Connectedness In Pythagorean Fuzzy Topological Spaces

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Abstract — *In this paper, the concept of Pythagorean fuzzy connectedness is introduced and studied. Also we investigate some interrelations between these types of pythagorean fuzzy connectedness . We Show that the fuzzy continuous image of Pythagorean fuzzy connected space is Pythagorean fuzzy connected.*

Keywords — *Pythagorean fuzzy subsets, Pythagorean fuzzy topology, Pythagorean fuzzy continuity, Pythagorean fuzzy connectedness.*

I. INTRODUCTION

The concept of fuzzy set was introduced by Zadeh in his classic paper [13]. The theory of fuzzy topological spaces was introduced and developed by C.L.Chang [4]. Since Atannasov [2,3] introduced the notion of intuitionistic fuzzy sets, Çoker [6] defined the intuitionistic fuzzy topological spaces.Turanlı and Çoker[11,12] introduced the investigate fuzzy connectedness in intuitionistic fuzzy topological spaces. In 2014, Yager [9,10] introduced the notion of Pythagorean fuzzy subsets which is a new class of non-standard fuzzy subsets and which has many effective applicants natural and social sciences. Some authors [8] studied the concept of Pythagorean fuzzy topological space by following the idea of Chang [4]. The aim of this paper is to extended the notions of fuzzy connectedness[1,5] and intuitionistic fuzzy connectedness by introducing the notion of Pythagorean fuzzy connectedness. This construction is based on the idea of Pythagorean fuzzy set developed by Yager [9,10].

II. PRELIMINARIES

We give the concept of Pythagorean fuzzy set defined by Yager [9,10] as a generalization of the concept of an intuitionistic fuzzy set given by Atannasov [2,3].

Definition 2.1. [2,3] Let X be a non-empty fixed set and I the closed interval [0,1]. An intuitionistic fuzzy set (IFS) A is an object of following form $A = \{< x, \mu_A(x), \vartheta_A(x) > | x \in X\}$ where the mappings $\mu_A : X \to I$ and $\vartheta_A : X \to I$ denote the degree of membership (namely) $\mu_A(x)$ and the degree of non-membership (namely) $\vartheta_A(x)$ for each element $x \in X$ to the set A respectively, and $0 \le \mu_A(x) + \vartheta_A(x) \le 1$ for each $x \in X$.

Definition 2.2. [2,3] Let A and B be IFS'S of the form $A = \{< x, \mu_A(x), \vartheta_A(x) > | x \in X\}$ and $B = \{< x, \mu_B(x), \vartheta_B(x) > | x \in X\}$. Then

(i) $A \subseteq B$ iff $\mu_A(x) \le \mu_B(x)$ and $\vartheta_A(x) \ge \vartheta_B(x)$;

(ii) $\bar{A} = \{< x, \vartheta_A(x)(x), \mu_A(x) > | x \in X\}$;

(iii) $A \cap B = \{ < x, \mu_A(x) \wedge \mu_B(x), \vartheta_A(x) \vee \vartheta_B(x) > : x \in X \}$

(iv) $A \cup B = \{ < x, \mu_A(x) \vee \mu_B(x), \vartheta_A(x) \wedge \vartheta_B(x) > : x \in X \}$.

Definition 2.3. [2] $0_\sim = \{< x, 0, 1 > x \in X\}$ and $1_\sim = \{< x, 1, 0 > x \in X\}$.

Definition 2.4. [6] An intuitionistic fuzzy topology (IFT for short) in Çoker's sense on a nonempty set X is a family τ of intuitionistic fuzzy sets in X satisfying the following axioms:

(T_1) $0_\sim, 1_\sim \in \tau$,

(T_2) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$,

(T_3) $\bigcup_{i \in I} G_i \in \tau$ for any arbitrary family $\{G_i : i \in I\} \subseteq \tau$.

In this case the pair (X,τ) is called an intuitionistic fuzzy topological space (IFTS for short) and each IFS in τ is known as a intuitionistic fuzzy open set (IFOS for short) in X.

Definition 2.5. [6] The complement $\bar{A}$ of an IFOS A in an IFTS (X,τ) is called an intuitionistic fuzzy closed set (IFCS for short) in X.

