

# A Mapping for $S$ - $g$ -Closedness

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**Abstract:** We define a concept almost contra  $S$ - $g$ -continuity. Then, we obtain some properties and characterizations of it. We also give the relations with some other types of continuity. Finally, we obtain that it is implied by  $al.cont.g-con.$  with  $(\theta,s)$ -  $S-con.$  and  $cont. S-g-con.$  implied by it.

**Keywords :** Almost contra- $S$ - $g$ -continuity,  $g$ -closed sets, ideal, topology.

## I. INTRODUCTION

Levine([15]) defined and examined a new closed set namely  $g$ -closed. Since *continuity* is important concept of topology, many topologist have studied the several kind of generalizations of continuity such as *contra continuity*[4]. Modifications of contra continuity have been introduced and investigated in many authors. Such as in ([13]), authors have defined a new contra continuity related to  $g$ -closed sets. Since *ideal* which is another important notion in topology, it is investigated in [14] and [19] respectively.

This paper is consist of three part. In third section, we give a definition namely almost contra- $S$ - $g$ -continuity. Then we got some properties of it and the relationships with some others. At last, we obtain that it is implied by  $al.cont.g-con.$  with  $(\theta,s)$ -  $S-con.$  and  $cont. S-g-con.$  implied by it.

## II. PRELIMINARIES

We will use the symbol  $(P, \lambda)$  for any topological space. If  $Y \subseteq P$  is any subset of  $P$ ,  $Cl(Y)$  and  $Int(Y)$  are denote the closure and interior of  $Y$  in  $P$  respectively.  $\lambda'$  is the class of all closed sets in  $P$ .

Let  $\wp(P)$  is power set of  $P$ . A nonempty subclass  $S$  of  $\wp(P)$  said to be an ideal on  $P$ , if it satisfies (1)  $Y \in S, Z \subseteq Y \Rightarrow Z \in S$  and (2)  $Y \in S, Z \in S \Rightarrow (Y \cup Z) \in S$ . Let a topological space  $(P, \lambda)$  with an ideal  $S$  on  $P$ . Then for every  $Y \subseteq P$ ,  $(.)^* : \wp(P) \rightarrow \wp(P)$  is defined as  $Y^*(S, \lambda) = \{p \in P : (K \cap Y) \notin S \text{ for every } Y \in \lambda_{(p)}\}$ ,  $\lambda_{(p)}$  is open neighborhood of  $P$ .  $Y^*(S, \lambda)$  is given a name a *local mapping of  $M$*  with related to  $S$  and  $\lambda$  ([14]). Briefly,  $Y^*$  is used instead of  $Y^*(S, \lambda)$ . In [19] author introduced a new closure function, where  $cl^* : \wp(P) \rightarrow \wp(P)$  is defined by  $cl^*(Y) = Y \cup Y^*$ .  $Y^*(S) = \{Y \subset P : cl^*(Y) = (P - Y)\}$  is a topology on  $P$  by using  $cl^*$  and it is finer than  $\lambda$  ([9]). If there are one topology  $\lambda$  and one ideal  $S$  on  $P$  is said to be an *ideal topological space*. It will be used by  $(P, \lambda, S)$  ([9]).

Now, we recall two lemmas and one definition which are need for this study.

**Lemma 2.1**([9]). For  $Z, Y$  are two subsets of  $(P, \lambda, S)$ , then

- (1)  $Z \subset Y \Rightarrow Z^* \subset Y^*$ ;
- (2)  $(Z \cup Y)^* = Z \cup Y^*$ ;
- (3)  $Y^* = cl^*(Y) \subset cl(Y)$ ;
- (4)  $(Y^*)^* \subset Y^*$ .

