Bayesian Analysisof Stress-Strength Reliability For Inverse Exponential Distribution Under Various Loss Functions

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ABSTRACT
In this paper, the estimation of stress – strength reliability(R)is considered, when strength and stress variables are assumed to be independently distributed inverse exponential random variables. The maximum likelihood and Bayes estimators of R are obtained. Bayesian estimation of R is studied under non-informative and Gamma priors with different loss functions using Lindley's approximation technique. The simulation study is performed, to compare estimator by evaluating mean squared errors.The real data analysis is conducted

Keywords: *Stress-strength model, Inverse exponential distribution, Reliability, maximum likelihood estimator, Bayes estimator, Lindley's approximation.*

I. INTRODUCTION

The stress-strength models are widely used models to study the reliability of systems. The stress-strength reliability is the probability that the strength of the system is greater than the stress. Many researchers have studied the problem of estimation of stress-strength reliability for different distributions. Ghitany et al.[3] studied the reliability of stress- strength system for power Lindley distribution. The reliability of multi-component stress-strength system under Pareto distribution is considered by Ashok et. al.[1]. The reliability of the system under stress-strength set up when stress and strength have exponentiated Pareto distribution was studied by Chen and Cheng[2] and Singh et al [13] considered the estimation of stress-strength reliability for inverse exponential distribution.

In this paper, the Bayes estimators of stress- strength reliability are derivedusing Lindley's approximation technique, assuming both strength, X and stress, Y variables follow independent inverse exponential distribution. Recently, many researchers have given their attention on obtaining the Bayes estimators using Lindley's [9] approximation technique for different life-time distributions. Kizilaslan and Nadar [6] studied the Bayes estimation of multicomponent stress-strength reliability for Weibull distribution. Estimation of parameters for constant shape Bi-Weibull and Lindley distribution are studied by Lavanya and Alexander[7] and Metiri et al [10], respectively. The estimators of stress-strength reliability under multi-component setup for inverse Chen and bivariate-generalized Rayleigh distribution, respectively arestudied by Joshi and Pandit[4] and Pandit and Joshi [12].

The inverse exponential distribution was introduced by Keller and Kamath[5] which possesses non-monotone failure rate and a widely used lifetime distribution.

The probability density function (pdf) is given by

$$f(x)=\frac{\alpha}{x^2}e^{-\frac{\alpha}{x}},x,\alpha>0\,. \qquad (1)$$

The rest of the paper is organised as below. Section 2 deals with the derivation of stress strength reliability for IE distribution and maximum likelihood estimation of R. The Bayesian analysis of R is presented in Section 3. The real data analysis is given in section 4. Section 5 contains simulation study. Summary and conclusions are presented in Section 6.

II. Maximum Likelihood Estimation (MLE)

Let X and Y be two independent random variables having IE$(\alpha_1)$and IE(α_2)distributions respectively.Then the stress-strength reliability R (Singh et al [13]) is given by

$$R=P(X>Y)$$

$$R=\frac{\alpha_1}{\alpha_1+\alpha_2} \qquad (2)$$

Let $\underline{X} = (X_1, X_2, ..., X_n)$ and $\underline{Y} = (Y_1, Y_2, ..., Y_m)$ be two independent random samples drawn from IE distribution. Then the log likelihood function of (α_1, α_2) given $(\underline{x}, \underline{y})$ is given by

$$\ell = n\log\alpha_1 - \sum_{i=1}^{n}\log x_i^2 - \sum_{i=1}^{n}\frac{\alpha_1}{x_i} + m\log\alpha_2 - \sum_{j=1}^{m}\log y_j^2 - \sum_{j=1}^{m}\frac{\alpha_2}{y_j}. \qquad (3)$$

The likelihood equations to get MLEs of parameters α_1 and α_2 are

$$\frac{n}{\alpha_1} - \sum_{i=1}^{n}\left(\frac{1}{x_i}\right) = 0 \qquad (4)$$

and $$\frac{n}{\alpha_2} - \sum_{j=1}^{m}\left(\frac{1}{y_j}\right) = 0 \qquad (5)$$

Solving (4) and (5), we get the MLE of α_1 and α_2 as

$$\hat{\alpha}_1 = \frac{n}{\sum_{i=1}^{n}\left(\frac{1}{x_i}\right)} \text{ and } \hat{\alpha}_2 = \frac{m}{\sum_{j=1}^{m}\left(\frac{1}{y_j}\right)}.$$

By using invariance property, the MLE of R is

$$\hat{R} = \frac{\hat{\alpha}_1}{\hat{\alpha}_1 + \hat{\alpha}_2}. \qquad (6)$$

III. Bayesian estimation of R

In this section, the Bayesian analysis of stress-strength reliability R is studied using Lindley's approximation under different loss functions such as squared error loss function (SELF), Linear exponential (LINEX) loss function and general entropy loss function (GELF). The prior distributions of the parameters α_i's are assumed to be gamma(c_i, d_i), i=1, 2.