Definition 2.6.[6] Let (X,τ) be an IFTS and $A = \{< x, \mu_A(x), \vartheta_A(x) > | x \in X\}$ be an IFS in X. Then the fuzzy interior and fuzzy closure of A are defined by

IF$cl(A) = \cap \{K: K$ is and IFCS in X and $A \subseteq K\}$ and IF$int(A) = \cup \{G: G$ is an IFOS in X and $G \subseteq A\}$.

Proposition 2.7. [6] Let (X,τ) be an IFTS and A be IFS'S in X. Then the following properties hold:

(i) $1_{\sim}$- IFcl (A)= IF int($1_{\sim}$- A),

(ii) $1_{\sim}$- IFint(A)= IF cl($1_{\sim}$- A).

Definition 2.8. [9,10] A Pythagorean fuzzy subset A of a non-empty set X is a pair

A=(μ_A, ϑ_A) of a membership function $\mu_A: X \to I$ and a non-membership function $\vartheta_A: X \to I$ with $\mu_A^2 + \vartheta_A^2 = r_A^2$ for each element $x \in X$ where $r_A: X \to [0,1]$ is a function which is called the strength of commitment at point x.

The notion of the Pythagorean fuzzy subsets has a basic geometric mean: (r_A, θ_A) be the polar coordinates of (μ_A, ϑ_A) for a point x∈X. If we define $d_A(x)=(1-\theta_A(x))\pi/2$, then the function d_A: X→[0,1] can be considered the direction of commitment at point x. Therefore, Pythagorean fuzzy subsets are more effective than intuitionistic fuzzy subsets as well as fuzzy subsets as shown in Fig. 1. [8,9].

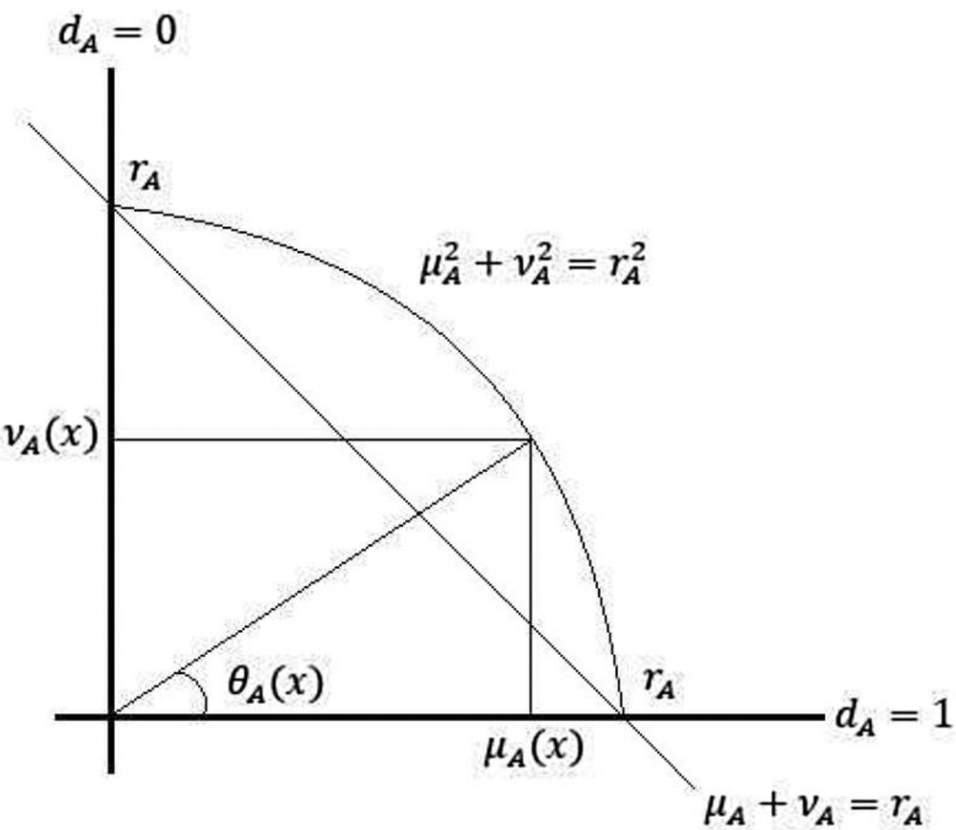

Fig. 1. Comparison of space of Pythagorean and intuitionistic membership grades.