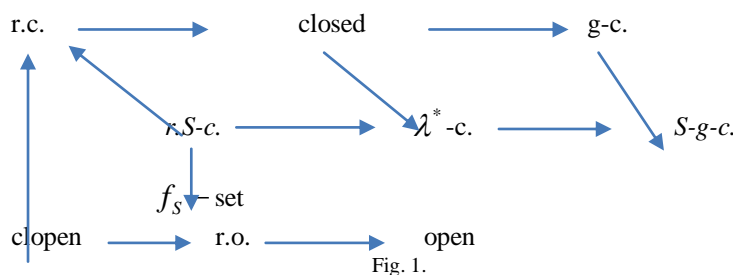
**Definition 2.1.** A subset  $M$  of  $(P, \lambda, S)$  given a name

- (1) clopen[14]  $M \in \lambda$  and  $M \in \lambda'$ ;
- (2) regular open (i.e.; r.o.) [18] if  $M = Int(Cl(M))$ ;
- (3) regular closed (i.e.; r.c.) [18] if  $M = Cl(Int(M))$ ;

- (4)  $g$ -closed(i.e.;  $g$ -c.) [15] if  $cl(M) \subset K$ , whenever  $M \subset K$  and  $K \in \lambda$ ;
- (5)  $\lambda^*$ -closed(i.e.;  $\lambda^*$ -c.) [9] if  $M^* \subset M$ ;
- (6)  $S$ - $g$ -closed(i.e.;  $S$ - $g$ -c.) [5]  $M^* \subset K$  whenever  $M \subseteq K$  and  $K \in \lambda$ ;
- (7) regular  $S$ -closed(i.e.;  $r$ - $S$ -c.) [11] if  $M = (Int(M))^*$ ;
- (8)  $f_s$ - [12]  $M \subset (Int(M))^*$ .

A subset  $M$  is given a name  $S$ - $g$ -open(i.e.;  $S$ - $g$ -o.) if  $(P - M)$  is an  $S$ - $g$ -c.. The class of all clopen (resp. r.o., r.c.,  $g$ -c.,  $S$ - $g$ -c.,  $S$ - $g$ -o.,  $r$ - $S$ -c.,  $f_s$ -) sets in  $(P, \lambda, S)$  is denoted by  $CO(P, \lambda)$  (resp.  $RO(P, \lambda)$ ,  $RC(P, \lambda)$ ,  $GC(P, \lambda)$ ,  $SGC(P, \lambda)$ ,  $SGO(P, \lambda)$ ,  $RSC(P, \lambda)$ ,  $f_s(P, \lambda)$ ).

By using above definition, one can obtain the following figure.



### III. ALMOST CONTRA S-G-CONTINUITY

In this part, we give a definition of almost contra- $S$ - $g$ -continuous mapping and obtain some conditions of it.

**Definition 3.1.** A subset  $K$  of  $(P, \lambda, S)$  is given a name  $mk$ -open set if  $K$  is open and  $\lambda^*$ -c.. The collection of all  $mk$ -open sets in  $(P, \lambda, S)$  is will be used by  $MK(P, \lambda)$ .

**Theorem 3.1.** Every clopen set is a  $mk$ -open.

**Proof.** Let  $K$  be a clopen set in  $(P, \lambda, S)$ . So,  $K$  is open and closed. As every closed set is a  $\lambda^*$ -c., then  $K$  is  $mk$ -open.

**Remark 3.1.** The reversible of Theorem 3.1 isn't right generally.

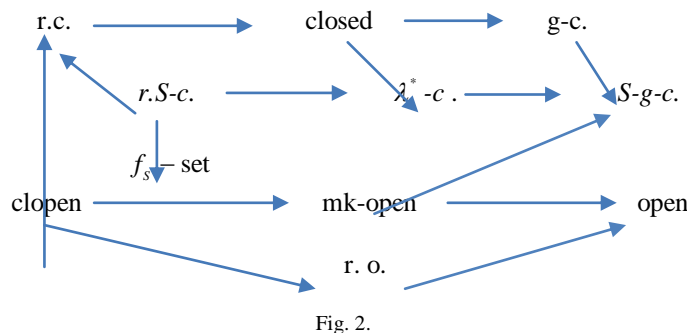
**Example 3.1.** For  $(P, \lambda, S)$  let  $P = \{1, 2, 3, 4\}$ ,  $S = \{\emptyset, \{3\}, \{4\}, \{3, 4\}\}$  and