The pdf of α_1 is

$$\pi(\alpha_1) = \frac{d_1^{c_1}}{\Gamma c_1} e^{-d_1\alpha_1} \alpha_1^{c_1 - 1} \qquad (7)$$

The pdf of α_2 is

$$\pi(\alpha_2) = \frac{d_2^{c_2}}{\Gamma c_2} e^{-d_2\alpha_2} \alpha_2^{c_2 - 1} \qquad (8)$$

Assuming α_1 and α_2 are independent, the joint posterior distribution of (α_1, α_2) is

$$\pi(\alpha_1, \alpha_2 \mid \underline{x}, \underline{y}) = \frac{(d_1 + s_1)^{n+c_1}(d_2 + s_2)^{m+c_2}}{\Gamma(n + c_1)\Gamma(m + c_2)} \alpha_1^{n+c_1-1}\alpha_2^{m+c_2-1} \exp(-\alpha_1(d_1 + s_1) - \alpha_2(d_2 + s_2)),$$ where

$$s_1 = \sum_{i=1}^{n}\frac{1}{x_i} \text{ and } s_2 = \sum_{j=1}^{m}\frac{1}{y_j}. \qquad (9)$$

The posterior distribution of R is given by

$$\pi(r \mid \underline{x}, \underline{y}) = \frac{(d_1 + s_1)^{n+c_1}(d_2 + s_2)^{m+c_2}}{Beta(n + c_1, m + c_2)} \frac{r^{n+c_1}(1 - r)^{m+c_2-1}}{[r(d_1 + s_1) + (1 - r)(d_2 + s_2)]^{n+m+c_1+c_2}}, 0 \le r \le 1 \qquad (10)$$

If c_1=d_1=c_2=d_2=0, then equation (10) gives the posterior distribution of R with non-informative prior. For more details on posterior distribution see Singh et al [13].

The different loss functions and their Bayes estimators aregiven below.

The SELF is $L(\hat{R}, R) = (R - \hat{R})^2$, where $\hat{R}$ is Bayes estimate of R.

The Bayes estimator of R under SELF is

$$\hat{R}_S = E(R \mid \underline{x}, \underline{y}), \text{ where } E(R) = \int_0^1 r\, \pi(r \mid \underline{x}, \underline{y}) dr.$$

The LINEX loss function is $L(\hat{R}, R) = e^{a(\hat{R}-R)} - a(\hat{R}-R) - 1, a \neq 0$.
The Bayes estimator of R under LINEX loss function is

$$\hat{R}_L = -\frac{1}{a}\ln\left(E\left(e^{-aR}\right)\right), \text{ where } E\left(e^{-aR}\right) = \int_0^1 e^{-ar}\, \pi(r \mid \underline{x}, \underline{y}) dr$$

The GELF is $L(\hat{R}, R) \propto \left(\frac{\hat{R}}{R}\right)^k - k\ln\left(\frac{\hat{R}}{R}\right) - 1$.

The Bayes estimator of R under GELF is

$\hat{R}_{GELF} = \left(E\left(R^{-k}\right)\right)^{-\frac{1}{k}}$, provided $E\left(R^{-k}\right)$ exists and finite. Here, $E\left(R^{-k}\right) = \int_0^1 r^{-k}\, \pi(r \mid \underline{x}, \underline{y}) dr$.

The closed form expressions for above integrals are not tractable. Hence, theintegrals are approximated using the Lindley's approximation technique.

A. The Lindley's approximation technique

The Lindley's [9] approximation technique is one of the methods of solving ratio of two integrals. Theprocedure of this technique is as described below.
Consider the ratio of integrals of the form

$$E(w(\theta) \mid \underline{x}) = \frac{\int w(\theta) e^{Q(\theta)} d\theta}{\int e^{Q(\theta)} d\theta}, \qquad (11)$$

Where $\theta = (\theta_1, \theta_2, \ldots, \theta_m)$ is the vector of parameters, $Q(\theta) = l(\theta) + \rho(\theta)$, $l(\theta)$ is the log-likelihood function of underlined distribution, $\rho(\theta)$ is the logarithm of prior density of θ and $w(\theta)$ being some function of parameters. Then the Lindley's approximation to posterior expectation given in (10) is

$$E(w(\theta) \mid \underline{x}) = \left[w + \frac{1}{2}\sum_i \sum_j \left(w_{ij} + 2w_i \rho_j\right)\sigma_{ij} + \frac{1}{2}\sum_i \sum_j \sum_k \sum_l L_{ijk}\sigma_{ij}\sigma_{kl} w_l \right]_{\hat{\theta}} + \text{terms of order } n^{-2}.$$

Here, $\hat{\theta}$ is the MLE of θ, w =w(θ), $w_i = \frac{\partial w}{\partial \theta_i}$, $w_{ij} = \frac{\partial^2 w}{\partial \theta_i \partial \theta_j}$, $L_{ijk} = \frac{\partial^3 l}{\partial \theta_i \partial \theta_j \partial \theta_k}$, $\rho_j = \frac{\partial \rho}{\partial \theta_j}$ and σ_{ij} is the (i, j)th element in the inverse of the observed information matrix.
For two parameter case $\theta = (\theta_1, \theta_2)$, the approximation to posterior expectation isgiven by

$$E(w(\theta) \mid x) = w + \left[w_1(\rho_1\sigma_{11} + \rho_2\sigma_{12}) + w_2(\rho_1\sigma_{21} + \rho_2\sigma_{22}) + w_{12}\sigma_{12} + \frac{1}{2}(w_{11}\sigma_{11} + w_{22}\sigma_{22}) \right] +$$

$$\frac{1}{2}\left[A(w_1\sigma_{11} + w_2\sigma_{12}) + B(w_1\sigma_{21} + w_2\sigma_{22})\right], \qquad (12)$$

where, $A = \sigma_{11}L_{111} + 2\sigma_{12}L_{121} + \sigma_{22}L_{221}$ and $B = \sigma_{11}L_{112} + 2\sigma_{12}L_{122} + \sigma_{22}L_{222}$.