Definition 2.9. [8,9] Let A=(μ_A,ϑ_A) and B=(μ_B,ϑ_B) be two Pythagorean fuzzy subsets of a set X. Then ,

(a) the complement of A is defined by $A^c := (\vartheta_A, \mu_A)$,
(b) the intersection of A and B is defined by

$$A \cap B = (\min\{\mu_A, \mu_B\}, \max\{\vartheta_A, \vartheta_B\}),$$

(c) the union of A and B is defined by

$$A \cup B = (\max\{\mu_A, \mu_B\}, \min\{\vartheta_A, \vartheta_B\}),$$

(d) we say A is a subset of B or B contains A and write $A \subseteq B$ if $\mu_A \leq \mu_B$ and $\vartheta_A \geq \vartheta_B$.

Throughout this paper, we use the notation 1_X for the Pythagorean fuzzy subsets (1,0) and 0_x for the Pythagorean fuzzy subsets (0,1), i.e. $\mu_{1x} = 1$ and $\vartheta_{1x} = 0$, $\mu_{0x} = 0$ and $\vartheta_{0x} = 1$ [8,9].

Definition 2.10. [8] Let X be a non-empty fixed set and let τ be a family of Pythagorean fuzzysubsets of X . If

(T_1) $0_x, 1_x \in \tau$
(T_2) for any $A_1, A_2 \in \tau$, we have $A_1 \cap A_2 \in \tau$
(T_3) $\bigcup_{i \in I} G_i \in \tau$ for any arbitrary family $\{G_i : i \in I\} \subseteq \tau$.

In this case the pair (X,τ) is called a Pythagorean fuzzy topological space (PFTS for short) and each Pythagorean fuzzy subsets in τ is known as a Pythagorean fuzzy open set (PFOS for short) in X. The complement of an open Pythagorean fuzzy subsets is called a closed Pythagorean fuzzy subsets.As any fuzzy subset or intuitionistic fuzzy subset of a set can be considered as a Pythagorean fuzzysubsets, we observe that any fuzzy topological space or intuitionistic fuzzy topological space is a Pythagorean fuzzy topological spaceas well. On the other hand , it is obvious that a Pythagorean fuzzy topological space needs not to be a fuzzy topological space or intuitionistic fuzzy topological space. Even an open Pythagorean fuzzy subsets may be neither a fuzzy subset nor an intuitionistic fuzzy subset [8].

Definition 2.11. [8] Let X and Y be two non-empty sets and $f : X \to Y$ be a function and let A and B be Pythagorean fuzzy subsets of X and Y, respectively. Then, the membership and non-membership functions of image of A with respect to f that is denoted by f [A] are defined by

$$\mu_{F[A]}(y) := \begin{cases} \sup \mu_A(z), if\ f^{-1}(y) is\ non-empty \\ z \in f^{-1}(y) \\ 0, otherwise \end{cases}$$

and

$$\vartheta_{f[A]}(y) := \begin{cases} \inf v_A(z), if\ f^{-1}(y)\ is\ non-empty \\ z \in f^{-1}(y) \\ 1, otherwise \end{cases}$$

respectively. The membership and non-membership functions of pre-image of B with respect to f that is denoted by $f^{-1}[B]$ are defined by

$$\mu_{f^{-1}[B]}(x) := \mu_B(f(x))$$

and

$$\vartheta_{f^{-1}[B]}(x) := \vartheta_B(f(x))$$

respectively. Note that f [A] and $f^{-1}[B]$ are Pythagorean fuzzy subsets.

Proposition 2.12. [8] Let X and Y be two non-empty sets and $f : X \to Y$ be a function. Then, we have

(i) $f^{-1}[B^c] = f^{-1}[B]^c$ for any Pythagorean fuzzy subset B of Y.
(ii) if $f[A]^c \subseteq f[A^c]$ for any Pythagorean fuzzy subset Aof X.