$\lambda = \{\emptyset, P, \{3\}, \{1, 3\}, \{2, 3\}, \{3, 4\}, \{1, 3, 4\}, \{1, 2, 3\}, \{2, 3, 4\}\}$ . For  $K = \{3, 4\}$ ,  $K$  is a  $mk$ -open, but isn't clopen. It is seen that  $K \in \lambda$  Since  $K^* = \emptyset \subset \{3, 4\} = K$ ,  $K$  is  $mk$ -open. Since  $Cl(K) = P \not\subset \{3, 4\} = K$ ,  $K$  isn't closed and hence not clopen.

**Proposition 3.1.**  $MK(P, \lambda)$  is closed finite union.

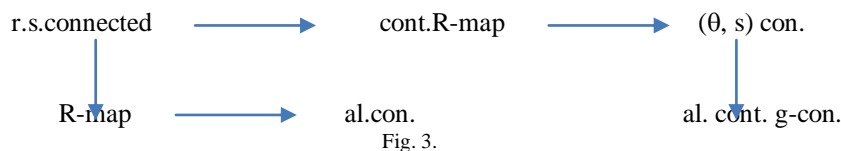
**Proof.** It is obtained from Lemma 2.1(2) and Definition 2.1(5).

By using Fig. 1 and Definition 3.1, the next figure is observed.



**Definition 3.2.** A map  $f : (P, \lambda) \rightarrow (Q, \mu)$  is given a name *regular set-connected* (i.e.; *r.s.connected*) [5], (resp. *R-map* [3], *almost continuous* (i.e.; *al.con.*) [18], *almost contra g-continuous* (i.e.; *al.cont.g-con.*) [13], *contra R-map* (i.e.; *cont. R-map*) [6],  $(\theta, s)$ -continuous (i.e.;  $(\theta, s)$ -con.) ([10],[16]))  $f^{-1}(Z)$  is clopen (resp. r.o., open, g-c., r.c., closed) in  $P$  for every r.o. set  $Z$  of  $Q$ .

The next figure is obvious from Fig. 1 with Definitions 3.2.



**Definition 3.3.** A map  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  is given a name *almost contra S-g-continuous* (i.e.; *al. cont. S-g-con.*) if  $f^{-1}(Z)$  is *S-g-c.* in  $P$  for every  $Z \in RO(Q, \mu)$ .

Two next theorem are related to characterizations of *al.cont. S-g-con. mappings*.

**Theorem 3.2.** Let  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  be a mapping. Then next four conditions are mutually equivalent:

- (1)  $f$  is *al.cont. S-g-con.*;
- (2)  $f^{-1}(F) \in IGO(P, \lambda)$  for every  $F \in RC(Q, \mu)$ ;
- (3)  $f^{-1}(Int(Cl(T))) \in IGC(P, \lambda)$  for every  $T \in \mu$ ;
- (4)  $f^{-1}(Cl(Int(R))) \in IGO(P, \lambda)$  for every  $R \in \mu'$ .

**Proof.** (1) $\Rightarrow$ (2) Let  $F \in RC(Q, \mu)$  Then  $(Q - F) \in RO(Q, \mu)$  and according to (1),  $f^{-1}((Q - F)) \in IGC(P, \lambda)$ . Hence,  $f^{-1}(F) \in IGO(P, \lambda)$ .

(2) $\Rightarrow$ (1) It is got similar to proof of (1) $\Rightarrow$ (2).

(1) $\Rightarrow$ (3) Let  $T \in \mu$ . Since  $Int(Cl(T)) \in RO(Q, \mu)$ , we have  $f^{-1}(Int(Cl(T))) \in IGC(P, \lambda)$  by using (1).

(3) $\Rightarrow$ (1) It is clear.

(2) $\Rightarrow$ (4) It is got as proof of (1) $\Rightarrow$ (3).

(4) $\Rightarrow$ (2) It is got as proof of (3) $\Rightarrow$ (1).