B. Lindley's approximation for different loss functions

In this section, the Bayes estimators of (α_1, α_2) for different loss functions are considered.

a) Squared error loss function

The Bayes estimator under SELF is given by

$$\hat{R}_S = u + u_1\rho_1\sigma_{11} + u_2\rho_2\sigma_{22} + \frac{1}{2}(u_{11}\sigma_{11} + u_{22}\sigma_{22}) + \frac{1}{2}\left(u_1\sigma_{11}^2 L_{111} + u_2\sigma_{22}^2 L_{222}\right),$$

where $u=\frac{\hat{\alpha}_1}{\hat{\alpha}_1+\hat{\alpha}_2}$, $u_1=\frac{\hat{\alpha}_2}{(\hat{\alpha}_1+\hat{\alpha}_2)^2}$, $u_2=\frac{-\hat{\alpha}_1}{(\hat{\alpha}_1+\hat{\alpha}_2)^2}$, $u_{11}=\frac{-2\hat{\alpha}_2}{(\hat{\alpha}_1+\hat{\alpha}_2)^3}$,
$u_{12}=\frac{\hat{\alpha}_1-\hat{\alpha}_2}{(\hat{\alpha}_1+\hat{\alpha}_2)^3}$, $u_{22}=\frac{2\hat{\alpha}_1}{(\hat{\alpha}_1+\hat{\alpha}_2)^3}$ and $u_{21}=u_{12}$

$L_{11}=-\frac{n}{\hat{\alpha}_1^2}$, $L_{22}=-\frac{m}{\hat{\alpha}_2^2}$, $L_{12}=L_{21}=0$, $L_{111}=\frac{2n}{\hat{\alpha}_1^3}$, $L_{222}=\frac{2m}{\hat{\alpha}_2^3}$, $L_{112}=L_{211}=L_{121}=0$ and
$L_{212}=L_{221}=L_{122}==0$.

Here, $\rho_1=\left(\frac{c_1-1}{\alpha_1}\right)-d_1$, $\rho_2=\left(\frac{c_2-1}{\alpha_2}\right)-d_2$, $\sigma_{11}=\frac{1}{L_{11}}$ and $\sigma_{22}=\frac{1}{L_{22}}$.

b) LINEX loss function

The Bayes estimator under LINEX loss function is given by

$$\hat{R}_L=-\frac{1}{a}Log\left(v+v_1\rho_1\sigma_{11}+v_2\rho_2\sigma_{22}+\frac{1}{2}(v_{11}\sigma_{11}+v_{22}\sigma_{22})+\frac{1}{2}\left(v_1\sigma_{11}^2L_{111}+v_2\sigma_{22}^2L_{222}\right)\right),$$

where $v=\exp\left(-a\frac{\hat{\alpha}_1}{\hat{\alpha}_1+\hat{\alpha}_2}\right)$, $v_1=-a\left(\frac{\hat{\alpha}_2}{(\hat{\alpha}_1+\hat{\alpha}_2)^2}\right)\exp\left(-a\frac{\hat{\alpha}_1}{\hat{\alpha}_1+\hat{\alpha}_2}\right)$, $v_2=a\left(\frac{\hat{\alpha}_1}{(\hat{\alpha}_1+\hat{\alpha}_2)^2}\right)\exp\left(-a\frac{\hat{\alpha}_1}{\hat{\alpha}_1+\hat{\alpha}_2}\right)$

$v_{11}=a\hat{\alpha}_2\left(\frac{2\hat{\alpha}_1+\hat{\alpha}_2(2+a)}{(\hat{\alpha}_1+\hat{\alpha}_2)^4}\right)\exp\left(-a\frac{\hat{\alpha}_1}{\hat{\alpha}_1+\hat{\alpha}_2}\right)$, $v_{22}=a\hat{\alpha}_1\left(\frac{\hat{\alpha}_1(a-2)+2\hat{\alpha}_2}{(\hat{\alpha}_1+\hat{\alpha}_2)^4}\right)\exp\left(-a\frac{\hat{\alpha}_1}{\hat{\alpha}_1+\hat{\alpha}_2}\right)$,

$v_{12}=a\left(\frac{\hat{\alpha}_2{}^2-\hat{\alpha}_1{}^2+a\hat{\alpha}_1\hat{\alpha}_2)}{(\hat{\alpha}_1+\hat{\alpha}_2)^4}\right)\exp\left(-a\frac{\hat{\alpha}_1}{\hat{\alpha}_1+\hat{\alpha}_2}\right)$ and $v_{21}=v_{12}$.