(iii) $B_1 \subseteq B_2$ then $f^{-1}[B_1] \subseteq f^{-1}[B_2]$ where B_1 and B_2 are Pythagorean fuzzy subset of Y.
(iv) if $A_1 \subseteq A_2$, then$f[A_1] \subseteq f[A_2]$ where A_1 and A_2 are Pythagorean fuzzy subset of X.
(v) $f[f^{-1}[B]] \subseteq B$ for any Pythagorean fuzzy subset B of Y.
(vi) $A \subseteq f^{-1}[f[A]]$ for any Pythagorean fuzzy subset Aof X.

Definition 2.13. [8] Let (X, τ_1) and (Y, τ_2) be two Pythagorean fuzzy topological spaces and let $f: X \to Y$ be a function.Then , f is said to be Pythagorean fuzzy continuous if for any Pythagorean fuzzy subset A of X and for any neighbourhood V of f[A] there exists a neighbourhood U of A such that $f[U] \subseteq V$.

Theorem 2.14. [8] Let (X, τ_1) and (Y, τ_2) be two Pythagorean fuzzy topological spaces. A function $f: X \to Y$ is Pythagorean fuzzy continuous iff for each open Pythagorean fuzzy subset B of Y we have $f^{-1}[B]$ is an open Pythagorean fuzzy subset of X.

Definition 2.15. [7] Let (X,τ) be a Pythagorean fuzzy topological space and $A=(\mu_A,\vartheta_A)$ be a Pythagorean fuzzy subset in X. Then the fuzzy closure and fuzzy interior of A are defined by

$$cl(A) = \cap \{K: K \text{ is a Pythagorean fuzzy closed subset in X and } A \subseteq K\}$$

and

$$int(A) = \cup \{G: G \text{ is a Pythagorean fuzzy open subset in X and } G \subseteq A\}.$$

Definition 2.16. [7] A Pythagorean fuzzy subsets A is called a Pythagorean fuzzy regular open set iff A=int(cl(A)); A Pythagorean fuzzy subsets B is called a Pythagorean fuzzy regular closed set iff B=cl(int (B)).

III. PYTHAGOREAN FUZZY CONNECTEDNESS

Here we generalize the concept of intuitionistic fuzzy connected topological space, first initiated by Çoker [6], to the case of Pythagorean fuzzy topological space.

Definition 3.1. Let A be aPythagorean fuzzy subset in (X,τ).

(a) If there exist Pythagorean fuzzy open sets U and V in X satisfying the following properties, then A is called Pythagorean fuzzy C_i-disconnected (i=1,2,3,4):
C_1: $A \subseteq U \cup V$, $U \cap V \subseteq A^c$, $A \cap U \neq 0_x$, $A \cap V \neq 0_x$,
C_2: $A \subseteq U \cup V$, $A \cap U \cap V = 0_x$,$A \cap U \neq 0_x$, $A \cap V \neq 0_x$,
C_3: $A \subseteq U \cup V$, $U \cap V \subseteq A^c$, $U \not\subseteq A^c$, $V \not\subseteq A^c$,
C_4: $A \subseteq U \cup V$, $A \cap U \cap V = 0_x$, $U \not\subseteq A^c$, $V \not\subseteq A^c$.

(b) A is said to be Pythagorean fuzzy C_i-connected (i=1,2,3,4) , if A is not Pythagorean fuzzy C_i-disconnected (i=1,2,3,4) .

It is clear that in Pythagorean fuzzy topological spaces we have the following implications:

$$\begin{array}{ccc} C_1\text{-connectedness} & \to & C_2\text{-connectedness} \\ \downarrow & & \downarrow \\ C_3\text{-connectedness} & \to & C_4\text{-connectedness.} \end{array}$$

