The Concept of Fuzzy Number

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Abstract — *Fuzzy numbers agree to us to representation in a simple method these non-probabilistic reservations. This justifies the growing concentration on hypothetical in addition to sensible aspects of fuzzy calculation in the previous existence, particularly heading for to: operations in excess of fuzzy statistics and properties, position of fuzzy numbers and canonical demonstration of fuzzy statistics. Usually, the description of adding together and multiplication of fuzzy numbers are based on the expansion standard. A main drawback of the multiplicative operation in this case is that by multiplication the shape of L - R type fuzzy numbers (so triangular or trapezoidal numbers) is not preserved.*

Keywords — *Fuzzy numbers, L - R type fuzzy numbers, trapezoidal numbers.*

I. INTRODUCTION

The concept of uncertain of fuzzy numbers may be presented in many ways. We consider a fuzzy number to be an extension of the concept of the interval of confidence which is familiar to anyone who has computed using imprecise data in simple or complex systems. This extension is based on a natural and very simple idea. Instead of considering the interval of confidence at one unique level, it is considered at several levels and more generally at all levels from 0 to 1. The word Fuzzy was first introduced by Zadeh in his famous paper **"Fuzzy Sets"** he used this word to generalize the mathematical concept of set of one of fuzzy set or fuzzy subset, where in a fuzzy set, a membership function is defined for each element of the referential set. The membership function takes its value in the interval $[0, 1] \subset R^+$ instead of $\{0, 1\}$ as in Boolean algebra. In another point of view, in many applications the measurements cannot be repeated. This situation occurs mainly in Geosciences, since in this case we cannot have two holes in the same place in order to repeat the measurement so every experiment can be considered as unique. This shows us that uncertainties on the measurements in geological data are more of possibility type than of probabilistic type, since in order to obtain the statistical distribution of a variable we need several experiments.

II. BASIC CONCEPTS OF FUZZY NUMBERS

Interval of confidence

Consider a situation in which the value is uncertain. Suppose that the information available in such that we can accept that the uncertain value belongs to the referential set r (a set of real numbers). In many situations encountered by scientists it is possible to locate the value inside a closed interval of r.

That is, an interval of confidence of r : $[a_1, a_2]$. We are thus certain that the value is greater or equal to a_1 and smaller or equal to a_2. This kind of statement often occurs in science and engineering. In this case we use the symbol

$$a = [a_1, a_2]$$

generally the numbers a_1 and a_2 are finite, but is some cases it is useful, or even necessary to consider that $a_1 = -\infty$ and/or $a_2 = \infty$.In other cases, instead of considering a closed interval, we consider open intervals by using the notations,

$]a_1, a_2]$ or $(a_1, a_2]$	:	open at the left
$[a_1, a_2[$ or $[a_1, a_2)$	:	open at the right
$]a_1, a_2[$ or (a_1, a_2)	:	open at the left and at the right

Note also that the symbol $x \varepsilon [a_1, a_2]$ denotes the fact that x is an uncertain value.

Operations for the Interval of Confidence

Addition:

Assume two intervals of confidence in R:

$$A = [a_1, a_2] \quad \ldots (1.1)$$

$$\text{And} \quad B = [b_1, b_2] \quad \ldots (1.2)$$

$$\text{Hence, if} \quad x \varepsilon [a_1, a_2] \quad \ldots (1.3)$$

$$\text{and} \quad y \varepsilon [b_1, b_2] \quad \ldots (1.4)$$

$$\text{then} \quad x + y \varepsilon [a_1 + b_1, a_2 + b_2] \quad \ldots (1.5)$$

Symbolically, we write

$$A (+) B = [a_1, a_2] (+) [b_1, b_2] = [a_1 + b_1, a_2 + b_2] \quad \ldots (1.6)$$

The proof of (1.5) is trivial, because if $x \geq a_1$, and $y \geq b_1$

$\therefore (x + y) \geq (a_1 + b_1)$

And because $x \leq a_2$ and $y \leq b_2$ then

$\therefore (x + y) \leq (a_2 + b_2)$

Subtraction:

If $x \,\varepsilon\, [a_1, a_2]$ and $y \,\varepsilon\, [b_1, b_2]$ then

$(x - y) \,\varepsilon\, [a_1 - b_2, a_2 - b_1]$

In fact, we must subtract the largest value in $[b_1, b_2]$ from a_1 and the smallest value in $[b_1, b_2]$ from a_2. Again,

Symbolically, we can write as follows:

$$A (-) B = [a_1, a_2] (-) [b_1, b_2] = [a_1 - b_2, a_2 - b_1]$$

Image:

If $x \,\varepsilon\, [a_1, a_2]$ then $- x \,\varepsilon\, [- a_2, - a_1]$

Hence, if A is an interval of confidence, its image is defined as

$$A^- = [- a_2, - a_1]$$

and note that

$$A (+) A^- = [a_1, a_2] (+) [- a_2, - a_1] = [a_1 - a_2, a_2 - a_1]$$

$$A (+) A^- \neq 0 \quad \ldots (1.7)$$

Properties and Structure:

Let $A = [a_1, a_2]$, $B = [b_1, b_2]$, $C = [c_1, c_2]$, $0 = [0, 0]$

- $A (+) B = B (+) A$ (Commutative) . . . (1.8)
- $A (+) (B (+) C) = (A (+) B) (+) C$(Associative) . . . (1.9)
- $A (+) 0 = 0 (+) A = A$ (Identity) . . . (1.10)
- $A (+) A^- = A^- (+) A \neq 0$ . . . (1.11)

Multiplication:

If the intervals of confidence belong to R and if $x \,\varepsilon\, [a_1, a_2]$ and $y \,\varepsilon\, [b_1, b_2]$ then $x.\, y \,\varepsilon\, [a_1.\, b_1, a_2.\, b_2]$

We can therefore write

$$A (.) B = [a_1, a_2] (.) [b_1, b_2] = [a_1 . b_1, a_2 . b_2] \quad \ldots (1.12)$$

Division:

Division is defined as only in R^+,

$$A (:) B = [a_1, a_2] (:) [b_1, b_2] = [a_1 / b_2, a_2 / b_1]$$

If $b_1 = 0$, the upper bound increases to $+ \infty$. If $b_1 = b_2 = 0$ then the interval of confidence is extended to $- \infty$.

Inverse:

If $x \,\varepsilon\, [a_1, a_2]$ then $1/x = [1/a_2, 1/a_1]$ and $A^{-1} = [a_1, a_2]^{-1} = [1/a_2, 1/a_1]$

***Properties and Structure*:**

Let $A = [a_1, a_2]$, $B = [b_1, b_2]$, $C = [c_1, c_2]$, $1 = [1, 1]$

- For all $A, B \subset R^+$

  $A (.) B= B (.) A$ (Commutative) . . . (1.13)
- For all A, B and $C \subset R^+$

  $(A (.) B) (.) C = A (.) (B (.) C)$ (Associative) . . . (1.14)

  $A (.) 1 = 1 (.) A = A$ (Identity) . . . (1.15)
- The inverse is not symmetric in the sense of set theory, that is

$$A (.) A^{-1} = [a_1, a_2] (.) [1/a_2, 1/a_1] = [a_1 / a_2, a_2 / a_1] \neq 1$$

Multiplication by a Non-Negative Number:

Suppose that $k \,\varepsilon\, R^+$, we may then write $k = [k, k]$ and for all $A \,\varepsilon\, R^+$ then

$$k . A = k . [a_1, a_2] = [k, k] (.)[a_1, a_2] = [ka_1, ka_2]$$

Division by $k > 0$ is equivalent to multiplication by $1/k$.

Maximum, Minimum of two intervals of confidence:

Consider two numbers in R, such that by definition

$$a \wedge b = \text{Min}(a, b)$$
$$= a \quad \text{if } a \leq b$$
$$= b \quad \text{if } b \leq a$$
$$a \vee b = \text{Max}(a, b)$$
$$= b \quad \text{if } a \leq b$$
$$= a \quad \text{if } b \leq a$$

Now let us consider two intervals of confidence in R,

$A = [a_1, a_2]$ and $B = [b_1, b_2]$, we introduce two operators $(\wedge)$ and $(\vee)$ as follows

$$A\ (\wedge)\ B = [a_1, a_2]\ (\wedge)\ [b_1, b_2] = [a_1 \wedge b_1, a_2 \wedge b_2]$$

$$A\ (\vee)\ B = [a_1, a_2]\ (\vee)\ [b_1, b_2] = [a_1 \vee b_1, a_2 \vee b_2]$$

We note that $(\wedge)$ and $(\vee)$ satisfy the commutative and associative properties.

We call the operation $(\wedge)$ is the minimum of A and B and the operation $(\vee)$ is the maximum of A and B.

