To Study Fuzzy on algebra

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Abstract — *Fuzzy on algebra, proposed to study provides various types of results on (Q, L) fuzzy ideals of ring, & (Q, L) fuzzy normal ideals of ring so that it will be easier to understand the concepts in the material, we have given the list of references from where we have collected the details for this dissertation. I hope that whatever the things that are discussed in the dissertation will be clear to the reader.*

Keywords — *Fuzzy on algebra, dissertation,fuzzy normal ideals.*

I. INTRODUCTION

Fuzzy subset concept was introduced by Zadeh in 1965 to represent manipulate data and information possessing non-statistical uncertainties. It was specifically designed to mathematically represent uncertainty and vagueness and to provide formalized tools for dealing with the imprecision intrinsic to many problems. The first publication in fuzzy subset theory by Zadeh (1965) and then by Goguen (1967, 1969) show the intention of the authors to generalize the classical set. In classical set theory, a subset A of a set X can be defined by its characteristic function A: X → {0, 1} which is defined by A(x) = 0, if x A and A(x) = 1, if x A. The mapping may be represented as a set of ordered pairs{(x, A(x))} with exactly one ordered pair present for each element of X. The first element of the ordered pair is an element of the set X and the second is its value in {0, 1}. The value ‘0’ is used to represent non-membership and the value ‘1’ is used to represent membership of the element in A. The truth or falsity of the statement “x is in A” is determined by the ordered pair. The statement is true, if the second element of the ordered pair is ‘1’, and the statement is false, if it is ‘0’.The following papers have motivated us to work on this thesis. In 1971 Azriel Rosenfeld had introduced fuzzy subgroups. Antony.J.M and Sharewood.H. redefined fuzzy groups and investigated some properties of them. Choudhury. F. P, Chakraborty A.B. and Khare S.S gave a note on fuzzy subgroups and fuzzy homomorphism. Mustafa Akgul has produced some properties of fuzzy groups. Dixit V.N., Rajeshkumar, Naseem Ajmal have introduced the concept of level subgroups and union of fuzzy subgroups and obtained some results. Gopalakrishnamoorthy.G defined the concept of anti-homomorphism in groups and obtained some results. Palaniappan.N and Arjunan.K have defined the homomorphism and anti-homomorphism on a fuzzy and anti-fuzzy ideals. G.Jayanthi, M.Simaringa, K.Arjunan have introduced and defined notes on (Q, L)-fuzzy ideals of a ring & homomorphism in (Q, L)-fuzzy normal ideals of a ring.

II. BASIC DEFINITIONS AND EXAMPLES

2.1 Definition:

Let X be a non-empty set and L = (L, ≤) be a lattice with least element 0 and greatest element 1 and Q be a non-empty set. A **(Q,L)-fuzzy subset** A of X is a function A: X×Q→L.

2.2 Definition:

Let (R, +, ·) be a ring and Q be a non empty set. A (Q, L)-fuzzy subset A of R is said to be a (Q, L)-**fuzzy ideal (QLFI)** of R if thefollowing conditions are satisfied:

(i) $A(x+y, q) \geq A(x, q) \wedge A(y, q)$,

(ii) $A(-x, q) \geq A(x, q)$,

(iii) $A(xy, q) \geq A(x, q) \wedge A(y, q)$, for all x and y in R and q in Q.

2.3 Definition:

Let (R, +, ·) and (R^{l}, +, ·) be any two rings and Q be a non empty set. Let $f : R \rightarrow R^{l}$ be any function and A be a (Q, L)-fuzzy ideal in R, V be a (Q, L)-fuzzy ideal in f (R) = R^{l}, Defined by

$V(y, q) = \sup_{x \in f^{-1}(Y)} A(x, q)$, for all x in R and y in R^{l} and q in Q.

Then A is called a pre-image of V under f and is denoted by $f^{-1}(V)$.