**Theorem 3.3.** If  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  is *al.cont. S-g-con.* mapping, then the next two conditions are mutually equivalent:

- (1) For every  $p \in P$ , every r.c.  $F$  in  $Q$  containing  $f(p)$ ; there exists an *S-g-o.* set  $K$  in  $P$  containing  $p$  while  $f(K) \subset F$ ,
- (2) For every  $p \in P$ , every r.o.  $K$  in  $Q$  non-containing  $f(p)$ ; there exists an *S-g-c.* set  $V$  in  $P$  non-containing  $p$  while  $f^{-1}(K) \subset V$ .

**Proof.** We will prove only (1) because of it is evident that (1) and (2) are equivalent to each other. Let  $F$  be any r.c. in  $Q$  containing  $f(p)$ . According to Theorem 3.2(2),  $f^{-1}(F) \in IGO(P, \lambda)$  and  $p \in f^{-1}(F)$ . Taking  $K = f^{-1}(F)$ , we observe immediately  $f(K) \subset F$ .

**Definition 3.4.** A mapping  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  is given a name *almost  $f_s$ -continuous* (i.e.; *al.  $f_s$ -con.*) (resp. *contra R-S-map* (i.e.; *cont. R-S-map*), *regular set\*-connected* (i.e.; *r.s.\*-connected*),  $(\theta, s)$ -*S-continuous* (i.e.;  $(\theta, s)$ -*S-con.*) if  $f^{-1}(Z)$ ,  $f_s$ - (resp. r. *S-c.*, *mk-open*,  $\lambda^*$ -*c.*) set is in  $P$  for every  $Z \in RO(Q, \mu)$ .

**Theorem 3.4.** If  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  is *cont. R-S-map*, then it is  $(\theta, s)$ -*S-con.*.

**Proof.** It is observed from Fig. 1.

**Remark 3.2.** We give next example which is the reversible of Theorem 3.4 isn't true.

**Example 3.2.** For  $(P, \lambda, S)$  let  $P = \{1, 2, 3\}$ ,  $\lambda = \{\emptyset, P, \{1\}, \{1, 3\}, \{1, 2\}\}$  and  $S = \{\emptyset, \{1\}, \{3\}, \{1, 3\}\}$ . For  $(Q, \mu)$ , let  $Q = \{1, 2, 3\} = P$  and  $\mu = \{\emptyset, Q, \{1, 2\}, \{3\}\}$ . Let  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  is an identity function. For  $Z = \{3\} \in RO(Q, \mu)$ ,  $(f^{-1}(Z))^* = (\{3\})^* = \emptyset \subset \{3\} = f^{-1}(Z)$ . So  $f^{-1}(Z)$  is a  $\lambda^*$ -c. .Because of  $Z = \{3\} \in \mu$   $(Int(f^{-1}(Z)))^* = (Int(\{3\}))^* = (\{3\})^* = \emptyset \subset \{3\} = f^{-1}(Z)$ . Hence isn't r.S-c. and hence  $f$  is  $(\theta, s)$ -S-con., but isn't cont. R-S-map.

**Theorem 3.5.** If  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  is  $(\theta, s)$ -S-c., then al. cont. S-g-con..

**Proof.** It is observed from Fig. 1.

**Remark 3.3.** We have next example in which is denoted the reversible of Theorem 3.4 isn't true.

**Example 3.3.** For  $(P, \lambda, S)$  let  $P = \{x, y, z\}$ ,  $\lambda = \{\emptyset, P, \{x\}, \{y, z\}\}$  and  $S = \{\emptyset, \{x\}, \{z\}, \{x, z\}\}$  For  $(Q, \mu)$ , let  $Q = \{x, y, z\} = P$  and  $\mu = \{\emptyset, Q, \{x, y\}, \{z\}\}$ . Let  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  is a function, defined by:  $f(x) = f(z) = z$  and  $f(y) = y$  Set  $Z = \{z\} \in RO(Q, \mu)$ . Therefore, we have  $f^{-1}(Z) = \{x, z\} \subset \{x, y, z\} = P \in \lambda$  and  $(f^{-1}(Z))^* = (\{x, y\})^* = \{y, z\} \subset P$ . Then  $f^{-1}(Z) = \{y, z\}$  is S-g-c., but it is not  $\lambda^*$ -c. . Therefore  $f$  is al. cont. S-g-con., but it is not  $(\theta, s)$ -S con..