III. ARITHMETIC OPERATORS OF FUZZY NUMBER

Addition of Fuzzy Numbers:

The addition of fuzzy numbers follows the same process (In section 1.2 we showed how to add two intervals of confidence), but level by level. For example, let A and B be two fuzzy numbers and A_α and B_α their intervals of confidence for the level of presumption α, $\alpha \in [0, 1]$, then we can write

$$A_\alpha\ (+)\ B_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right] + \left[b_1^{(\alpha)}, b_2^{(\alpha)}\right]$$
$$= \left[a_1^{(\alpha)} + b_1^{(\alpha)}, a_2^{(\alpha)} + b_2^{(\alpha)}\right] \quad \ldots (2.1)$$

If A, B $\subset$ R, then for the intervals of confidence at the level α we may define the following ordinary subsets A_α, B_α.

$$A_\alpha = \{x/\mu_A(x) \geq \alpha\} \quad \ldots (2.2)$$
$$B_\alpha = \{x/\mu_B(x) \geq \alpha\} \quad \ldots (2.3)$$

Let us now consider another method for the addition of fuzzy numbers.

Let A, B $\subset$ R, for all x, y and z $\in$ R;

$$\mu_{A\,(+)\,B}(z) = \bigvee_{z = x + y} \left(\mu_A(x) \wedge \mu_B(y)\right) \quad \ldots (2.4)$$

We will now prove that (2.1) and (2.4) describe the same operation.

From (1.14) and (1.15) we can write by nesting that

$$A = \cup\, \alpha A_\alpha = \cup\, \alpha \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right] \quad \ldots (2.5)$$
$$B = \cup\, \alpha B_\alpha = \cup\, \alpha \left[b_1^{(\alpha)}, b_2^{(\alpha)}\right] \quad \ldots (2.6)$$

It is easy to prove that equation (2.1) and (2.4) are as valid for numbers in z as in N, Two examples will clarify this situation

$$\mu_A(x) = 1 \quad \text{if } x \in \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right]$$
$$= 0 \quad \text{if } x \notin \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right]$$
$$\mu_B(x) = 1 \quad \text{if } x \in \left[b_1^{(\alpha)}, b_2^{(\alpha)}\right]$$
$$= 0 \quad \text{if } x \notin \left[b_1^{(\alpha)}, b_2^{(\alpha)}\right] \quad \ldots (2.7)$$

Note that we have used symbolic writing in (2.5), (2.6) and (2.7). We can also write that

$$\mu_{A\,(+)\,B}(z) = 1 \quad \text{if } z \in \left[a_1^{(\alpha)} + b_1^{(\alpha)}, a_2^{(\alpha)} + b_2^{(\alpha)}\right]$$

Now let us return to (2.4) and apply it to every level α. We find that

For all x, y and z $\in$ R:

$$\mu_{A\alpha\,(+)\,B\alpha}\,(z) = \bigvee_{z=x+y} \left(\mu_{A\alpha}(x) \wedge \mu_{B\alpha}(y)\right) \quad \text{. . . (2.8)}$$

For all values of x and y such that

$\mu_{A\alpha}(x) = 1$ and

$\mu_{B\alpha}(x) = 1$

The right side of (2.8) gives 1. If this is not true, it gives 0 and since z = x + y, we write

$$z\ \varepsilon \left[a_1^{(\alpha)} + b_1^{(\alpha)}, a_2^{(\alpha)} + b_2^{(\alpha)}\right]$$

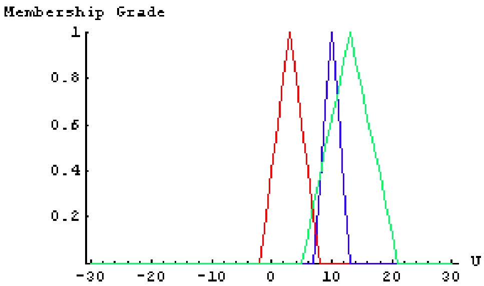
Referring to (1.17) and (1.18) we write

$$A_\alpha\ (+)\ B_\alpha = \bigcup_\alpha \left[a_1^{(\alpha)} + b_1^{(\alpha)},\ a_2^{(\alpha)} + b_2^{(\alpha)}\right] \text{. . . (2.9)}$$

Figure (7) shows the addition of two fuzzy numbers.

Figure – 2.1

Fuzzy Addition of Two Numbers



Subtraction:

The definition of addition can also be extended to the definition of subtraction. Consider the following definitions and symbols.