Example 3.2. Let X ={1,2,3} and define the Pythagorean fuzzy subsetsA, B, C, D as follows ;
$\mu_A(1)=0{,}5$, $\mu_A(2)=0{,}5$, $\mu_A(3)=0{,}4$, $\vartheta_A(1)=0{,}2$, $\vartheta_A(2)=0{,}4$, $\vartheta_A(3)=0{,}4$,
$\mu_B(1)=0{,}4$, $\mu_B(2)=0{,}6$, $\mu_B(3)=0{,}2$, $\vartheta_B(1)=0{,}5$, $\vartheta_B(2)=0{,}3$, $\vartheta_B(3)=0{,}3$,
$\mu_C(1)=0{,}5$, $\mu_C(2)=0{,}6$, $\mu_C(3)=0{,}4$, $\vartheta_C(1)=0{,}2$, $\vartheta_C(2)=0{,}3$, $\vartheta_C(3)=0{,}3$,
$\mu_D(1)=0{,}4$, $\mu_D(2)=0{,}5$, $\mu_D(3)=0{,}2$, $\vartheta_D(1)=0{,}5$, $\vartheta_D(2)=0{,}4$, $\vartheta_D(3)=0{,}4$.

Then $\tau = \{0x, 1x, A, B, C, D\}$ is a Pythagorean fuzzy topological space on X , and consider the Pythagorean fuzzy set E given below

$\mu_E(1)=0{,}6$, $\mu_E(2)=0{,}5$, $\mu_E(3)=0{,}4$, $\vartheta_E(1)=0{,}2$, $\vartheta_E(2)=0{,}2$, $\vartheta_E(3)=0{,}3$
in X. Then E is Pythagorean fuzzy C_1-connected , and E is also Pythagorean fuzzy C_2-, C_3-, C_4- connected.

Example 3.3. Consider the Pythagorean fuzzy topological space (X,τ) given in the Example 3.2 and consider the Pythagorean fuzzy subset F given below

$\mu_F(1)=0{,}2$, $\mu_F(2)=0{,}3$, $\mu_F(3)=0{,}2$, $\vartheta_F(1)=0{,}4$, $\vartheta_F(2)=0{,}6$, $\vartheta_F(3)=0{,}4$.

Since $F\cap A\cap B\neq 0_x$, $F\cap A\cap C\neq 0_x$, $F\cap A\cap D\neq 0_x$, $F\cap B\cap C\neq 0_x$, $F\cap B\cap D\neq 0_x$, and $F\cap C\cap D\neq 0_x$, F is Pythagorean fuzzy C_1-disconnected, and hence not Pythagorean fuzzy C_1-connected.

Definition 3.4. (a) Let (X,τ) be a Pythagorean fuzzy topological space. If there exists a Pythagorean fuzzy regular open set A in X such that $0_x\neq A\neq 1_x$, then X is called Pythagorean super disconnected.

(b)X is called Pythagorean fuzzy super connected, if X is not Pythagorean fuzzy super disconnected.

Example 3.5. Let $X=\{1,2,3\}$ and define the Pythagorean fuzzy subsets A, B, C as follows ;

$\mu_A(1)=0{,}4$, $\mu_A(2)=0{,}6$, $\mu_A(3)=0{,}2$, $\vartheta_A(1)=0{,}5$, $\vartheta_A(2)=0{,}3$, $\vartheta_A(3)=0{,}3$,
$\mu_B(1)=0{,}5$, $\mu_B(2)=0{,}6$, $\mu_B(3)=0{,}4$, $\vartheta_B(1)=0{,}2$, $\vartheta_B(2)=0{,}3$, $\vartheta_B(3)=0{,}3$,
$\mu_C(1)=0{,}4$, $\mu_C(2)=0{,}5$, $\mu_C(3)=0{,}2$, $\vartheta_C(1)=0{,}5$, $\vartheta_C(2)=0{,}4$, $\vartheta_C(3)=0{,}4$.

Then $\tau=\{0x, 1x, A, B, C\}$ is a Pythagorean fuzzy topological space on X , and (X,τ) is Pythagorean fuzzy super connected.

Proposition 3.6. In a Pythagorean fuzzy topological space (X,τ) the following conditions are equivalent:

(i) X is Pythagorean fuzzy super connected.
(ii) For each Pythagorean fuzzy open set $A\neq 0_x$ in X we have $cl(A)=0_x$.
(iii) For each Pythagorean fuzzy closed set $A\neq 1_x$ in X we have $int(A)=0_x$.
(iv) There exist no Pythagorean fuzzy open set A and B in X such that $A\neq 0_X\neq B$ and $A\subseteq B^c$.
(v) There exist no Pythagorean fuzzy open set A and B in X such that $A\neq 0_X\neq B$, $B=[cl(A)]^c$, $A=[cl(B)]^c$.
(vi) There exist no Pythagorean fuzzy closed set A and B in X such that $A\neq 1_x\neq B$,

$B=[intl(A)]^c$, $A=[int(B)]^c$.