**Theorem 3.6.** If  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  is cont. R-S-map, then it is al. S-con..

**Proof.** This proof is observed from Fig. 3 and Theorem 3.2.

**Remark 3.4.** The reversible of Theorem 3.6 isn't true as observed in below.

**Example 3.4.** For  $(P, \lambda, S)$  let  $P = \{x, y, z\}$ ,  $\lambda = \{\emptyset, P, \{x\}, \{y, z\}\}$  and  $S = \{\emptyset, \{z\}\}$ . For  $(Q, \mu)$ , let  $Q = \{x, y, z\} = P$  and  $\mu = \{\emptyset, Q, \{x, y\}, \{z\}\}$ . Let  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  is a function, defined by :  $f(x) = f(z) = x$  and  $f(y) = z$ . Set  $Z = \{z\} \in RO(Q, \mu)$ . Then  $f^{-1}(Z) = \{y\}$  is  $f_s$ -set, but it isn't S-g-c.. This shows that  $f$  is al. S-con., but it is not cont. R-S-map.

**Theorem 3.7.** If  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  is al. cont. g-con. mapping, then it is al. cont. S-g-con..

**Proof.** This proof is got from Fig. 1, Definition 2.2 and Definition 3.2.

**Remark 3.5.** The reversible of Theorem 3.7 isn't true as observed in below.

**Example 3.5.** For  $(P, \lambda, S)$  let  $P = \{x, y, z, t\}$ ,  $\lambda = \{\emptyset, P, \{x\}, \{y, z\}, \{x, y, z\}\}$  and  $S = \{\emptyset, \{x\}, \{z\}, \{x, z\}\}$ . For  $(Q, \mu)$ , let  $Q = \{x, y, z\}$  and  $\mu = \{\emptyset, Q, \{x, y\}, \{z\}\}$ . Let  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  is a mapping, defined by  $f(x) = f(z) = z$ ,  $f(y) = x$  and  $f(t) = y$  Set  $Z = \{z\} \in RO(Q, \mu)$ . Then  $f^{-1}(Z) = \{x, z\}$ . Also  $Y = \{x, y, z\} \in \lambda$   $(f^{-1}(Z))^* = (\{x, z\})^* = \emptyset \subset Y$  and hence  $f^{-1}(Z)$  is S-g-c. set. Since  $Cl(f^{-1}(Z)) = P \subsetneq Y$  we observe that  $f^{-1}(Z)$  is not g-c..It is observed that  $f$  is al. cont. S-g-con., but isn't al. cont. g-con..

**Theorem 3.8.** If  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  is r.s.connected, then it is r.s.\*-connected.

**Proof.** This proof is got from Fig. 1, Theorem 3.1.

**Remark 3.6.** The reversible of Theorem 3.8 isn't true as denoted below.

**Example 3.6** For  $(P, \lambda, S)$  let  $P = \{x, y, z, t\}$ ,  $\lambda = \{\emptyset, P, \{x, y\}, \{z, t\}\}$  and  $S = \{\emptyset, \{z\}, \{t\}, \{z, t\}\}$ . For  $(Q, \mu)$ , let  $Q = \{x, y, z\}$  and  $\mu = \{\emptyset, Q, \{x, y\}, \{z\}\}$ . Let  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  is a function, defined by :  $f(x) = x$ ,  $f(y) = y$  and  $f(z) = f(t) = z$ . If  $Z = \{z\} \in RO(Q, \mu)$ , then  $f^{-1}(Z) = \{z, t\}$  is mk-open.