c) General entropy loss function

Similarly, the Bayes estimator under GELF is given by

$$\hat{w}_G=\left(w+w_1\rho_1\sigma_{11}+w_2\rho_2\sigma_{22}+\frac{1}{2}(w_{11}\sigma_{11}+w_{22}\sigma_{22})+\frac{1}{2}\left(w_1\sigma_{11}^2L_{111}+w_2\sigma_{22}^2L_{222}\right)\right)^{-\frac{1}{k}},$$

where $w=\left(\frac{\hat{\alpha}_1}{\hat{\alpha}_1+\hat{\alpha}_2}\right)^{-k}$, $w_1=-k\alpha_2\frac{(\hat{\alpha}_1)^{-(k+1)}}{(\hat{\alpha}_1+\hat{\alpha}_2)^{1-k}}$, $w_2=k\hat{\alpha}_1\frac{(\hat{\alpha}_1)^{-(k)}}{(\hat{\alpha}_1+\hat{\alpha}_2)^{1-k}}$,

$w_{11}=k\hat{\alpha}_2(\hat{\alpha}_1)^{-(k+2)}\left(\frac{\hat{\alpha}_2(k+1)+2\hat{\alpha}_1}{(\hat{\alpha}_1+\hat{\alpha}_2)^{2-k}}\right)$, $w_{12}=-k(\hat{\alpha}_1)^{-(k+1)}\left(\frac{\hat{\alpha}_2k+\hat{\alpha}_1}{(\hat{\alpha}_1+\hat{\alpha}_2)^{2-k}}\right)$,

$w_{22}=k(k-1)\left(\frac{(\hat{\alpha}_1)^{-(k)}}{(\hat{\alpha}_1+\hat{\alpha}_2)^{2-k}}\right)$ and $w_{21}=w_{12}$.

IV. Real Data analysis

In this section, the real data given in Lawless [8] is analysed, which was recently, used by Mokhlis et al [11]. The two data sets represent failure times, in minutes, for two types of electrical insulation in an experiment. Here, the insulation is subjected to a continuously increasing voltage stress. The recorded values of twelve electrical insulation of each type are given below.

The failure times of first type (X): 21.8, 70.7, 24.4, 138.6, 151.9, 75.3, 12.3, 95.5, 98.1, 43.2, 28.6 and 46.9.

The failure times of second type (Y): 219.3, 79.4, 86.0, 150.2, 21.7, 18.5, 121.9, 40.5, 147.1, 35.1, 42.3 and 48.7.

The Kolmogorov- Smirnov(KS) and Anderson- Darling (AD) test are conducted to check the goodness of fit for the underlined distribution. The p-values for data set 1 and data set 2 using KS test are 0.2553 and 0.3452, respectively.The p-values for data set 1 and data set 2 using AD test are 0.7076 and 0.7083, respectively.The estimates of parameters using different methods of estimation like method of maximum likelihood and Bayesian method with non-informative and gamma priors, are presented in table 1.

Table 1: Maximum likelihood estimate and Bayes estimates of the parameters of inverse exponential distribution α_1 and α_2 for data set 1 and data set 2

Name of the distribution	MLE	Bayes estimates					
		Gamma prior			Non-informative prior		
		$\hat{\alpha}_S$	$\hat{\alpha}_L$	$\hat{\alpha}_G$	$\hat{\alpha}_S$	$\hat{\alpha}_L$	$\hat{\alpha}_G$
IED (data set 1)	38.7732	19.8389	12.5056	15.0738	38.7732	23.0754	36.1960
IED (data set 2)	48.3998	23.2389	28.7079	17.1165	48.3998	26.4996	45.1828
Estimate of R	0.5501	0.5638	0.5663	0.5601	0.5537	0.5512	0.5500

V. Simulation Study

A simulation study is conducted for 10000 observations to compare the performances of different estimators of reliability considered here. For simulation study different combinations of sample sizes (n, m) = (10,10), (10,15), (20, 20), (20,25), (30,30),(30,35),(40,40), (40,45) and (50,50) are considered with the parameter values (α_1, α_2) = (2,3), (3,2) and (0.4,0.1).The stress-strength reliability R is computed by substituting these values of the parameters in (2).The different values of loss parameters (a, k) are (0.5, 0.6) and (-0.5, -0.6). The parameters values of the prior distributions are (c_1,d_1) = (1, 1.4), (3, 0.55) and (c_2,d_2) = (0.5, 1) , (2, 0.45).The values of mean squared error (MSE) are computed for both estimators. The performance of the estimators are compared based on the values of MSEs.

Table 2: The estimates and corresponding MSE's of MLE and Bayes estimator with non-informative priors and loss functions