Proof. (i=>ii): Assume that there exists a Pythagorean fuzzy open set $A\neq 0_X$ such that $cl(A)\neq 1_x$. Since $B=int(cl(A))$ is a Pythagorean fuzzy regular open set in X, and this is a contradiction with the Pythagorean fuzzy super connectedness of X.

(ii=>iii): Let $A\neq 1_x$ be a Pythagorean fuzzy closed set in X. If we take $B=A^c$, then B is a Pythagorean fuzzy open set in X and $B\neq 0_X$. Hence, $cl(B)=1_x \Rightarrow [cl(B)]^c=0_X \Rightarrow int(B^c)=0_x \Rightarrow int(A)=0_x$.

(iii=>iv): Let A and B be Pythagorean fuzzy sets in X such that $A\neq 0_X\neq B$ and $A\subseteq B^c$. Since B^c is a Pythagorean fuzzy closed set in X and $B\neq 0_x \Rightarrow B^c\neq 1_x$, we obtain $int(B^c)=0_x$. Now, we have $0_x\neq A=int(A)\subseteq int(B^c)=0_x$, which is a contradiction.

(iv=>v): Obvious.

(i=>v): Let A and B be Pythagorean fuzzy open sets in X such that $A\neq 0_X\neq B$ and $B=[cl(A)]^c$, $A=[cl(B)]^c$. Now, we have $int(cl(A))=int(B^c)=[cl(B)]^c=A$ and $A\neq 0_X$, $A\neq 1_X$. This is a contradiction.

(v=>i): Obvious.

(v=>vi): Let Aand B Pythagorean fuzzy closed set in X such that $A\neq 1_X\neq B$,
$B=[int(A)]^c$, $A=[int(B)]^c$. Taking $U=A^c$ and $V=B^c$, U and V become Pythagorean fuzzy open sets and $U\neq 0_x\neq V$, $[cl(U)]^c=V$ and $[cl(V)]^c=U$. This is a contradiction.

(vi=>v): This is similar to (v=>vi).

Definition 3.7. Let (X,τ) be a Pythagorean fuzzy topological space.

(i) X is said to be Pythagorean fuzzy C_5-disconnected if there exists a Pythagorean fuzzy open and fuzzy closed set G such that $G\neq 1_x$ and $G\neq 0_x$.
(ii) X is said to be Pythagorean fuzzy C_5-connected if it is not Pythagorean fuzzy C_5-disconnected.

Example 3.8. Let X ={1,2 } and define the Pythagorean fuzzy subsets A, B, C, D as follows ;

$\mu_A(1)=0,4$, $\mu_A(2)=0,2$, $\vartheta_A(1)=0,3$, $\vartheta_A(2)=0,7$,
$\mu_B(1)=0,3$, $\mu_B(2)=0,7$, $\vartheta_B(1)=0,4$, $\vartheta_B(2)=0,2$,
$\mu_C(1)=0,3$, $\mu_C(2)=0,2$, $\vartheta_C(1)=0,4$, $\vartheta_C(2)=0,7$,
$\mu_D(1)=0,4$, $\mu_D(2)=0,7$, $\vartheta_D(1)=0,3$, $\vartheta_D(2)=0,2$.

Then the family $\tau=\{0x, 1x, A, B, C, D\}$ is a Pythagorean fuzzy topology on X and (X,τ) is a Pythagorean fuzzy C_5-disconnected, since A is a nonzero Pythagorean fuzzy open and Pythagorean fuzzy closed set in X.

Definition 3.9. Let (X,τ) be a Pythagorean fuzzy topological space.
(İ) X is called Pythagorean fuzzy disconnected, if there exist Pythagorean fuzzy open sets $A\neq 0_x$ and $B\neq 0_x$ such that $AUB=1_x$ and $A\cap B=0_x$.
(ii) X is called Pythagorean fuzzy connected, if X is not Pythagorean fuzzy disconnected.

