A Study in Detailed About Fuzzy Algebra and Fuzzy Near Ring.

S. Geetha [#1], Dr.V. Ramadoss [*2]

[#1]*M.Phil (Scholar) & Mathematics & PRIST University*
Vallam, Thanjavur, Tamilnadu, India.
[*2]*M.Sc.,M.Phil.,Phd., Asst.Prof & M.Phil (Mathematics) & PRIST University*
Vallam, Thanjavur, Tamilnadu, India.

Abstract — *a fuzzy set is class of objects in which the transition from membership to no membership is gradual rather than abrupt. Such a class is characterized by a membership function which assigns to an element a grade or degree of membership between 0 and 1. The nearer the values of an element to unity, the higher the grade of its membership. Hence in fuzzy set theory everything is a matter of degree or, to put it metaphorically, everything has elasticity. Although for some time, the simplicity of the definition of a fuzzy set on the one hand, and the existence of analogy between algebra of fuzzy sets with that of the algebra of ordinary sets with on the other, lead to scepticism among mathematical community.*

Keywords — *Galois Theory, group theory, roots of polynomials.*

I. INTRODUCTION

This project work is designed and written to study in detailed about Fuzzy Algebra and Fuzzy near Ring. The concept of fuzzy set was introduced by zadeh. Since then these ideal have been applied to other algebraic structure like groups, rings, modules, topologies and so on. The notations of fuzzy sub near – ring and ideals where introduced by S.Abou – Zaid in 1991. However, the fantastic growth of literature, the vitality of the field, and the applications of this theory provided a denial for this impression. This is well evident from contributions in many international journals such as fuzzy sets and systems, information sciences, information and control, journal of Mathematical Analysis and Applications, Rocky Mountain Journal of Mathematics, etc., Fuzziness means different things depending upon the domain of application and the way it is measured. By means of fuzzy sets, vague notions can be described mathematically in their very abstractness. Fuzzy set theory has been widely acclaimed as offering greater richness in application than the ordinary set theory. It is now a vigorous area of research with manifold applications. It should be mentioned that these are natural ways to fuzzily various mathematical structures such as topological spaces, algebraic structures, etc. In this paper we attempt to characterize some properties of fuzzy subgroups and fuzzy ideals of near rings. We introduce the notations of fuzzy near – ring subgroups and fuzzy ideals of near ring subgroups. We also investigate some characteristic of fuzzy subgroups and ideals of a strongly regular near – ring.

II. BASIC DEFINITIONS AND EXAMPLES

Definition: 2.1

Let S be any non-empty set. A mapping $A: S\rightarrow [0, 1]$ is called a fuzzy subset of S.

A fuzzy subset A of a ring R is called a fuzzy sub ideal of R if for all $x,y \in R$, the following conditions are satisfied

i). $A(x-y) \geq \min(A(x),A(y))$

ii). $A(xy) \geq \max(A(x),A(y))$

Theorem: 2.1

i). Let μ be any fuzzy ideal of a ring R, and let $t = \mu(0)$, then the fuzzy subset μ^* of R/μ_t defined by

$\mu^*(x+\mu_t) = \mu(x)$ for all $x \in R$ is a fuzzy ideal of R/μ_t ……….(a)

ii). If A is an ideal of R and θ is a fuzzy ideal of R/A such that

$\theta(x+A) = \theta(A)$ ………………..(b) only when $x \in A$ then there exists a fuzzy ideal μ of R such that, $\mu_t = A(t=\mu(0))$ and $\theta = \mu^*$.

Proof:

Part : (i)

Let μ be a fuzzy ideal of R and μ_t is an ideal of R.

To prove that,

μ^* is well-defined

By condition,

$\mu^*(x+\mu_t) = \mu(x)$

Then,

$x+\mu_t = y+\mu_t$ for all $x,y \in R$
$x-y \in \mu_t$ where $t \in \mu(0)$
$\mu(x-y) \in \mu(0)$
$\mu(x) = \mu(y)$
$\mu^*(x+\mu_t) = \mu^*(y+\mu_t)$ [by using (a)]
μ^* is well defined.