For all $\alpha\ \varepsilon\ [0, 1]$

$$A\ (-)\ B = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right] (-) \left[b_1^{(\alpha)}, b_2^{(\alpha)}\right]$$

$$= \left[a_1^{(\alpha)} - b_2^{(\alpha)}, a_2^{(\alpha)} - b_1^{(\alpha)}\right]$$

(or)

For all x, y and z ε R :

$$\mu_{A\,(-)\,B}\,(z) = \bigvee_{z=x-y} \left(\mu_A(x) \wedge \mu_B(y)\right) \quad \text{. . . (2.10)}$$

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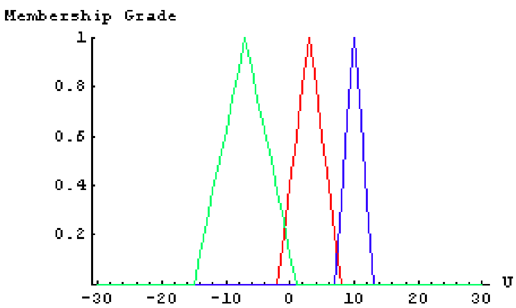
Subtraction is, in fact, the addition of the image of B^- to A, where

For all $\alpha\ \varepsilon\ [0, 1]$

$$B_\alpha^{-1} = [-b_2(\alpha),\ -b_1(\alpha)]$$

Subtraction is neither commutative nor associative. It is defined in z as in R, but not in R^+ or N because negative numbers could appear.

Fuzzy Subtraction



Multiplication of Fuzzy Numbers:

At this point we consider multiplication in R^+ and N. Let us consider two fuzzy numbers A and B in R^+. Form level α of presumption, we can write,

$$A_\alpha\ (.)\ B_\alpha = \left[a_1^{(\alpha)} + a_2^{(\alpha)}\right] (.) \left[b_1^{(\alpha)} + b_2^{(\alpha)}\right]$$

$$= \left[a_1^{(\alpha)} .\ b_1^{(\alpha)}\right], \left[a_2^{(\alpha)} .\ b_2^{(\alpha)}\right] \quad ...(2.11)$$

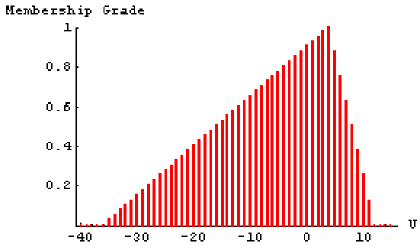
Multiplication can also be given by

For all x, y and z ε R:

$$\mu_{A\,(.)\,B}(z) = \bigvee_{z\,=\,x.\,y} \left(\mu_A(x) \wedge \mu_B(y)\right) \quad ...(2.12)$$

Equations (2.11) and (2.12) are equivalent.

Fuzzy Multiplication of Two Numbers



Division of Fuzzy Numbers:

Division of two fuzzy numbers is defined in R^+ by

$$A\ (:)\ B = \left[a_1^{(\alpha)} + a_2^{(\alpha)}\right] (:) \left[b_1^{(\alpha)} + b_2^{(\alpha)}\right]$$

$$= \left[a_1^{(\alpha)}/b_2^{(\alpha)}, a_2^{(\alpha)}/b_1^{(\alpha)}\right], b_2^{(\alpha)} > 0, \text{ for all } \alpha\ \varepsilon[0, 1]$$

(or)

For all x, y and z ε R^+

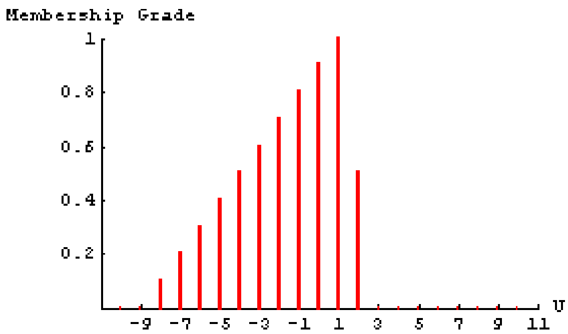
$$\mu_{A\,(:)\,B}(z) = \bigvee_{z\,=\,x\,/\,y} \left(\mu_A(x) \wedge \mu_B(y)\right) \quad ...(2.13)$$

Division, as has been mentioned previously, is a multiplication by the inverse, that is given by

$$B_\alpha^{-1} = [1/b_2(\alpha),\ 1/b_1(\alpha)], b_2(\alpha) > 0, \text{ for all } \alpha\ \varepsilon\ [0, 1] \quad ...(2.14)$$

Figure – 2.2.1

Fuzzy Division



Multiplication of a Fuzzy number with by an Ordinary Number:

Let A be a fuzzy number in R and k an ordinary number, k ε R_0^+

For all A ⊂ R:

$$k\ .\ A_\alpha = [k, k]\ (.) \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right]$$