R=0.4, $\alpha_1 = 2, \alpha_2 = 3, c_1 = 1, d_1 = 1.4, c_2 = 0.5$ and $d_2 = 1$, $a = \pm 0.5$ and $k = \pm 0.6$						
Sample Size	MLE	Bayes Estimates with Non-informative Prior				
		$\hat{R}_S$	$\hat{R}_L$	$\hat{R}_{L1}$	$\hat{R}_G$	$\hat{R}_{G1}$
n=10,m=10	0.405607 0.011767	0.409544 0.010921	0.406892 0.010796	0.412181 0.011066	0.388646 0.011373	0.404170 0.010950
n=10,m=15	0.408622 0.009611	0.408034 0.008913	0.405778 0.008829	0.410293 0.009009	0.390609 0.009243	0.403525 0.008939
n=20,m=20	0.403215 0.005849	0.405375 0.005632	0.403991 0.005594	0.406756 0.005675	0.394413 0.005734	0.402596 0.005634
n=20,m=25	0.404103 0.005234	0.404875 0.005035	0.403622 0.005006	0.406127 0.005067	0.395003 0.005127	0.402366 0.005039
n=30,m=30	0.402372 0.004007	0.403862 0.003904	0.402927 0.003886	0.404795 0.003923	0.396436 0.003953	0.401988 0.003906
n=30,m=35	0.402852 0.003558	0.403672 0.003467	0.402800 0.003452	0.404542 0.003484	0.396770 0.003506	0.401928 0.003467
n=40,m=40	0.401694 0.002991	0.402834 0.002932	0.402128 0.002922	0.403538 0.002942	0.397215 0.002961	0.401419 0.002933
n=40,m=45	0.401951 0.002641	0.402698 0.002589	0.402030 0.002581	0.403366 0.002599	0.397388 0.002614	0.401361 0.002591
n=50,m=50	0.401678 0.002288	0.402597 0.002253	0.402029 0.002247	0.403164 0.002261	0.398079 0.002268	0.401461 0.002253

Table 3: The estimates and corresponding MSE's of MLE and Bayes estimator with Gammapriors and loss functions

R=0.4, $\alpha_1 = 2, \alpha_2 = 3, c_1 = 1, d_1 = 1.4, c_2 = 0.5$ and $d_2 = 1$, $a = \pm 0.5$ and $k = \pm 0.6$						
Sample Size	MLE	Bayes Estimates with Gamma Prior				
		$\hat{R}_S$	$\hat{R}_L$	$\hat{R}_{L1}$	$\hat{R}_G$	$\hat{R}_{G1}$
n=10,m=10	0.405607 0.011767	0.422748 0.005712	0.420508 0.005536	0.424965 0.005905	0.404244 0.005313	0.418315 0.005490
n=10,m=15	0.408622 0.009611	0.398695 0.004547	0.396725 0.004554	0.400684 0.004554	0.383064 0.005364	0.394668 0.004672
n=20,m=20	0.403215 0.005849	0.412834 0.004263	0.411507 0.004203	0.414152 0.004326	0.402028 0.004171	0.410178 0.004205

n=20,m=25	0.404103 0.005234	0.405916 0.003757	0.404695 0.003729	0.407135 0.003790	0.396176 0.003849	0.403468 0.003754
n=30,m=30	0.402372 0.004007	0.409007 0.003252	0.408093 0.003224	0.409916 0.003282	0.401598 0.003218	0.407174 0.003229
n=30,m=35	0.402852 0.003558	0.405803 0.002875	0.404943 0.002857	0.406661 0.002895	0.398917 0.002888	0.404081 0.002866
n=40,m=40	0.401694 0.002991	0.406796 0.002561	0.406101 0.002545	0.407488 0.002578	0.401174 0.002544	0.405401 0.002549
n=40,m=45	0.401951 0.002641	0.404928 0.002259	0.404265 0.002248	0.405589 0.002272	0.399612 0.002261	0.403600 0.002253
n=50,m=50	0.401678 0.002288	0.405775 0.002031	0.405213 0.002020	0.406335 0.002042	0.401249 0.002015	0.404649 0.002022

Table 4: The estimates and corresponding MSE's of MLE and Bayes estimator with non-informative priors and loss functions

R = 0.6, $\alpha_1 = 3, \alpha_2 = 2, c_1 = 1, d_1 = 1.4, c_2 = 0.5$ and $d_2 = 1$, $a = \pm 0.5$ and $k = \pm 0.6$						
Sample Size	MLE	Bayes Estimates with Non-informative Prior				
		$\hat{R}_S$	$\hat{R}_L$	$\hat{R}_{L1}$	$\hat{R}_G$	$\hat{R}_{G1}$
n=10,m=10	0.595898 0.011787	0.591891 0.010926	0.589256 0.011057	0.594536 0.010810	0.577276 0.012387	0.588134 0.011257
n=10,m=15	0.600791 0.009351	0.593364 0.008841	0.591156 0.008937	0.595588 0.008754	0.581403 0.009837	0.590277 0.009069
n=20,m=20	0.598324 0.005964	0.596119 0.005728	0.594743 0.005763	0.597497 0.005698	0.588638 0.006116	0.594222 0.005816
n=20,m=25	0.599791 0.005164	0.596596 0.004993	0.595354 0.005024	0.597841 0.004967	0.589893 0.005305	0.594894 0.005064
n=30,m=30	0.599054 0.003986	0.597545 0.003880	0.596614 0.003897	0.598477 0.003864	0.592512 0.004058	0.596275 0.003920
n=30,m=35	0.599904 0.003540	0.597920 0.003485	0.597055 0.003469	0.598788 0.003443	0.593262 0.003603	0.596744 0.003488
n=40,m=40	0.599227 0.002946	0.598079 0.002887	0.597375 0.002897	0.598784 0.002878	0.594286 0.002989	0.597124 0.002910
n=40,m=45	0.599766 0.002614	0.598342 0.002566	0.597676 0.002574	0.599009 0.002559	0.594767 0.002652	0.597442 0.002585
n=50,m=50	0.599777 0.002881	0.598846 0.002243	0.598281 0.002249	0.599413 0.002238	0.595808 0.002304	0.598082 0.002257

L1:LINEX loss function with loss parameter a=-0.5 and G1:General entropy loss function with loss parameter k=-0.6.