2.4 Definition:

Let (R, +, ·) be a ring and Q be a non empty set. A (Q, L)-fuzzy subset A of R is said to be a (Q, L)-**anti-fuzzy ideal (QLAFI)** of R if the following conditions are satisfied:

(i) $A(x+y, q) \leq A(x, q) \wedge A(y, q)$,(ii) $A(-x, q) \leq A(x, q)$,

(iii) $A(xy, q) \leq A(x, q) \wedge A(y, q)$, for all x and y in R and q in Q.

2.5 Definition:

Let A be a (Q, L)-fuzzy ideal of a ring (R, +, ·) and a in R.Then the **pseudo (Q, L)-fuzzy coset** $(aA)^p$ is defined by $((aA)^p)(x,q) = p(a)A(x, q)$, for every x in R and for some p in P and q in Q.

2.6 Definition:

Let A be a (Q, L)-fuzzy ideal of a ring (R, +, ·). For any a in R, a+A defined by $(a+A)(x, q) = A(x-a, q)$, for every x in R and q in Q, is called a **(Q, L)-fuzzy coset** of R.

Some Properties:

2.7 Theorem:

Let (R, +, ·) and (R^{l}, +, ·) be any two rings and Q be a non-empty set. The homomorphic image of a (Q, L)-fuzzy ideal of R is a (Q, L)-fuzzy ideal of R^{l}.

Proof:

Let (R, +, ·) And (R^{l}, +, ·) be any two rings and Q be a non-emptyset and f : R→R^{l} be a homomorphism.That is $f(x+y) = f(x)+f(y)$, $f(xy) = f(x)f(y)$,for all x and y in R. Let A be a (Q, L)-Fuzzy Ideal of R. Let V be the homomorphism image of A under F. We have to prove that V is a (Q, L)-Fuzzy Ideal of F(R) = R^{l}.

Now, for f(x) and f(y) in R^{l}, We have, $V(f(x)+f(y), q) = V(f(x+y), q)$

$\geq A(x+y, q)$

$\geq A(x, q) \wedge a(y, q)$

$\geq V(f(x), q) \wedge V(f(y), q)$. For f(x) in R^{l},

We have, $V(-f(x), q) = V(f(-x), q)$

$\geq A(-x, q) \wedge A(x, q)$

$\geq V(-f(x), q) \wedge V(f(x), q)$.

And, $V(f(x) f(y), q) = V(f(xy), q)$

$\geq A(xy, q)$

$\geq A(x, q) \wedge A(y, q)$

$\geq V(f(x), q) \wedge V(f(y), q)$. Hence V is a (Q, L)-fuzzy ideal of a ring R^{l}.

2.8 Theorem:

Let (R, +, ·) and (R^{l}, +, ·) be any two rings and Q be a non-empty set. The homomorphic pre-image of a (Q, L)-fuzzy ideal of R^{l} is a (Q, L)-fuzzy ideal of R.

Proof:

Let (R, +, ·) and (R^{l}, +, ·) be any two rings and Q be a non-empty set and f: R→R^{l} be a homomorphism. That is $f(x+y) = f(x) +f(y)$, $f(xy) = f(x) f(y)$, for all x and y in R.

Let V be a (Q, L)-fuzzy ideal of f(R) = R^{l} and A be the pre-image of V under f. We have to prove that A is a (Q, L)-fuzzy ideal of R.

Let x and y in R and q in Q. Then, $A(x+y, q) = V(f(x+y), q) = V(f(x)+f(y), q) \wedge V(f(x), q) \wedge V(f(y), q)$

$= A(x, q) \wedge A(y, q)$, $\Longrightarrow A(x+y, q) \geq A(x, q) \wedge A(y, q)$, for x and y in R and q in Q.

And, $A(-x, q) = V(f(-x), q) = V(-f(x), q) \wedge V(f(x), q) = A(x, q)$, $\Longrightarrow A(-x, q) \wedge A(x, q)$, for x in R and q in Q.