Since  $(f^{-1}(Z))^* = (\{z, t\})^* = \emptyset \subset \{z, t\}$ ,  $Cl(f^{-1}(Z)) = P \not\subset \{z, t\}$  is  $\lambda^*$ -c. set but it is not closed and clopen. So,  $f$  is r.s.\*-connected but isn't r.s. connected.

**Theorem 3.9.** If  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  is cont.R-S-map, then it is cont.R-map.

**Proof.** This proof is got from Fig. 1, Definition 2.2 and Definition 3.3.

**Remark 3.7.** The reversible of Theorem 3.8 isn't true as as denoted below.

**Example 3.7** For  $(P, \lambda, S)$  let  $P = \{x, y, z, t\}$ ,  $\lambda = \{\emptyset, P, \{x, z\}, \{y, t\}\}$  and  $S = \{\emptyset, \{y\}, \{t\}, \{y, t\}\}$  For  $(Q, \mu)$ , let  $Q = \{x, y, z\}$  and  $\mu = \{\emptyset, Q, \{x, y\}, \{z\}\}$ . Let  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  is a function defined by:  $f(x) = x$ ,  $f(z) = y$  and  $f(y) = f(t) = z$ . If  $Z = \{z\} \in RO(Q, \mu)$ , then  $f^{-1}(Z) = \{y, t\}$ . Since  $Int(f^{-1}(Z)) = Int(\{y, t\}) = \{y, t\}$  and  $Cl(Int(f^{-1}(Z))) = Cl(Int(\{y, t\})) = \{y, t\}$ ,  $f^{-1}(Z)$  is r.c.set. Besides since  $(Int(f^{-1}(Z)))^* = (Int(\{y, t\}))^* = \emptyset \neq \{y, t\}$ .  $f^{-1}(Z)$  is not r. S-c.. This shows that  $f$  is cont.R-map but isn't cont.R-S-map.

**Definition 3.4.** A mapping  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  is given a name *contra S-g-continuous* (i.e.; cont. S-g-con.) if  $f^{-1}(Z)$  is S-g-c. in  $(P, \lambda, S)$  for every open set  $Z$  of  $Q$ .

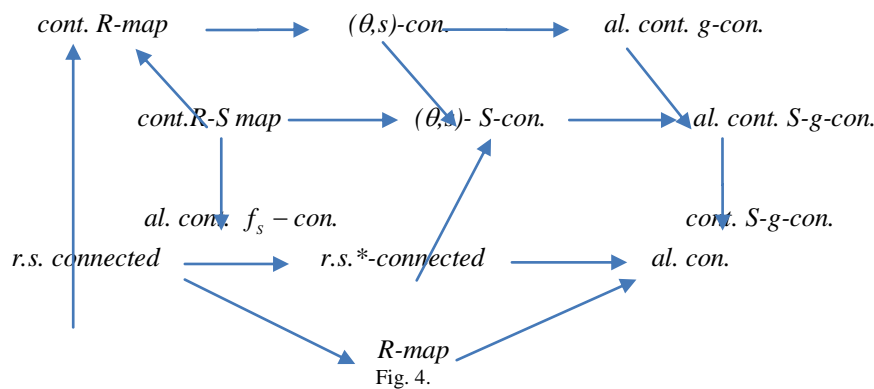
**Theorem 3.10.** If is  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  al.cont. S-g-con. mapping, then it is cont. S-g-con..

**Proof.** For  $Z \in RO(Q, \mu)$  since  $Z \in \mu$  and  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  is al.cont.S-g-con.,  $f^{-1}(Z) \in SGC(P, \lambda)$ . Thus,  $f$  is cont.S-g-con..