To prove that,
μ^* is a fuzzy ideal of R/μ_t for all $x,y \in R$.

We have,
$\mu^*((x+\mu_t)-(y+\mu_t)) = \mu^*(x-y+\mu_t)$
$= \mu(x-y)$ [by using (a)]
$\geq \min(\mu(x),\mu(y))$ [by definition of fuzzy ideal]
$= \min(\mu^*((x+\mu_t),\mu^*(y+\mu_t)))$

And,
$\mu^*((x+\mu_t)(y+\mu_t)) = \mu^*(xy)+\mu_t$
$= \mu(xy)$ [by using (a)]
$\geq \max(\mu(x),\mu(y))$[by definition of fuzzy ideal]
$= \max(\mu^*((x+\mu_t),\mu^*(y+\mu_t)))$
μ^* is a fuzzy ideal of R/μ_t.

Part : (ii)

Define,
$\mu : R \rightarrow [0,1]$
$\mu(x) = \theta(x+A)$ …………….(ii)

To prove that,
μ is an fuzzy ideal of R for all $x,y \in R$.

Consider,
$\mu((x+A)-(y+A)) = \mu(x-y)+A$
$= \mu(x-y)$
$\geq \min(\mu(x),\mu(y))$
$= \min(\theta(x+A),\theta(y+A))$ [by (i)]
$\mu((x+A)-(y+A)) \geq \min(\theta(x+A),\theta(y+A))$
$\mu((x+A)(y+A)) = \mu(xy+A)$
$= \mu(xy)$
$\geq \max(\mu(x),\mu(y))$
$= \max(\theta(x+A),\theta(y+A))$ [by (i)]
$\mu((x+A)(y+A)) \geq \max(\theta(x+A),\theta(y+A))$
μ is an fuzzy ideal of R.

Given $\mu_t = A$
Since $x \in \mu_t \leftrightarrow \mu(x) = \mu(0)$ where $t = \mu(0)$
$\theta(x+A) = \theta(0+A)$
$\theta(x+A) = \theta(A)$
$x \in A$

Finally, we get
$\mu^* = 0$

Since,
$\mu^*(x+\mu_t) = \mu(x)$
$= \theta(x+A)$ [by (i)]
$= \theta(x+\mu_t)$ where $A=\mu_t$
$\mu^*(x+\mu_t) = \theta(x+\mu_t)$
$\mu_t = 0$
Hence the proof.

Definition: 2.3

Let μ be any fuzzy ideal of a ring R. The fuzzy subset μ_x of R defined by
$\mu_x{}^*(r) = \mu(r-x)$ for all $x \in R$ is termed as the fuzzy coset determined by x and μ.

Theorem: 2.2

Let μ be any fuzzy ideal of R. Then R_μ the set of all fuzzy coset of μ in R is a ring under the compositions
$\mu^*_x+\mu^*_y = \mu^*_{x+y}$ and
$\mu^*_x \mu^*_y = \mu^*_{xy}$ for all $x,y \in R$.

Proof:

First we have to show that binary operation is well defined.
Let a, b, c and d are the elements of R,
Such that,

$\mu^*_a = \mu^*_b$ and $\mu^*_c = \mu^*_d$

Then,

$\mu(r\text{-}a) = \mu(r\text{-}b)$ for all $r \in R$ …………(1)

$\mu(r\text{-}c) = \mu(r\text{-}d)$ for all $r \in R$ …………(2)

Put r = a+c-d in eqn (1)

$\mu(a+c-d-a) = \mu(a+c-d-b)$

$\mu(c-d) = \mu(a+c-d-b)$ …………(3)

Put r = c in eqn (2)

$\mu(c-c) = \mu(c-d)$

$\mu(0) = \mu(c-d)$ ……………..(4)

Put r = a in eqn (1)

$\mu(a-a) = \mu(a-b)$

$\mu(0) = \mu(a-b)$ …………….(5)