Table 5: The estimates and corresponding MSE's of MLE and Bayes estimator with Gamma priors and loss functions

R = 0.6, $\alpha_1 = 3, \alpha_2 = 2, c_1 = 1, d_1 = 1.4, c_2 = 0.5$ and $d_2 = 1$, $a = \pm 0.5$ and $k = \pm 0.6$						
Sample Size	MLE	Bayes Estimates with Gamma Prior				
		$\hat{R}_S$	$\hat{R}_L$	$\hat{R}_{L1}$	$\hat{R}_G$	$\hat{R}_{G1}$
n=10,m=10	0.595898 0.011787	0.549876 0.007892	0.548168 0.008219	0.551641 0.007572	0.540223 0.010125	0.54728 0.008437
n=10,m=15	0.600791 0.009351	0.536064 0.008829	0.535191 0.009090	0.536988 0.008563	0.531273 0.001040	0.534687 0.009246
n=20,m=20	0.598324 0.005964	0.575197 0.004784	0.574011 0.004880	0.576409 0.004692	0.568899 0.005433	0.573543 0.004946
n=20,m=25	0.599791 0.005164	0.571564 0.004516	0.570568 0.004609	0.572588 0.004425	0.566362 0.005109	0.570181 0.004665
n=30,m=30	0.599054 0.003986	0.583634 0.003423	0.582781 0.003468	0.584501 0.003380	0.579118 0.003736	0.582466 0.003499
n=30,m=35	0.599904 0.003540	0.582087 0.003149	0.581317 0.003191	0.582870 0.003107	0.578049 0.003433	0.581036 0.003218
n=40,m=40	0.599227 0.002946	0.587700 0.002634	0.587038 0.002659	0.588370 0.002609	0.584193 0.002813	0.586800 0.002677
n=40,m=45	0.599766 0.002614	0.586585 0.002387	0.586242 0.002412	0.587482 0.002364	0.583614 0.004551	0.586023 0.002427
n=50,m=50	0.599777 0.002881	0.590502 0.002076	0.589962 0.002092	0.591046 0.002061	0.587648 0.002188	0.589773 0.002103

Table 6: The estimates and corresponding MSE's of MLE and Bayes estimator with non-informative priors and loss functions

R = 0.8, $\alpha_1 = 0.4, \alpha_2 = 0.1, c_1 = 1, d_1 = 1.4, c_2 = 0.5$ and $d_2 = 1$, $a = \pm 0.5$ and $k = \pm 0.6$						
Sample Size	MLE	Bayes Estimates with Non-informative Prior				
		$\hat{R}_S$	$\hat{R}_L$	$\hat{R}_{L1}$	$\hat{R}_G$	$\hat{R}_{G1}$
n=10,m=10	0.790323 0.006033	0.781662 0.006213	0.780326 0.006364	0.783008 0.003064	0.776042 0.006984	0.780225 0.006402
n=10,m=15	0.795167 0.004508	0.795145 0.004773	0.784073 0.004872	0.786227 0.004675	0.780724 0.005246	0.784014 0.004892
n=20,m=20	0.795421 0.002936	0.790858 0.002989	0.790204 0.003027	0.791515 0.002951	0.788117 0.003178	0.790179 0.003035
n=20,m=25	0.796440 0.002402	0.792005 0.002471	0.791423 0.002501	0.792591 0.002443	0.789633 0.002612	0.791405 0.002506
n=30,m=30	0.797115 0.001851	0.794019 0.001879	0.793586 0.001895	0.794454 0.001863	0.792257 0.001958	0.793575 0.001898
n=30,m=35	0.797937 0.001637	0.794672 0.001667	0.794273 0.001681	0.795073 0.001654	0.793053 0.001732	0.794265 0.001683
n=40,m=40	0.797837 0.001334	0.795494 0.001351	0.795170 0.001361	0.795818 0.001342	0.794183 0.001394	0.795165 0.001362
n=40,m=45	0.798381 0.001186	0.79594 0.00120	0.795635 0.001211	0.796245 0.001195	0.794771 0.001240	0.795631 0.001212
n=50,m=50	0.798574 0.001035	0.796689 0.001045	0.796432 0.001051	0.796948 0.001039	0.795648 0.001071	0.796428 0.001051

Table 7: The estimates and corresponding MSE's of MLE and Bayes estimator with Gamma priors and loss functions