**Remark 3.8.**The reversible of Theorem 3.10 is false in generally.

**Example 3.8.** For  $(P, \lambda, S)$  let  $P = \{x, y, z, t\}$ ,  $\lambda = \{\emptyset, P, \{x\}, \{y\}, \{x, z\}, \{x, y\}, \{x, y, z\}\}$  and  $S = \{\emptyset, \{y\}\}$ . For  $(Q, \mu)$  let  $Q = \{x, y, z\}$  and  $\mu = \{\emptyset, Q, \{x\}, \{y, z\}\}$ . Let  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  be a mapping defined:  $f(x) = y$ ,  $f(y) = f(z) = x$ . For  $Z = \{x\} \in \mu$ . Then,  $f^{-1}(Z) = \{y, z\} \subset \{x, y, z\} = K \in \lambda$ ,  $(f^{-1}(Z))^* = (\{y, z\})^* = \{z\} \subset K$  is S-g-c. set. So,  $f$  is cont. S-g-con.. For  $N = \{y, z\} \in RO(Y, \varphi)$ ,  $f^{-1}(N) = \{x\} = L \in \lambda$ .  $(f^{-1}(N))^* = (\{x\})^* = \{x, z\} \not\subset L$ ,  $N$  is not S-g-c.. Therefore,  $f$  isn't al. cont. S-g-con..

By using Fig. 3 and above theorems, the next figure is given.



Recall that a function  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  is given a name *completely continuous* (i.e.; *compl. con.*)[1] (resp. continuous (i.e.;*con.*) if  $f^{-1}(Z)$  is r. o. (resp. open) in  $P$  for every open set  $Z$  of  $Q$ .

**Theorem 3.11.** For two mappings  $f : (P, \lambda, S) \rightarrow (Q, \mu, D)$  and  $g : (Q, \mu, D) \rightarrow (R, \kappa)$  , let  $gof : (P, \lambda, S) \rightarrow (R, \kappa)$  is a composite mapping. The next conditions valid:

- (1) If  $f$  is al. cont.  $S$ -g-con.,  $g$  is an R-map then  $gof$  is al. cont.  $S$ -g-con.;
- (2) If  $f$  is cont.  $S$ -g-con.,  $g$  is al. con. then  $gof$  is al. cont.  $S$ -g-con.;
- (3) If  $f$  is al. cont.  $S$ -g-con.,  $g$  is completely con. then  $gof$  is cont.  $S$ -g-con.;
- (4) If  $f$  is cont.  $S$ -g-con.,  $g$  is con. then  $gof$  is cont.  $S$ -g-con.

**Proof.** (1) Let  $V$  be any r. o. set in  $R$ . Since  $g$  is an R-map and  $f$  is al. cont.  $S$ -g-con.,  $g^{-1}(V) \in RO(Q, \mu)$  and  $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V) \in SGC(P, \lambda)$ , respectively. This shows that,  $gof$  is al. cont.  $S$ -g-con.

(2) Let  $V$  be any r.o. set in  $R$ . Since  $g$  is al. con. and  $f$  is cont.  $S$ -g-con. ,  $g^{-1}(V) \in \mu$  and  $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V) \in SGC(P, \lambda)$ , respectively. Hence,  $gof$  is al. cont.  $S$ -g-con.

(3) It is easily observe similar to (1).

(4) It is easily observe similar to (2).

**Theorem 3.12.** If  $f : (P, \lambda, S) \rightarrow (Q, \mu)$  is al. cont.  $S$ -g-con. and al. con., then  $f$  is r.s.\*-connected.

**Proof.** Let  $V$  be any r.o in  $Q$ .  $f$  is both al. cont.  $S$ -g-con. and al. con.,  $f^{-1}(V)$  is both  $S$ -g-c. and open. So,  $f^{-1}(V)$  is  $\lambda^*$ -c. and mk-open set. Hence  $f$  is r.s.\*-connected.

**Definition 3.5.** A mapping  $f : (P, \lambda, S) \rightarrow (Q, \mu, D)$  is given a name

- (1)  $S$ -g-open if  $f(K)$  is  $D$ -g-o. in  $Q$  for every  $S$ -g-o. set  $K$  of  $P$ ;
- (2)  $S$ -g-closed if  $f(K)$  is  $D$ -g-c. in  $Q$  for every  $S$ -g-c. set  $K$  of  $P$ .

