le Is\ int\ \Big(h\big(h^{-1}(f, B)\big)\Big) \le Is\ int\,(f, B)$. Hence, $int\,\big(h^{-1}(f, B)\big) \le h^{-1}\big(Is\ int(f, B)\big)$.

$(iii) \Rightarrow (iv)$: Consider $(f, B) \in (Y, \mu, E)$ and $(f, B)^c \in (Y, \mu, E)$. Put $(f, B)^C = (f, B)$ in (iii), then we get $int\,\big(h^{-1}((f, B)^c)\big) \le h^{-1}\big(Is\ int((f, B)^c)\big)$, implies $\Big(cl\,\big(h^{-1}(f, B)\big)\Big)^c \le \Big(h^{-1}\big(Is\ cl(f, B)\big)\Big)^c$. Hence, $h^{-1}\big(Is\ cl\,(f, B)\big) \le cl\,\big(h^{-1}(f, B)\big)$.

$(iv) \Rightarrow (v)$: Let $(f, A) \in (X, \tau, E)$, then $\big(h(f, A)\big)^c \in (Y, \mu, E)$.

Using (iv), we have, $h^{-1}\Big(Is\ cl\,\big(h(f, A)\big)^c\Big) \le cl\,\Big(h^{-1}\big(h(f, A)\big)^c\Big)$

$$\Rightarrow \Big(h^{-1}\Big(Is\ int\,\big(h(f, A)\big)\Big)\Big)^c \le \Big(int\,\Big(h^{-1}\big(h(f, A)\big)\Big)\Big)^c$$

Then, $int(f, A) \le h^{-1}\Big(Is\ int\,\big(h(f, A)\big)\Big)$ and $h\big(int(f, A)\big) \le \Big(Is\ int\,\big(h(f, A)\big)\Big) \le int^{*}cl\Big(Is\ int\,\big(h(f, A)\big)\Big)$. Hence, $h\big(int\,(f, A)\big) \le int^{*}cl\,\big(h(f, A)\big)$.

$(v) \Rightarrow (i)$: Let $(f,A) \in (X,\tau,E)$. Using (v), we have $h(f,A) \leq int^{*}cl\left(h(f,A)\right)$, which implies h is a fuzzy soft infra-semi open function.

Corollary 4.12: Let (X,τ,E) and (Y,μ,E) be two fuzzy soft topological space and $h: (X,\tau,E) \rightarrow (Y,\mu,E)$, then the following properties are equivalent

i) h is a fuzzy soft infra-semi closed.
ii) $h\left(Is\ cl\ (f,B)\right) \leq cl\left(h(f,B)\right), \forall (f,B) \in (Y,\mu,E)$.
iii) $int\left(h^{-1}(f,B)\right) \leq h^{-1}\left(Is\ int\ (f,B)\right), \forall (f,B) \in (Y,\mu,E)$.
iv) $int\left(h(f,B)\right) \leq h^{-1}\left(int^{*}\ cl\ (f,B)\right), \forall (f,B) \in (Y,\mu,E)$.

Definition 4.13: A fuzzy soft set $(f,A) \in (X,\tau,E)$ is said to be fuzzy soft infra-semi connected if and only if (f,A) cannot be expressed as the union of two fuzzy soft infra-semi separated sets.

Theorem 4.14: Let $h: (X,\tau,E) \rightarrow (Y,\mu,E)$ be a surjective fuzzy soft infra-semi continuous function. If (f,A) is a fuzzy soft infra-semi connected subset in (X,τ,E) then, $h(f,A)$ is a fuzzy soft infra-semi connected space in (Y,μ,E).

Proof: Assume that $h(f,A) \in (Y,\mu,E)$ is not a fuzzy soft infra-semi connected. Then, there exists a fuzzy soft infra-semi separated subsets (f,B) and (f,C) in (Y,μ,E) such that $h(f,A) = (f,B) \cup (f,C)$. Since, h is a surjective fuzzy soft infra-semi continuous function $h^{-1}(f,B)\ \&\ h^{-1}(f,C) \in \mathrm{FSIso}(X,\tau,E)$ and $(f,A) = h^{-1}\left(h(f,A)\right) = h^{-1}\left((f,B) \cup (f,C)\right) = h^{-1}(f,B) \cup h^{-1}(f,C)$. Thus, it is clear that $h^{-1}(f,B)\ \&\ h^{-1}(f,C)$ are the fuzzy soft infra-semi separated in (X,τ,E), then (f,A) is not fuzzy soft infra-semi connected in (X,τ,E), which is a contradiction. Hence, (f,A) is a fuzzy soft infra-semi connected space.

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