And, $A(xy, q) = V(f(xy), q)$

$= V(f(x) f(y), q) \wedge V(f(x), q) \wedge V(f(y), q)$

$= A(x, q) \wedge A(y, q)$,

$\Longrightarrow A(xy, q) \geq A(x, q) \wedge A(y, q)$, for x and y in R and q in Q.

Hence A is a (Q, L)-fuzzy ideal of the ring R.

2.9 Theorem:

Let (R, +, ·) and (R^{l}, +, ·) be any two rings and Q be a non-empty set. The anti-homomorphic image of a (Q, L)-fuzzy ideal of R is a (Q, L)-fuzzy ideal of R^{l}.

Proof:

Let (R, +, ·) and (R^{l}, +, ·) be any two rings and Q be a non-empty set and f: R → R^{l} be a anti-homomorphism. That is f(x+y) = f(y)+f(x), f(xy) = f(y)f(x), for all x and y in R and q in Q.

Let A be a (Q, L)-fuzzy ideal of R. Let V be the homomorphic image of A under f. We have to prove that V is a (Q, L)-fuzzy ideal of f(R) = R^{l}.

Now, let f(x) and f(y) in R^{l},

We have, $V(f(x)+f(y), q) = V(f(y+x), q)$

$\geq A(y+x, q)$

$\geq A(x, q) \wedge A(y, q)$,

$\Rightarrow V(f(x) + f(y), q) \geq V(f(x), q) \wedge V(f(y), q)$. For x in R and q in Q,

$V(-f(x), q) = V(f(-x), q)$

$\geq A(-x, q) \wedge A(x, q)$,

$\Rightarrow V(-f(x), q) \wedge V(f(x), q)$, for x in R and q in Q.

And, $V(f(x)f(y), q) = V(f(yx), q)$

$\geq A(yx, q)$

$\geq A(x, q) \wedge A(y, q)$,

$\Rightarrow V(f(x)f(y), q) \geq V(f(x), q) \wedge V(f(y), q)$. Hence V is a (Q, L)-fuzzy ideal of R^{l}.

2.10 Theorem:

Let (R, +, ·) and (R^{l}, +, ·) be any two rings and Q be a non-empty set. The anti-homomorphic pre-image of a (Q, L)-fuzzy ideal of R^{l} is a (Q, L)-fuzzy ideal of R.

Proof:

Let (R, +, ·) and (R^{l}, +, ·) be any two rings and Q be a non-empty set and f: R → R^{l} be a anti-homomorphism.

That is f(x + y) = f(y) + f(x), f(xy) = f(y)f(x), for all x and y in R and q in Q.

Let V be a (Q, L)-fuzzy ideal of f(R) = R^{l}. Let A be the pre-image of V under f. We have to prove that A is a (Q, L)-fuzzy ideal of R.

Let x and y in R and q in Q.

Now, $A(x+y, q) = V(f(x+y), q)= V(f(y)+f(x), q) \wedge V(f(x), q) \wedge V(f(y), q) = A(x, q) \wedge A(y, q)$,

$\Rightarrow A(x+y, q) \geq A(x, q) \wedge A(y, q)$, for all x and y in R and q in Q.

And, $A(-x, q) = V(f(-x), q)$

$= V(-f(x), q) \wedge V(f(x), q)$

$= A(x, q)$,

$\Rightarrow A(-x, q) \wedge A(x, q)$, for x in R and q in Q.

Now, $A(xy, q) = V(f(xy), q)$

$= V(f(y)f(x), q) \wedge V(f(x), q) \wedge V(f(y), q)$

$= A(x, q) \wedge A(y, q)$,

$\Rightarrow A(xy, q) \geq A(x, q) \wedge A(y, q)$, forall x and in R and q in Q. Hence A is a (Q, L)-fuzzy ideal of the ring R.

In the following Theorem is the composition (◦) operation of functions:

2.11 Theorem:

Let A be a (Q, L)-fuzzy ideal of a ring H and f is an isomorphism from a ring R onto H. Then A◦f is a (Q, L)-fuzzy ideal of R.