To prove that,

$\mu^*_a+\mu^*_c = \mu^*_b+\mu^*_d$

To show that,

$\mu^*_a+\mu^*_c \subseteq \mu^*_b+\mu^*_d$

Then,

$(\mu^*_a+\mu^*_b)(r) = \mu^*_{a+b}(r)$

$= \mu(r-a-c)$ [where $r \in R$ by the defn of fuzzy coset]

$= \mu((r-b-d)-(a+c-b-d))$

$\geq \min(\mu(r-b-d), \mu(a+c-b-d))$

$= \min(\mu(r-b-d), \mu(0))$ [by using eqn 3,4]

$= \mu(I-b-d)$

$= \mu^*_{b+d}$ [by defn of fuzzy coset]

$= (\mu^*_b+\mu^*_d)(r)$

$(\mu^*_a+\mu^*_c)(r) \subseteq (\mu^*_b+\mu^*_d)(r)$

$(\mu^*_a+\mu^*_c) \subseteq (\mu^*_b+\mu^*_d)$ ……………..(i)

Next to prove,

$(\mu^*_b+\mu^*_d) \subseteq (\mu^*_a+\mu^*_c)$

Then,

$\mu(r\text{-}a) = \mu(r\text{-}b)$ for all $r \in R$ …………(6)

$\mu(r\text{-}c) = \mu(r\text{-}b)$ for all $r \in R$ …………(7)

Put r = c+a-d in eqn (6)

$\mu(c+a-d-a) = \mu(c+a-d-b)$

$\mu(c-d) = \mu(c+a-d-b)$ ……….(8)

Put r = b in eqn (6)

$\mu(b-a) = \mu(b-b)$

$\mu(b-a) = 0$ …………….(9)

Put r = d in eqn (7)

$\mu(d-c) = \mu(d-d)$

$\mu(d-c) = \mu(0)$ …………..(10)

$(\mu^*_a+\mu^*_b)(r) = (\mu^*_{a+b})(r)$ [by definition of fuzzy coset]

$= \mu(r-b-d)$ where $r \in R$

$= \mu((r-a-c)-(b+d-a-c))$

$\geq \min(\mu(r-a-c),\mu(b+d-a-c))$

$= \min(\mu(r-a-c),\mu(0))$ [by using eqn 9 & 10]

$= \mu(I-a-c)$

$= \mu^*_{a+c}$

$= (\mu^*_a+\mu^*_c)(r)$

$(\mu^*_b+\mu^*_d)(r) \subseteq (\mu^*_a+\mu^*_c)(r)$

$(\mu^*_b+\mu^*_d) \subseteq (\mu^*_a+\mu^*_c)$ …………(ii)

From (i) and (ii)

$(\mu^*_a+\mu^*_c) \subseteq (\mu^*_b+\mu^*_d)$

Binary operation is well-defined. To prove, multiplication is well-defined. To prove that,

$\mu^*_a \mu^*_c = \mu^*_b \mu^*_d$

First of all to prove that,

$\mu^*_a \mu^*_c \subseteq \mu^*_b \mu^*_d$

Since,

$(\mu^*_a \mu^*_c)(r) = \mu^*_{ac}$

$= \mu^*(r\text{-}ac)$ where $r \in R$

$= \mu((r\text{-}bd)\text{-}(ac\text{-}bd))$

$= \min(\mu(r\text{-}bd), \mu((a\text{-}b)c\text{-}b(d\text{-}c)))$

$\geq \min(\mu(r\text{-}bd), \min(\mu(a\text{-}b),\mu(d\text{-}c)))$

$= \min(\mu(r\text{-}bd), \min(\mu(0),\mu(0)))$ [by using (5) & (6)]

$= \mu(r\text{-}bd)$

$=\mu^*_{bd}(r)$

$= (\mu^*_b \mu^*_d)(r)$

$(\mu^*_a+\mu^*_c)(r) \subseteq (\mu^*_b \mu^*_d)(r)$

$(\mu^*_a+\mu^*_c) \subseteq (\mu^*_b \mu^*_d)$ ……………(iii)

Similarly,

$(\mu^*_b \mu^*_d) \subseteq (\mu^*_a \mu^*_c)$ ………..(iv)

From (iii) and (iv)

$(\mu^*_a \mu^*_c) \subseteq (\mu^*_b \mu^*_d)$

Multiplication are well defined Clearly,

$\mu^*_0(=\mu)$ acts as the additive identity

μ^*_c acts as the multiplicative identity and

μ^*_{-x} as the additive inverse of μ^*_x

Hence the proof.