R = 0.8, $\alpha_1 = 0.4, \alpha_2 = 0.1, c_1 = 1, d_1 = 1.4, c_2 = 0.5$, $d_2 = 1$, $a = \pm 0.5$ and $k = \pm 0.6$						
Sample Size	MLE	Bayes Estimates with Gamma Prior				
		$\hat{R}_S$	$\hat{R}_L$	$\hat{R}_{L1}$	$\hat{R}_G$	$\hat{R}_{G1}$
n=10,m=10	0.790323 0.006033	0.782128 0.005791	0.780794 0.005939	0.783473 0.005647	0.776471 0.006555	0.780691 0.005977
n=10,m=15	0.795167 0.004508	0.787533 0.004311	0.786451 0.004404	0.788623 0.004221	0.783035 0.004775	0.786791 0.004424
n=20,m=20	0.795421 0.002936	0.791113 0.002882	0.790457 0.002919	0.791768 0.002845	0.788419 0.003069	0.790432 0.002927
n=20,m=25	0.796440 0.002402	0.792867 0.002359	0.792284 0.002387	0.793454 0.002332	0.790481 0.002495	0.792266 0.002393
n=30,m=30	0.797115 0.001851	0.794194 0.001833	0.793761 0.001849	0.794628 0.001816	0.79243 0.001912	0.793750 0.001852
n=30,m=35	0.797937 0.001637	0.795139 0.001618	0.794739 0.001631	0.795540 0.001604	0.793515 0.001682	0.794731 0.001633
n=40,m=40	0.797837 0.001334	0.795630 0.001326	0.795306 0.001335	0.795955 0.001317	0.794318 0.001369	0.795301 0.001336
n=40,m=45	0.798381 0.001186	0.796247 0.001177	0.795942 0.001185	0.796552 0.001170	0.795015 0.001213	0.795938 0.001186
n=50,m=50	0.798574 0.001035	0.796796 0.001028	0.796538 0.001034	0.797054 0.001023	0.795754 0.001055	0.796534 0.001035

Table 8: The estimates and corresponding MSE's of Bayes estimator with Gamma prior under different loss functions

R = 0.4, $\alpha_1 = 2, \alpha_2 = 3, c_1 = 3, d_1 = 0.55, c_2 = 2$, $d_2 = 0.45$, $a = \pm 0.5$ and $k = \pm 0.6$					
Sample Size	Bayes Estimates with Gamma Prior				
	$\hat{R}_S$	$\hat{R}_L$	$\hat{R}_{L1}$	$\hat{R}_G$	$\hat{R}_{G1}$
n=10,m=10	0.404342 0.007998	0.401789 0.007922	0.408906 0.008095	0.384206 0.008738	0.399174 0.008069
n=10,m=15	0.418549 0.007195	0.409143 0.007105	0.413564 0.007301	0.393861 0.007466	0.406920 0.007182
n=20,m=20	0.402588 0.004873	0.401217 0.004846	0.403963 0.004905	0.391776 0.005052	0.399836 0.004891
n=20,m=25	0.404949	0.403704	0.406195	0.395096	0.402425

	0.004484	0.004456	0.004516	0.004587	0.004487
n=30,m=30	0.401961 0.003550	0.401031 0.003537	0.402892 0.003566	0.394596 0.003633	0.400095 0.003559
n=30,m=35	0.403177 0.003197	0.402308 0.003184	0.404046 0.003213	0.396245 0.003247	0.401438 0.003199
n=40,m=40	0.401401 0.002731	0.400698 0.002724	0.402106 0.002739	0.395817 0.002779	0.399991 0.002736
n=40,m=45	0.402107 0.002433	0.401439 0.002425	0.402774 0.002441	0.396810 0.002465	0.400771 0.002435
n=50,m=50	0.401434 0.002131	0.400867 0.002126	0.402006 0.002137	0.396937 0.002157	0.400299 0.002134

Table 9: The estimates and corresponding MSE's of Bayes estimator with different priors under different loss functions

R = 0.6, $\alpha_1 = 3, \alpha_2 = 2, c_1 = 1, d_1 = 1.4, c_2 = 0.5,\ d_2 = 1,\ a = \pm 0.5$ and $k = \pm 0.6$					
Sample Size	Bayes Estimates with Gamma Prior				
	$\hat{R}_S$	$\hat{R}_L$	$\hat{R}_{L1}$	$\hat{R}_G$	$\hat{R}_{G1}$
n=10,m=10	0.596765 0.007992	0.594211 0.008083	0.599309 0.007924	0.582167 0.009241	0.593117 0.008237
n=10,m=15	0.605149 0.006509	0.602992 0.006541	0.607293 0.006492	0.592963 0.007177	0.602125 0.006631
n=20,m=20	0.598749 0.004955	0.597382 0.004981	0.600115 0.004935	0.591236 0.005298	0.596863 0.005026
n=20,m=25	0.60157 0.004321	0.600334 0.004336	0.602804 0.004309	0.594791 0.004561	0.599873 0.004369
n=30,m=30	0.599351 0.003523	0.598422 0.003536	0.600279 0.003513	0.594298 0.003683	0.598083 0.003557
n=30,m=35	0.60087 0.003138	0.600005 0.003147	0.601734 0.003132	0.596177 0.003257	0.599694 0.003163
n=40,m=40	0.599471 0.002688	0.598767 0.002696	0.600174 0.002682	0.595665 0.002779	0.598516 0.002707
n=40,m=45	0.600414 0.002389	0.599748 0.002395	0.601078 0.002385	0.596819 0.002460	0.599513 0.002404
n=50,m=50	0.59995 0.002118	0.599384 0.002123	0.600515 0.002115	0.596903 0.002173	0.599186 0.002130

Table 10: The estimates and corresponding MSE's and Bayes estimator with different priors under different loss functions