Proposition 3.10. Pythagorean fuzzy C_5-connectedness implies Pythagorean fuzzy connectedness.

Proof. Suppose that there exist Pythagorean fuzzy open sets A and B such that $AUB=1_x$ and $A\cap B=0_x$.
In this case we have $\mu_A \vee \mu_B=1$, $\vartheta_A\wedge \vartheta_B=0$, $\mu_A \wedge \mu_B=0$ and $\vartheta_A \vee \vartheta_B=1$. Let U={xϵX: $\mu_A(x)>0$}, V= {xϵX: $\mu_A(x)=0$}. If xϵU , then $\mu_A(x)>0 \Rightarrow \mu_B(x)=0 \Rightarrow \mu_A(x)=1 \Rightarrow \vartheta_A(x)=0 \Rightarrow \vartheta_B(x)=1$. If xϵV, then $\mu_B(x)=1 \Rightarrow \vartheta_B(x)=0$. Hence $\mu_A= \vartheta_B$ and $\vartheta_A= \mu_B$, that is $B=A^c$. Therefore A is both Pythagorean fuzzy open and closed set, and since $B\neq 0_x \Rightarrow A \neq 1_x$, $0_x\neq A \neq 1_x$. Thus (X,τ) is Pythagorean fuzzy C_5-disconnected.

Proposition 3.11. Let (X,τ_1), (Y,τ_2) be two Pythagorean fuzzy topological spaces and let f :X→Y be a Pythagorean fuzzy continuous surjection. If (X,τ_1) is Pythagorean fuzzy connected, , then so is (Y,τ_2).

Proof. On the contrary, suppose that (Y,τ_2) is Pythagorean fuzzy disconnected. Then there exist Pythagorean fuzzy open sets $A\neq 0_Y$,$B\neq 0_Y$ in Y such that $AUB=1_y$, $A\cap B=0_Y$. Now, we see that $U=f^{-1}(A)$, $V= f^{-1}(B)$ are Pythagorean fuzzy open sets in X, since f is Pythagorean fuzzy continuous.
From $A\neq 0_Y$, we get $U=f^{-1}(A)\neq 0_x$. Similarly $V\neq 0_X$. Hence, $AUB=1_y \Rightarrow f^{-1}(A)Uf^{-1}(B)= f^{-1}(1_Y)=1_X \Rightarrow$

$U\cup V=1_X$; $A\cap B=0_Y \Rightarrow f^{-1}(A)\cap f^{-1}(B)=f^{-1}(0_Y)=0_X \Rightarrow U\cap V=0_X$. But this is a contradiction to our hypothesis, i.e. Y is Pythagorean fuzzy connected.

Corollary 3.12. Let (X,τ_1), (Y,τ_2) be two Pythagorean fuzzy topological spaces and let f :X→Y be a Pythagorean fuzzy continuous surjection. If (X,τ_1) is Pythagorean fuzzy C_5- connected, , then so is (Y,τ_2).

Definition 3.13. A Pythagorean fuzzy topological space (X,τ) is said to be Pythagorean fuzzy strongly connected , if there exists nonzero Pythagorean fuzzy closed sets A and B such that $\mu_A+ \mu_B\leq 1$ and $\vartheta_A+ \vartheta_B\geq 1$.

Example 3.14. Let X ={1,2 } and define the Pythagorean fuzzy subsets A, B as follows ;

$\mu_A(1)=0,3$, $\mu_A(2)=0,4$, , $\vartheta_A(1)=0,6$, $\vartheta_A(2)=0,5$,
$\mu_B(1)=0,4$, $\mu_B(2)=0,1$, $\vartheta_B(1)=0,5$, $\vartheta_B(2)=0,3$.

Then the family $\tau=\{0x, 1x, A, B, AUB, A\cap B\}$ is a Pythagorean fuzzy topology on X, and (X,τ) is a Pythagorean fuzzy strongly connected.

Proposition 3.15. Let (X,τ_1), (Y,τ_2) be two Pythagorean fuzzy topological spaces and let f :X→Y be a Pythagorean fuzzy continuous surjection. If (X,τ_1) is Pythagorean fuzzy strongly connected, then so is (Y,τ_2).

Proof. This is analogous to proof of Proposition 3.11.