Lemma: 2.1

If μ is any fuzzy ideal of a ring R then the following holds

$\mu(x) = \mu(0) \leftrightarrow \mu^*_x = \mu^*_0$ where $x \in R$.

Proof:

Assume that,

$\mu(x) = \mu(0)$

To prove,

$\mu^*_x = \mu^*_0$ where $x \in R$

Given,

$\mu(x) = \mu(0)$ ………………(i)

Then,

$\mu(r) < \mu(x)$

$= \mu(0)$ for all $r \in R$ [by (i)]

If $\mu(r) < \mu(x)$

$\mu(r\text{-}x) = \mu(r)$ ………(ii)

By using the lemma,

"If μ is any fuzzy subring (fuzzy ideal) of a ring R and if $\mu(r) < \mu(x)$ for some $x,y \in R$, then $\mu(x\text{-}y) = \mu(x) = \mu(y\text{-}x)$"

We have, if $\mu(r) =\mu(0)$ then $x,r \in \mu_t$ where $t = \mu(0)$

$\mu(r\text{-}x) = \mu(0)$

$\mu(r\text{-}x) = \mu(x)$ ………(iii)

From (ii) and (iii)

$\mu(r\text{-}x) = \mu(x) = \mu(r)$

$\mu(r\text{-}x) = \mu(r)$

$\mu^*_x = \mu^*_0$ [by definition of fuzzy coset]

Conversely,

Consider $\mu^*_x = \mu^*_0$

$\mu(r\text{-}x) = \mu(r)$

$\mu(x) = \mu(0)$

Hence the proof.

Theorem: 2.3

For any fuzzy ideal μ of a ring R the following holds

$R/\mu_t \cong R_\mu$ where $t = \mu(0)$.

Proof:

Let us consider the map,

$f : R \rightarrow R_\mu$

The function is defined by

$f(x) = \mu^*_x$ for all $x \epsilon R$ …………….(i) is an onto homomorphism.

Then,

$f(x) = \mu^*_0$

$\mu^*_x = \mu^*_0$ [by (i)]

By using the lemma,

"If μ is any fuzzy ideal of a ring R then the following holds

$\mu(x) = \mu(0) \leftrightarrow \mu^*_x = \mu^*_0$ where $x \epsilon R$"

$\mu(x) = \mu(0)$

This shows that ker $f = \mu_t$

Accordingly,

$$R/\mu_t \cong R_\mu$$

Hence the proof.

Definition: 2.4

If μ is any fuzzy ideal of a ring R then the fuzzy ideal μ' of R_μ defined $\mu'(\mu^*_s) = \mu(x)$ for all $x \epsilon R$ is designated by μ.

Theorem: 2.4

If μ is any fuzzy ideal of a ring R then the fuzzy subset μ' of R_μ defined by $\mu'(\mu^*_s) = \mu(x)$ where $x \epsilon R$ is a fuzzy ideal of R_μ.

Proof:

Let μ is any fuzzy ideal of a ring R

Let μ' be a fuzzy subset of R_μ is defined by

$\mu'(\mu^*_x) = \mu(x)$ ……………….(i)

To prove,

μ' is well defined.

$\mu^*_x = \mu^*_y$

$\mu^*_{x-y} = \mu^*_0$ [by definition of fuzzy coset]

$\mu(x-y) = \mu(0)$

$\mu(x) = \mu(y)$

$\mu'(\mu^*_x) = \mu'(\mu^*_y)$ [by (i)]

μ' is well defined.