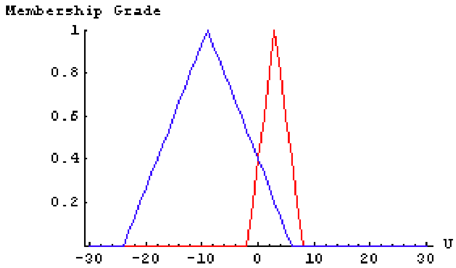
$$= \left[k.\,a_1^{(\alpha)}, k.\,a_2^{(\alpha)}\right]$$

(Or)

For all x ε R :

$$\mu_{k.A}(x) = \mu_A(x/k) \quad ...(2.15)$$

Fuzzy Number with Constant Multiplication



Distributive in R^+ :

We now prove that for all A, B, C $\subset R^+$:

(A (+) B) (.) C = ((A (.) C) (+) ((B (.) .C)

Where

$$A = \left[a_1^{(\alpha)} + a_2^{(\alpha)}\right]$$

$$B = \left[b_1^{(\alpha)} + b_2^{(\alpha)}\right]$$

$$C = \left[c_1^{(\alpha)} + c_2^{(\alpha)}\right]$$

$$(A (+) B) (.) C = \left\{\left[a_1^{(\alpha)} + a_2^{(\alpha)}\right] + \left[a_1^{(\alpha)} + a_2^{(\alpha)}\right] + \left[a_1^{(\alpha)} + a_2^{(\alpha)}\right]\right\}$$

$$= \left[a_1^{(\alpha)} + b_1^{(\alpha)}, a_2^{(\alpha)} + b_2^{(\alpha)}\right] (.) \left[c_1^{(\alpha)} + c_2^{(\alpha)}\right]$$

$$= \left[a_1^{(\alpha)}.c_1^{(\alpha)} + b_1^{(\alpha)}.c_1^{(\alpha)}, a_2^{(\alpha)}.c_2^{(\alpha)} + b_2^{(\alpha)}.c_2^{(\alpha)}\right] - *$$

And

$$(A (.) C) (+) (B (.) C) = \left\{\left(\left[a_1^{(\alpha)}, a_2^{(\alpha)}\right] (.) \left[c_1^{(\alpha)}, c_2^{(\alpha)}\right]\right) (+) \left(\left[b_1^{(\alpha)}, b_2^{(\alpha)}\right] (.) \left[c_1^{(\alpha)}, c_2^{(\alpha)}\right]\right)\right\}$$

$$= \left[a_1^{(\alpha)}.c_1^{(\alpha)} + b_1^{(\alpha)}.c_1^{(\alpha)}, a_2^{(\alpha)}.c_2^{(\alpha)} + b_2^{(\alpha)}.c_2^{(\alpha)}\right] - *$$

From * we have (A (+) B) (.) C = ((A (.) C) (+) ((B (.) .C)

Positive and Non-Negative of Fuzzy Numbers:

A fuzzy number M is called positive, denoted by M > 0, if its membership function $\mu_M(x)$ satisfies $\mu_M(x) = 0$, x$\forall$ < 0. Now according to Definition may write the LR type fuzzy number M symbolically as M = (m, α, β), where m is the mean value of M and a and ß are left and right spreads, respectively. Any crisp number 'a' is a fuzzy number with this form $\tilde{a}$ = (a, 0, 0). Usually, $\tilde{a}$ is a positive (nonnegative) fuzzy number, if and only if a is positive (nonnegative) crisp number.

Definition:

A fuzzy number M is called positive, denoted by M > 0, if its membership function $\mu_M(x)$ satisfies $\mu_M(x) = 0$, x$\forall$ ≤ 0

Definition:

A fuzzy number M is called nonnegative, denoted by M = 0, if its membership function $\mu_M(x)$ satisfies $\mu_M(x) = 0$, $\forall$ x < 0

Role of fuzzy number

Exercise:

1. Let A, B be two fuzzy numbers whose membership functions are given by

$$\mu_A(x) = \begin{cases} 0 & \text{for } x \geq -1 \text{ and } x > 3 \\ (x + 1)/2 & \text{for } -1 < x \leq 1 \\ (3 - x)/2 & \text{for } 1 < x \leq 3 \end{cases} \quad (1)$$

And

$$\mu_B(x) = \begin{cases} 0 & \text{for } x \leq 1 \text{ and } x \geq 5 \\ (x - 1)/2 & \text{for } -1 < x \leq 3 \\ (5 - x)/2 & \text{for } 3 < x \leq 5 \end{cases} \quad (2)$$

Calculate the fuzzy numbers A + B, A – B

Solution:

First calculate fuzzy number A + B, to compute the intervals of confidence, from (1)

$$\alpha = \left[a_1^{(\alpha)} + 1\right]/2 \quad \text{and}$$

$$\alpha = \left[3 - a_2^{(\alpha)}\right]/2$$

Hence, the intervals of confidence

$A_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right]$ (3)

$= [2\alpha - 1, 3 - 2\alpha]$

From (2),

$\alpha = \left[b_1^{(\alpha)} - 1\right]/2$

And

$\alpha = \left[5 - b_2^{(\alpha)}\right]/2$

Hence, the intervals of confidence

$B_\alpha = \left[b_1^{(\alpha)},\ b_2^{(\alpha)}\right]$ (4)

$= [2\alpha + 1, 5 - 2\alpha]$

Adding (3) and (4) gives

$A_\alpha + B_\alpha = \left[a_1^{(\alpha)} + b_1^{(\alpha)},\ a_2^{(\alpha)} + b_2^{(\alpha)}\right]$

$= [2\alpha - 1 + 2\alpha + 1, - 2\alpha + 3 - 2\alpha + 5]$

$= [4\alpha, -4\alpha + 8]$ for $\alpha \,\varepsilon\, [0, 1]$

Thus, $a_1^{(\alpha)} + b_1^{(\alpha)} = 4\alpha$

$a_2^{(\alpha)} + b_2^{(\alpha)} = -4\alpha + 8$

We obtain,

$$\mu_{A\,(+)\,B}(x) = \begin{cases} 0 & \text{for } x \le 0 \text{ and } x > 8 \\ x/4 & \text{for } 0 < x \le 4 \\ (-x + 8)/4 & \text{for } 4 < x \le 8 \end{cases}$$

Next calculate the fuzzy number A – B

From (3)

$A_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right]$

$B_\alpha = \left[b_1^{(\alpha)},\ b_2^{(\alpha)}\right]$

We obtain,

$A_\alpha + B_\alpha = \left[a_1^{(\alpha)} - b_2^{(\alpha)},\ a_2^{(\alpha)} - b_1^{(\alpha)}\right]$

$= [2\alpha - 1 + 2\alpha - 5, - 2\alpha + 3 - 2\alpha - 1]$

$= [4\alpha - 6, -4\alpha + 2]$ for $\alpha \,\varepsilon\, [0, 1]$

$a_1^{(\alpha)} - b_2^{(\alpha)} = 4\alpha - 6$

$a_2^{(\alpha)} - b_1^{(\alpha)} = -4\alpha + 2$

We obtain,

for $x \le -6$ and $x > 2$

$\mu_{A\,(-)\,B}(x) = (x + 6)/4$ for $-6 \le x \le -2$

$(2 - x)/4$ for $-2 \le x \le 2$

Let A, B be two fuzzy numbers whose membership functions are given by

$(x + 2)/2$ for $-2 < x \le 0$

$\mu_A(x) = (2 - x)/2$ for $0 < x < 2$

Otherwise and $(x - 2)/2$ $2 < x \le 4$

$\mu_B(x) = (6 - x)/2$ for $0 < x \le 6$

Otherwise calculate the fuzzy numbers A (.) B, A / B

Solution:

First calculate fuzzy number A / B,

To compute the intervals of confidence,

From (1)

$\alpha = \left[2 + a_1^{(\alpha)}\right]/2$

And

$\alpha = \left[2 - a_2^{(\alpha)}\right]/2$

Hence, the intervals of confidence

$A_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right]$

$= [2\alpha - 2, -2\alpha + 2]$

From (2),

$\alpha = \left[b_1^{(\alpha)}, -2\right]/2$

And

$\alpha = \left[6, -b_2^{(\alpha)}\right]/2$

Hence, the intervals of confidence

$B_\alpha = \left[b_1^{(\alpha)},\ b_2^{(\alpha)}\right]$

$= [2\alpha + 2, -2\alpha + 6]$

From (3) and (4) gives

$A_\alpha\ (:)\ B_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right] (:) \left[b_1^{(\alpha)}, b_2^{(\alpha)}\right]$

$= \left[a_1^{(\alpha)}/b_2^{(\alpha)}, a_2^{(\alpha)}/b_1^{(\alpha)}\right]$

$= [(2\alpha - 2)/(-2\alpha + 6), (-2\alpha + 2)/(2\alpha + 2)]$

Thus,

$a_1^{(\alpha)} / b_2^{(\alpha)} = (2\alpha - 2)/(-2\alpha + 6)$

$a_2^{(\alpha)} / b_1^{(\alpha)} = (-2\alpha + 2)/(2\alpha + 2)$

We obtain,

$\mu_{A\ (:)\ B}(x) = \begin{cases} (6x + 2)/(2x + 2) & \text{for } x \leq 0 \\ (-2x + 2)/(2x + 2) & \text{for } 0 \leq x \leq ½ \\ \text{otherwise} & \end{cases}$