It is clear that in Pythagorean fuzzy topological spaces, Pythagorean fuzzy strongly connectedness does not imply Pythagorean fuzzy C_5- connectedness, and the same is true for its converse.

Example 3.16. Let X ={1,2 } and define the Pythagorean fuzzy subsets A, B, C, D as follows ;

$\mu_A(1)=0,4$, $\mu_A(2)=0,2$, $\vartheta_A(1)=0,3$, $\vartheta_A(2)=0,7$,
$\mu_B(1)=0,3$, $\mu_B(2)=0,7$, $\vartheta_B(1)=0,4$, $\vartheta_B(2)=0,2$,
$\mu_C(1)=0,3$, $\mu_C(2)=0,2$, $\vartheta_C(1)=0,4$, $\vartheta_C(2)=0,7$,
$\mu_D(1)=0,4$, $\mu_D(2)=0,7$, $\vartheta_D(1)=0,3$, $\vartheta_D(2)=0,2$.

Then the family $\tau = \{0x, 1x, A, B, C, D\}$ is a Pythagorean fuzzy topology on X and (X,τ) is a Pythagorean fuzzy strongly connected, but not Pythagorean fuzzy C_5- connected.

Example 3.17. Let X ={1,2 } and define the Pythagorean fuzzy subsets A, B as follows ;

$\mu_A(1)=0,5$, $\mu_A(2)=0,6$, $\vartheta_A(1)=0,4$, $\vartheta_A(2)=0,3$,
$\mu_B(1)=0,5$, $\mu_B(2)=0,4$, $\vartheta_B(1)=0,2$, $\vartheta_B(2)=0,4$.

Then the family $\tau = \{0x, 1x, A, B, A \cup B, A \cap B\}$ is a Pythagorean fuzzy topology on X, and (X,τ) is Pythagorean fuzzy C_5- connected, but not Pythagorean fuzzy strongly connected.

IV. CONCLUSIONS

In this paper, we introduce the notion of Pythagorean fuzzy connected space which extends the notions of both fuzzy topological space and intuitionistic fuzzy topological spaces. The presented concepts in this study are fundamental for further researches and will open a way to improve more applications on Pythagorean fuzzy topology.

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REFERENCES

[1] N. Ajmal, J.K.Kohli, Connectedness in fuzzy topological spaces, Fuzzy Sets and Systems 31(1989)369-388.
[2] K.Atanassov, Intuitionistic fuzzy sets, Fuzzy Sets and Systems 20(1986)87-96.
[3] K.Atanassov and S.Stoeva, Intionistic fuzzy sets, in:Polish Symp.on Interval and Fuzzy Mathematics, Poznan(1983)23-26.
[4] C.Chang, Fuzzy topological spaces, J.Math.Anal.Appl.24(1968)182-190.
[5] A.K.Chaudhuri, P.Das, Fuzzy connected sets in fuzzy topological spaces, Fuzzy Sets and Systems 49(1992)223-229.
[6] D.Çoker, An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and Systems 88(1997)81-89
[7] A.H.Eş, Compactness in Pythagorean fuzzy topological spaces (submitted) .
[8] M.Olgun, M.Ünver and Ş.Yardımcı, Pythagorean fuzzy topological spaces Complex and Intelligent Systems, (2019) doi org /10. 1007/s40747-019-0095-2. .
[9] R.R.Yager,Pythagorean fuzzy subsets.In:2013 joint IFSA World congress and NAFIPS annual meeting (IFSA/NAFIPS),IEEE, Edmonton, AB, Canada,24-28 JUNE 2013, 57-61.doi.org/10.1109/IFSA-nafips.2013.6608375.
[10] R.R. Yager,Pythagorean membership grades in multicriteria decision making,IEE Transactıons on fuzzy systems 22(2014)958-965.
[11] N.Turanlı and D.Çoker, On some types of fuzzy connectedness in fuzzy topological spaces, Fuzzy Sets and Systems 60(1993)97-102 .
[12] Fuzzy connectedness in intuitionistic fuzzy topological spaces,Fuzzy Sets and Systems 116(2000) 369-375.
[13] L.A.Zadeh, Fuzzy sets, Inf.Control 8 (1965) 338-353.