To prove,

μ' is fuzzy ideal of R_μ for all $x,y \epsilon R$, then we have

$\mu'(\mu^*_x - \mu^*_y) = \mu(x-y)$

$\geq \min(\mu(x),\mu(y))$

$= \min(\mu'(\mu^*_x), \mu'(\mu^*_y))$

$\mu'(\mu^*_{xy}) = \mu(xy)$

$\geq \max(\mu(x), \mu(y))$

$= \max(\mu'(\mu^*_x), \mu'(\mu^*_y))$

μ' is fuzzy ideal of R_μ.

Hence the proof.

Theorem: 2.5

If μ is any fuzzy ideal of a ring R. Then each fuzzy ideal of R_μ corresponds in a natural way to a fuzzy ideal of R.

Proof:

Let μ is any fuzzy ideal of a ring R

μ' be any fuzzy ideal of R_μ

Routine computations show that the fuzzy subset θ of R.

To prove,

θ is a fuzzy ideal of R is defined by

$\theta(x) = \mu'(\mu^*_x)$ for all $x \epsilon R$ ……………..(i)

$\theta(x-y) = \mu'(\mu^*_x - \mu^*_y)$

$\geq \min(\mu'(\mu^*_x, \mu'(\mu^*_y))$

$= \min(\theta(x),\theta(y))$ [by (i)] ……………(ii)

$\mu(xy) = \mu'(\mu^*_{xy})$

$\geq \max(\mu'(\mu^*_x, \mu'(\mu^*_y))$
$= \max(\theta(x), \theta(y))$

From (i) and (ii)

θ is a fuzzy ideal of R.

Hence the proof.

Lemma: 2.2

The induced mapping f' is an isomorphism $\leftrightarrow$ μ is f-invarient

Proof:

Assume that,

f' is an isomorphism

To prove,

μ is f-invarient

Consider f' be one-to-one and x,yϵR.

Then μ is f-invarient.

Since,

$f(x) = f(y)$
$f'(\mu^*_x) = f'(\mu^*_y)$
$\mu^*_x = \mu^*_y$ [$\because f(x) = f'(\mu^*_x)$]
$\mu^*_{x-y} = \mu^*_0$
$\mu(x-y) = \mu(0)$ [by definition of fuzzy coset]
$\mu(x) = \mu(y)$
f-invarient

To show that,

f' is one-to-one

Consider,

μ is f-invarient

Then f' is one-to-one

Since,

$f'(\mu^*_x) = f'(\mu^*_0)$ [by condition $f(x)=f'(\mu^*_x)$]
$f(x) = f(0)$
$\mu(x) = \mu(0)$ [since μ is f-invariant]
$\mu^*_x = \mu^*_0$
Hence the proof.

III. FUZZY ALGEBRA AND FUZZY NEAR RING.

Definition: 3.1

Let μ be any fuzzy ideal of a ring R, tϵ[0,1] and $t \leq \mu(0)$. The ideal μ_t is called a level ideal of μ.

Definition: 3.2

A fuzzy ideal μ of a ring R is called prime, if the ideal μ_t, where $\mu(0)=t$, is a prime ideal of R.

Theorem: 3.1

Let μ be any fuzzy ideal of R, such that each level ideal μ_t, $t \epsilon I_m\mu$ is prime.If $\mu(x) < \mu(y)$ for some x,y ϵ R. Then

$\mu(xy) = \mu(y)$

Proof:

Let μ be any fuzzy ideal of a ring R.

To prove that,

$\mu(xy) = \mu(y)$

Given by the condition

$\mu(x) < \mu(y)$ …………….(i)

Consider,

$\mu(x) = t$, $\mu(y) = t'$ and $\mu(xy) = S$

Then,

$S = \mu(xy)$
$\geq \max(\mu(x), \mu(y))$ [by definition of fuzzy ideal]
$= \mu(y)$
$= t'$

$S \geq t'$

Assume that,

$S > t'$ …………..(ii)

Consider,

$\mu(x) < \mu(y) < \mu(xy)$

That is,

$t < t' < S$

Then,

$X \in \mu_s$ (or) $Y \in \mu_s$

$XY \in \mu_s$, where μ_s is a prime ideal of R.