Next calculate the fuzzy number A . B

From (3)

$A_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right]$

$B_\alpha = \left[b_1^{(\alpha)},\ b_2^{(\alpha)}\right]$

We obtain,

$A_\alpha . B_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right] (.) \left[b_1^{(\alpha)},\ b_2^{(\alpha)}\right]$

$= \left[a_1^{(\alpha)}.b_1^{(\alpha)},\ a_2^{(\alpha)}.b_2^{(\alpha)}\right]$

$= [(2\alpha - 2)(2\alpha + 2), (-2\alpha + 2)(-2\alpha + 6)]$

$= [(4\alpha^2 + 4\alpha - 4\alpha - 4), (4\alpha^2 - 12\alpha - 4\alpha + 12)]$

$= [4\alpha^2 - 4, 4\alpha^2 - 16\alpha + 12]$

We have two quadratic equations, to solve the equation namely

$$4\alpha^2 - 4 - x = 0 \tag{5}$$

$$4\alpha^2 - 16\alpha + 12 = 0 \tag{6}$$

From (5)

$4\alpha^2 - 4 - x = 0$

$4\alpha^2 = 4 + x$

$\alpha^2 = \frac{4+x}{4} = 1 + \frac{x}{4}$

$\alpha = \sqrt{1 + \frac{x}{4}}$

From (6) $4\alpha^2 - 16\alpha + 12 = 0$

$\alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-16) \pm \sqrt{(-16)^2 - 4(4)(12 - x)}}{2(4)}$

$= \frac{16 \pm \sqrt{256 - 192 + 16x}}{8}$

$= \frac{16 \pm \sqrt{64 + 16x}}{8}$

$= \frac{8 \pm \sqrt{4 + x}}{2}$

$= 2 \pm \frac{\sqrt{4 + x}}{2}$

We will retain only two roots in the interval [0, 1]

From (5),

$$\alpha = \sqrt{1+\frac{x}{4}}$$

And from (6),

$$\alpha = 2-\frac{\sqrt{4+x}}{2}$$

Finally we obtain,

$$\mu_{A\,(:)\,B}(x) = \begin{cases} \sqrt{1+\frac{x}{4}} & \text{for } -4 \le x \le 0 \\ 2-\frac{\sqrt{4+x}}{2} & \text{for } 0 \le x \le 12 \end{cases}$$

Let A be a fuzzy number whose membership functions is given by

$$\mu_A(x) = \begin{cases} & \text{for } x \le -1 \text{ and } x > 3 \\ (x+1)/2 & \text{for } -1 < x \le 1 \\ (3-x)/2 & \text{for } 1 < x \le 3 \end{cases}$$

Calculate for multiplication of fuzzy number by on ordinary number.

Solution:

The formula for multiplication of fuzzy number by on ordinary number by for all x ε R;

$$\mu_{k.A}(x) = \mu_A(x/k)$$

Now let k = 3,

Using (1) we obtain,

For all x ε R:

$$\mu_{3.A}(x) = \begin{cases} & \text{for } x \le -3 \text{ and } x > 9 \\ (x+1)/2 & \text{for } -3 < x \le 3 \\ (3-x)/2 & \text{for } 3 < x \le 9 \end{cases}$$

Let A, B and C be there fuzzy numbers whose membership functions are given by

$$\mu_A(x) = \begin{cases} (x+2)/2 & \text{for } -2 < x \le 0 \\ (2-x)/2 & \text{for } 0 < x < 2 \end{cases}$$

$$\mu_B(x) = \begin{cases} (x-2)/2 & \text{for } 2 < x \le 4 \\ (6-x)/2 & \text{for } 0 < x \le 6 \end{cases}$$

$$\mu_C(x) = \begin{cases} (x-6)/2 & \text{for } 6 < x \le 8 \\ (10-x)/2 & \text{for } 8 < x \le 8 \end{cases}$$

Find the distributive in R^+

Solution:

To Prove:

(A (+) B) (.) C = (A (.) C) (+) (B (.) C)

First we find (A (+) B) (.) C:

From (1)

$\alpha = \left(a_1^{(\alpha)} + 2\right)/2$

And $\alpha = \left(2 - a_2^{(\alpha)}\right)/2$

Hence the intervals of confidence

$A_\alpha = \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right] = [2\alpha - 2, 2 - 2\alpha]$ . . . (4)