R = 0.8, $\alpha_1 = 0.4, \alpha_2 = 0.1, c_1 = 3, d_1 = 0.55, c_2 = 2,\ d_2 = 0.45,\ a = \pm 0.5$ and $k = \pm 0.6$					
Sample Size	Bayes Estimates with Gamma Prior				
	$\hat{R}_S$	$\hat{R}_L$	$\hat{R}_{L1}$	$\hat{R}_G$	$\hat{R}_{G1}$
n=10,m=10	0.794792 0.005257	0.793439 0.005367	0.796137 0.005155	0.788944 0.005876	0.793337 0.005402
n=10,m=15	0.808401 0.003804	0.807342 0.003843	0.809443 0.003768	0.803841 0.004055	0.807285 0.003859
n=20,m=20	0.797428 0.002744	0.796768 0.002774	0.798086 0.002716	0.794686 0.002898	0.798744 0.002781
n=20,m=25	0.801664 0.002214	0.801079 0.002231	0.802246 0.002197	0.799242 0.002306	0.801062 0.002235
n=30,m=30	0.798402 0.001771	0.797966 0.001784	0.798838 0.001759	0.796616 0.001835	0.797956 0.001786
n=30,m=35	0.800528 0.001555	0.800127 0.001564	0.800929 0.001546	0.798886 0.001601	0.800119 0.001566
n=40,m=40	0.798785 0.001292	0.798459 0.001298	0.799110 0.001285	0.797461 0.001326	0.798454 0.001299
n=40,m=45	0.800092 0.001143	0.799786 0.001148	0.800397 0.001137	0.798849 0.001169	0.799782 0.001149
n=50,m=50	0.799318 0.001008	0.799059 0.001013	0.799577 0.001004	0.798269 0.001029	0.799056 0.001013

VI. Summary andConclusions

Wepresent below the summary and conclusions in view of the above discussions.

1. The estimators of stress-strength reliability for inverse exponential distribution are derived using maximum likelihood and Bayesian methods.

2. The system reliability of stress-strength model is derived assuming inverse exponential distribution for strength and stress variables.
3. The stress-strength reliability is estimated using the method of maximum likelihood and Bayesian method under different loss functions with non-informative and gamma priors.
4. The different loss functions considered are SELF, LINEX loss function and GELF.
5. The estimators derived are illustrated with a real data analysis and IE distribution fits well to the data considered.
6. Based on the simulation study, the Bayes estimators are better, when compared to MLE in terms of MSE.
7. Bayes estimators with gamma prior is better than that with non-informative prior for all loss functions considered. However, Bayes estimator under LINEX loss function is better than under SELF and GELF for a particular loss parameter (a=-0.5).

References

[1] Ashok, P., Swathi, N., Tirumala Devi, M. And Sumathi Uma Maheswari, T. (2018). Reliability for multi-component system using Pareto distribution. International Journal of Mathematical Trends and Technology, Vol. 64, No.2, pp.109-114.

[2] Chen, J. and Cheng, C. (2016). Reliability of stress-strength model for exponentiated pareto distributions, Journal of Statistical Computation and Simulation, Vol. 87, N0. 4, pp. 791-805.

[3] Ghitaney, M.E., Al-Mutairi, D.K. and Aboukhamseen, S.M. (2014). Estimation of the reliability of a stress-strength system from power-Lindley distribution, Communication in Statistics–Simulation and Computation, Vol.44, No.1, pp.118-136.

[4] Joshi, Shubhashree and Pandit, P. V. (2018). Estimation of system reliability in multi-component stress-strength model for inverse Chen distribution.. International Journal of Agricultural and Statistical Sciences, Vol.14, No.2, pp825-831.

[5] Keller, A. Z. And Kamath, A. R. R. (1982). Alternative reliability models for mechanical systems. Third International Conference on Reliability and Maintainability. 18-21 October 1982, Toulse, France.

[6] Kizilaslan, F. and Nadar, M. (2015). Classical and Bayesian estimation of reliability in multicomponent stress-strength model based on Weibull distribution, Revista Colombiana de estadistica, Vol.38, No.2, pp.467-484.

[7] Lavanya, A. and Alexander, L. (2016). Estimation of parameters using Lindley's method, International Journal of Advanced Research, Vol. 4, No. 12, pp. 1767-1778.

[8] Lawless, J. (2003). Statistical models and methods for life time data, New Jersey: John wiley and sons.

[9] Lindley, D. V. (1980). Approximate Bayes method, Trabajos de Estadistica, Vol.3, pp. 281-288.

[10] Metiri, F., Zeghdoudi, H. and Remita, M. R. (2016). On Bayes estimates of Lindley distribution under Linex loss function: informative and non-informative priors, Global Journal of Pure and Applied Mathematics, Vol. 12, No. 1, pp. 391-400.

[11] Mokhlis, N. A., Ibrahim, E.J. and Gharieb, D.M. (2016). Stress-strength reliability with general form of distributions, Communication in Statictics – Theory and Methods.

[12] Pandit, P. V. and Joshi, S. (2019). Estimation of multicomponent system reliability for a bi-variate generalized Rayleigh distribution, International Journal of Computational and Theoretical Statistics, Vol. 6, No.1.

[13] Singh, S. K., Umesh Singh, Yadav, A.S and Vishwkarma, P. K. (2015). International Journal of Scientific World, Vol. 3, No.1, pp. 98-112.