Hence,

$t = \mu(x) \geq S$

(or)

$t = \mu(y) \geq S$

Therefore, $t \geq S$. [by condition (i)]

Which is contradiction from (ii)

$\mu(xy) = \mu(y)$

Hence the proof.

Result: 3.1

If μ is any fuzzy prime ideal of a ring R. Then $\mu(xy) = \max(\mu(x),\mu(y))$

Result: 3.2

If μ and θ are any fuzzy prime ideal of a ring R. Then $\mu\cap\sigma\theta$ is a fuzzy prime ideal of R $\leftrightarrow \mu \subseteq \theta$ (or) $\theta \subseteq \mu$

Theorem: 3.2

Let μ be any fuzzy prime ideal of a ring R, such that $I \in I_m\mu$ and let θ be any fuzzy prime ideal of R. Then $\mu\cap\theta$ is a fuzzy prime ideal of the ring.

$\mu_t = \{x \in R \,/\, \mu(x)=1\}$

Proof:

Let μ be any fuzz ieal of a ring R,and θ be any fuzzy prime ideal of R.

Case :(i)

If θ is constant

Let $\theta(r) = c$ for all $r \in R$. [since for all $x\in\mu_t$)]

Then , we have

$(\mu\cap\theta)x = \min(\mu(x),\theta(x))$

$= \min(1,c)$

$= c$

Hence $\mu\cap\theta$ is a fuzzy prime ideal of μ_t.

Case :(ii)

If θ is non-constant

Then there exists $\alpha \in [0,1]$ such that

$$\theta(x) = \begin{cases} 1 & if\ x\epsilon\theta_t \\ \alpha & if\ x\epsilon R\sim\theta_t \end{cases}$$

Where $\theta_t = \{x\in R \,/\, \theta(x) = 1\}$

Also θ_t is a primeideal of R

$\theta_t\cap\mu_t$ is a prime ideal of μ_t.

$$(\mu\cap\theta) = \begin{cases} 1 & if\ x\epsilon\theta_t\cap\mu_t \\ \propto & if\ x\epsilon\mu_t-(\theta_t \cap \mu_t) \end{cases}$$

Hence $\mu\cap\theta$ is a fuzzy prime ideal of μ_t

Hence the proof.

Theorem: 3.3

If f is a homomorphism from a ring R onto a ring R' and μ is any f-invarient fuzzy prime ideal of R. Then f(μ) is fuzzy prime ideal of R'.

(or)

f(μ) is a fuzzy prime ideal of R'. Prove that $\sigma' \subseteq f(\mu)$ (or) $\theta' \subseteq f(\mu)$.

Proof:

Let σ' and θ' be any two fuzzy ideal of R' such that $\sigma'\theta' \subseteq f(\mu)$

Then,

$f^{-1}(\sigma'\theta') \subseteq f^{-1}(f(\mu)) = \mu$ [by result $f^{-1}(f(\mu))=\mu$]

$f^{-1}(\sigma')\, f^{-1}(\theta') \subseteq \mu$

Either,

$\sigma' = f(f^{-1}(\sigma)) \subseteq f(\mu)$

$\sigma' \subseteq f(\mu)$ (or)

$\theta' = f(f^{-1}(\theta)) \subseteq f(\mu)$

$\theta' \subseteq f(\mu)$

$\sigma'\theta' \subseteq f(\mu)$

Accordingly,

$f(\mu)$ is fuzzy prime ideal of R.

Hence the proof.