From (2)

$\alpha = \left(b_1^{(\alpha)} - 2\right)/2$

And $\alpha = \left(6 - b_2^{(\alpha)}\right)/2$

Hence the intervals of confidence

$B_\alpha = \left[b_1^{(\alpha)}, b_2^{(\alpha)}\right]$

$= [2\alpha + 2, 6 - 2\alpha]$ . . . (5)

From (3)

$\alpha = \left(c_1^{(\alpha)} - 6\right)/2$

And $\alpha = \left(10 - c_2^{(\alpha)}\right)/2$

Hence the intervals of confidence

$C_\alpha = \left[c_1^{(\alpha)},\ c_2^{(\alpha)}\right]$

$= [2\alpha + 6, 10 - 2\alpha]$ . . . (6)

From (4), (5) and (6) we obtain

$$
\begin{aligned}
A\ (+)\ B &= \left[a_1^{(\alpha)}, a_2^{(\alpha)}\right] (.) \left[b_1^{(\alpha)},\ b_2^{(\alpha)}\right] \\
&= \left[a_1^{(\alpha)},\ b_1^{(\alpha)}\right] + \left[a_2^{(\alpha)},\ b_2^{(\alpha)}\right] \\
&= [2\alpha - 2 + (-2\alpha) +2, 2 - 2\alpha + 6\text{-}2\alpha)] \\
&= [4\alpha, 8 - 4\alpha] \\
(A\ (+)\ B)\ (.)\ C &= [4\alpha, 8 - 4\alpha]\ (.)\ [2\alpha + 6, 10 - 2\alpha] \\
&= [4\alpha\ (.)\ 2\alpha + 6, 8 - 4\alpha\ (.)\ 10 - 2\alpha] \\
&= [8\alpha^2 + 24\alpha, 80 - 16\alpha - 40\alpha + 8\alpha^2] \qquad \text{. . . (7)} \\
A\ (.)\ C &= \left[a_1^{(\alpha)},\ a_2^{(\alpha)}\right]\ (.)\ \left[c_1^{(\alpha)},\ c_2^{(\alpha)}\right] \\
&= \left[a_1^{(\alpha)}.c_1^{(\alpha)},\ a_2^{(\alpha)}.\ c_2^{(\alpha)}\right] \\
&= [2\alpha - 2\ (.)\ 2\alpha + 6, 2 - 2\alpha\ (.)\ 10 - 2\alpha] \\
&= [4\alpha^2 + 12\alpha - 4\alpha - 12, 20 - 4\alpha - 20\alpha + 4\alpha^2] \\
&= [4\alpha^2 + 8\alpha - 12, 4\alpha^2 - 24\alpha - 20] \\
B\ (.)\ C &= \left[b_1^{(\alpha)},\ b_2^{(\alpha)}\right]\ (.)\ \left[c_1^{(\alpha)},\ c_2^{(\alpha)}\right] \\
&= \left[b_1^{(\alpha)}.c_1^{(\alpha)},\ b_2^{(\alpha)}.\ c_2^{(\alpha)}\right] \\
&= [2\alpha + 2\ (.)\ 2\alpha + 6, 6 - 2\alpha\ (.)\ 10 - 2\alpha] \\
&= [4\alpha^2 + 12\alpha + 4\alpha + 12, 60 - 12\alpha - 20\alpha + 4\alpha^2] \\
&= [4\alpha^2 + 16\alpha + 12, 4\alpha^2 - 32\alpha + 60] \\
(A(.)C)\ (+)\ (B(.)C) &= [4\alpha^2 + 8\alpha - 12, 4\alpha^2 - 24\alpha - 20] + \\
&\quad [4\alpha^2 + 16\alpha + 12, 4\alpha^2 - 32\alpha + 60] \\
&= [8\alpha^2 + 24\alpha, 80 - 16\alpha - 40\alpha + 8\alpha^2] \qquad \text{. . . (8)}
\end{aligned}
$$

From (7) and (8)

$$(A\ (+)\ B)\ (.)\ C = (A\ (.)\ C)\ (+)\ (B\ (.)\ C)$$

IV. CONCLUSIONS

Here we have seen the concept of fuzzy number, interval of confidence, level of presumption, Arithmetic operators of fuzzy numbers. It describes their more specific application to typical problems of mechanical engineering design. In subjective terms, the designer may express the various dimension of the design artefact by appropriate fuzzy numbers representing linguistic description. It also rectifies the damages in building to happen often. Moreover, they are applied in civil engineering, computer engineering etc.

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