**Theorem 3.13.** If  $f : (P, \lambda, S) \rightarrow (Q, \mu, D)$  is surjection  $S$ -g-open (or  $S$ -g-closed),  $g : (Q, \mu, D) \rightarrow (R, \omega)$  is a mapping while  $gof : (P, \lambda, S) \rightarrow (R, \kappa)$  is al. cont.  $S$ -g-con., then  $g$  is al. cont.  $D$ -g-con..

**Proof.** Let  $V$  be any r. c. (resp. r. o.) in  $R$ .  $gof$  al. cont.  $S$ -g-con.,  $(gof)^{-1}(V) = f^{-1}(g^{-1}(V))$  is  $S$ -g-o. (resp.  $S$ -g-c.). Since  $f$  is both surjection and  $S$ -g-open (or  $S$ -g-closed),  $f(f^{-1}(g^{-1}(V))) = g^{-1}(V)$  is  $D$ -g-o. (or  $D$ -g-c.). Hence,  $g$  is al. cont.  $D$ -g-con..

#### IV. CONCLUSION

We obtain that it is implied by al.cont.g-con. with  $(\theta, s)$ -  $S$ -con. and cont.  $S$ -g-con. implied by it.

#### REFERENCES

- [1] S.P Arya., R Gupta., "On strongly continuous mappings", Kyungpook Math. J., 14,pp.131-143,1974.
- [2] N. Bourbaki. , General Topology, Part I. Reading, MA: Addison Wesley,1996.
- [3] D. A. Carnahan., Some Properties Related To Compactness in Topological spaces. PhD Thesis, Univ. of Arkansas,1973.
- [4] J. Dontchev., "Contra-continuous functions and strongly  $S$ -closed mappings", Int. J. Math. Math. Sci., 19,pp.303-310,1996.
- [5] J.Dontchev, M Ganster., T Noiri., "Unified Operation Approach of Generalized Closed Sets via Topological Ideals", Math. Japonica, 49(3),pp. 395-401,1999.
- [6] J. Dontchev ,M. Ganster, I. L Reilly., "More on almost  $s$ -continuity", Indian J. Math., 41,pp. 139-146,1999.
- [7] E. Ekici, "Another form of contra-continuity", Kochi J. Math., 1,pp.21-29. 2006.
- [8] E Hayashi., "Topologies defined by local properties", Math. Ann., 156,pp.205-215, 1964.
- [9] D. Jankovic., T.R. Hamlett, "New Topologies from Old via Ideals", Amer. Math. Monthly, 97(4), pp.295-310,1990.
- [10] J. K Joseph., M. K Kwak., "On  $S$ -closed spaces", Proc. Amer Math. Soc., 45,pp.65-87,1980.
- [11] A .Keskin., T. Noiri, S. Yuksel, " Idelizaton of A Decomposition Theorem", Acta. Math. Hungar., 102(4) ,pp.269-277,2004.

- [12] A Keskin., T. Noiri ,S. Yuksel , “  $f_r$ - Sets and Decomposition of RIC-Continuity”, Acta Math. Hungar., 107(4) ,pp.307-313,2004.
- [13] A. Keskin., T.Noiri, “Almost Contra  $g$ - Continuous Functions”, Chaos, Solitons and Fractals, 42,pp. 238-246.,2009.
- [14] K .Kuratowski., Topologie I, Warszawa.,1933.
- [15] N. Levine., “Generalized Closed Sets in Topology”, Rendicontidel Circolo Matematico di Palermo, 19(1), pp.89-96,1970.
- [16] T .Noiri., S. Jafari , “Properties of  $(\theta,s)$ -continuous functions”, Topol. Appl., 123,pp.167-179, 2002.
- [17] M. K Singal., A.R Singal., “Almost-continuous mappings”, Yokohama Math. J., 16,pp.63-73,1968.
- [18] M. H Stone., “Applications of the theory of Boolean rings to general topology”, TAMS, 41,pp.375-381,1937.
- [19] R. Vaidyanathaswamy., Set Topology, Chelsea Publishing New York,1960.