Theorem: 3.4

Let f be a homomorphism from a ring R onto a ring R' if μ' and θ' are any two fuzzy ideals of R' then the following holds,

$f^{-1}(\mu')\, f^{-1}(\theta') \subseteq f^{-1}(\mu'\theta')$

Proof:

Let x ϵ R and Ɛ > 0 be given the set

$\alpha = (f^{-1}(\mu')\ f^{-1}(\theta'))(x)$ and

$\beta = (f^{-1}(\mu'\theta'))(x)$

Then,

$\alpha\text{-}\varepsilon < \sup (\min((f^{-1}(\mu')), (f^{-1}(\theta'))(x_1)))$

$x = x_1\, x_2$

$= \sup(\min \mu'(f(x_1)), \theta'(f(x_1)))$

$x = x_1\, x_2$

$\alpha\text{-}\varepsilon < \min (\mu'(f(x_1)),\theta'(f(x_2)))$ for some $x_1x_2 \epsilon R$ such that $x=x_1x_2$

$\leq (\mu'\theta')(f(x_1x_2))$

$= f^{-1}(\mu'\theta')(x)$

$= \beta$

$\alpha\text{-}\varepsilon \leq \beta$

Since Ɛ is an arbitrary.

Hence,

$f^{-1}(\mu')f^{-1}(\theta') \subseteq f^{-1}(\mu'\theta')$

Hence proved.

Theorem: 3.5

If f is an homomorphism from a ring R onto a ring R' and μ' is any fuzzy prime ideal of R.

Proof:

Let μ and σ be any two fuzzy ideals of R such that,

$\mu\sigma \leq f^{-1}(\mu)$

Then,

$f(\mu\sigma) \subseteq f(f^{-1}(\mu')) = \mu'$

By using theorem,

" Let f be a homomorphism from a r in R onto a ring and R' if μ' and θ' are any two fuzzy ideals of R' then the following holds,

$f^{-1}(\mu')\, f^{-1}(\theta') \subseteq f^{-1}(\mu'\theta')$"

$f(\mu)\, f(\sigma) \subseteq \mu'$

Either $f(\mu) \subseteq \mu'$ (or)

$f(\sigma) \subseteq \mu'$

Since μ' is fuzzy prime,

Either,

$f^{-1}(f(\mu)) \subseteq f^{-1}(\mu')$

(or)

$f^{-1}(f(\sigma)) \subseteq f^{-1}(\mu')$

$\mu \subseteq f^{-1}(\mu')$

(or)

$\sigma \subseteq f^{-1}(\mu')$

Hence,

$f^{-1}(\mu)$ is fuzzy prime ideal of R'.

Hence the proof.

Definition: 3.3

A fuzzy ideal μ of R is called fuzzy prime if for all a,b∈R, either μ(ab) = μ(a) or μ(ab) = μ(b).

Definition: 3.4

Let μ and σ be fuzzy subsets of R. The cartesian product of μ and σ is defined by

μ×σ = (x,y) = min(μ(x),σ(y)) for all x,y ∈ R.

Theorem: 3.6

If μ and σ be fuzzy prime ideals of R then μ×σ is fuzzyprime ideal of R×R.

Proof:

We know that the cartesian product of any two fuzzy ideals is fuzzy ideal by (Malik and Mordeson,1991)

So it is enough to show that

∀ (a,b),(c,d) ∈ R×R

Either,

μ×σ((a,b)(c,d)) = μ×σ((a,b))

μ×σ((a,b)(c,d)) = μ×σ((c,d))

Since μ and σ is fuzzy prime ideals of R for all a,b,c,d ∈ R

Either μ(ac) = μ(a) or else μ(ac) = μ(c)

Either μ(bd) = μ(b) or else μ(bd) = μ(d)

Then,

μ×σ((a,c)(b,d)) = μ×σ(ac,bd)

= min(μ(ac),σ(bd))

= μ(ac) = μ(a) = μ×σ(a,b)

or

= μ(ac) = μ(c) = μ×σ(c,d)

Similarly,

μ×σ((a,c)(c,d)) = μ×σ(ac,bd)

= min(μ(ac),σ(bd))

= σ(bd) = σ(b) = μ×σ(a,c)

or

= σ(bd) = σ(d) = μ×σ(c,d)

Therefore if μ and σ are fuzzy prime ideals of R then μ×σ is a fuzzy prime ideal of R×R.

Hence the proof.

Theorem: 3.7

If μ and σ be fuzzy semi prime ideals of R then μ×σ is a fuzzy semi prime ideal of R×R.

Proof:

We know that the cartesian product of any two fuzzy ideals is fuzzy ideal by(Malik and Mordeson,1991).

So it is enough to show that for all (a,b) ∈ R×R.

Either,

$\mu\times\sigma((a,b)^n) = \mu\times\sigma(a,b)$

Since μ and σ is fuzzy semiprime ideals of R for all a,b ∈ R, ∀ n $\mathcal{E}^+$ ·

$\mu(a)^n = \mu(a)$ and $\sigma(b)^n = \mu(b)$

Then,

$\mu\times\sigma((a,b)^n) = \mu\times\sigma(a^n,b^n)$

$= \min(\mu(a^n), \sigma(b^n))$

$= \mu(a^n)$

$= \mu(a)$

$= \mu\times\sigma(a,b)$

.

$\mu\times\sigma((a,b)^n) = \mu\times\sigma(a^n,b^n)$

$= \min(\mu(a^n), \sigma(b^n))$

$= \sigma(b^n)$

$= \sigma(b)$

$= \mu\times\sigma(a,b)$

Therefore μ×σ is a fuzzy semi prime ideal of R×R.

Hence the proof.

Theorem: 3.8

If μ and σ be fuzzy semi primary ideals of R then μ×σ is a fuzzy semi primary ideal of R×R.

Proof:

Since, μ and σ be fuzzy semi primary ideals of R then for all a,b,c,d ϵ R.
Either $\mu(ab) \leq \mu(a^n)$ or else $\mu(ab) \leq \mu(b^m)$ for some m,n $\mathcal{E}^+$.
Either $\sigma(cd) \leq \sigma(d^k)$ or else $\sigma(cd) \leq \sigma(d^l)$ for some k/$\mathcal{E}^+$.
Let t = max(m,n) and
s = max(k,l)
Then,

$$\mu\times\sigma((a,c)(b,d)) = \mu\times\sigma(ab,cd)$$
$$= \min(\mu(ab),\sigma(cd))$$
$$= \mu(ab)$$
$$\leq \mu(a^t)$$
$$= \mu\times\sigma((a,c)^t)$$

Or else,

$$\mu\times\sigma((a,c)(b,d)) = \mu\times\sigma(ab,cd)$$
$$= \min(\mu(ab),\sigma(cd))$$
$$= \sigma(cd)$$
$$\leq \sigma(d^s)$$
$$= \mu\times\sigma((b,d)^s)$$

Therefore $\mu\times\sigma$ is fuzzy semi primary ideal of R×R.
Hence the proof.

Corollary:
A fuzzy ideal $\mu\times\sigma$ of R×R is fuzzy semi primary ideal if and only if the level ideals t ϵ $I_m\mu\times\sigma$, $(\mu\times\sigma)_t$ are semi primary ideals of R×R.s

IV. CONCLUSIONS

In this project work briefly discussed about the concept of Fuzzy Algebra and Near Ring. Using the idea of the new sort of fuzzy subnear-ring of a near-ring, fuzzy subgroups, and their generalizations defined by various researchers, we try to introduce the notion of ($\in, \in \vee q$)-fuzzy ideals of N-groups. These fuzzy ideals are characterized by their level ideals, and some other related properties are investigated.

REFERENCES

[1] Fuzzy algebra by A.R.Meenakshi.
[2] Fuzzy matrix: Theory and Applications by A.R.Meenakshi.
[3] Fuzzy algebra, Rajesh Kumar, University of Delhi Press.
[4] Fuzzy Algebra, Dr.W.B.Vasantha Kandasamy, Indian Institute of Technology of Chennai.
[5] Smarandache Fuzzy Algebra, W.B.Vasantha Kandasamy, American Research Press, Rehoboth,2003.