Proof:

Let x and y in R and A be a (Q,L)-fuzzy ideal of the ring Hand Q be a non-empty set.

Then we have,

$(A \circ f)(x-y, q) = A(f(x-y), q)$

$= A(f(x)-f(y), q)$

$\geq A(f(x), q) \wedge A(f(y), q)$

$\geq (A \circ f)(x, q) \wedge (A \circ f)(y, q)$,

$\Rightarrow (A \circ f)(x-y, q) \geq (A \circ f)(x, q) \wedge (A \circ f) (y, q)$.

Then we have,

$(A \circ f)(xy, q) = A(f(xy), q)$

$= A(f(x)f(y), q)$
$\geq A(f(x), q) \wedge A(f(y), q)$
$\geq (A\circ f)(x, q) \wedge (A\circ f)(y, q),$

$\Rightarrow (A\circ f)(xy, q) \geq (A\circ f)(x, q) \wedge (A\circ f)(y, q).$
Therefore $(A\circ f)$ is a (Q, L)-fuzzy ideal of a ring R.

2.12 Theorem:Let A be a (Q, L)-fuzzy ideal of a ring H and f is an anti-isomorphism from a ring R onto H. Then $A\circ f$ is a (Q, L)-fuzzy ideal of R.

Proof:

Let x and y in R and A be a (Q, L)-fuzzy ideal of the ringH and Q be a non-empty set.
Then we have, $(A\circ f)(x-y, q) = A(f(x-y), q)$
$= A (f(y) - f(x), q)$
$\geq A(f(x), q) \wedge A(f(y), q)$
$\geq (A\circ f) (x, q) \wedge (A\circ f)(y, q),$

$\Rightarrow (A\circ f)(x- y, q) \geq (A\circ f)(x, q) \wedge (A\circ f)(y, q).$
Then we have,
$(A\circ f)(xy, q) = A(f(xy), q)$
$= A (f(y) f(x), q)$
$\geq A(f(x), q) \wedge A(f(y), q)$
$\geq (A\circ f)(x, q) \wedge (A\circ f)(y, q),$

$\Rightarrow (A\circ f)(xy, q) \geq (A\circ f)(x, q) \wedge (A\circ f)(y, q).$
Therefore $(A\circ f)$ is a (Q, L)-fuzzy ideal of a ring R.

2.13 Theorem:

Let (R, +, $\cdot$) be a ring and Q be a non-empty set. A is a (Q, L)-fuzzy ideal of R $\Leftrightarrow A^c$ is a (Q, L)-anti-fuzzy ideal of R.

Proof:

Suppose A is a (Q, L)-fuzzy ideal of R. For all x and y in Rand q in Q, we have, $A(x – y, q) \geq A(x, q) \wedge A(y, q), \Rightarrow 1– A^c(x – y, q) \geq \{1– A^c(x, q)\} \wedge \{1– A^c (y, q)\},$

$\Rightarrow A^c(x–y, q) \leq 1–\{1–A^c(x, q)\wedge 1–A^{c} (y, q)\},$

$\Rightarrow A^c(x–y, q) \leq Ac(x)\wedge A^c (y).$

Also, $A (xy, q) \geq A(x, q) \wedge A(y, q),$

$\Rightarrow 1–A^c(xy, q) \geq \{1– A^c(x, q)\} \wedge\{1–A^c(y, q) \},$

$\Rightarrow A^c(xy, q) \leq 1– \{1– A^c(x, q) \wedge 1– A^c (y, q) \},$

$\Rightarrow A^c (xy, q) \leq A^c(x, q) \wedge A^c (y, q).$

Thus A^c is a (Q, L)-anti-fuzzy ideal of R. Converse also can be proved similarly.

2.14 Theorem:

Let A be a (Q, L)-fuzzy ideal of a ring R, then the pseudo (Q, L)-fuzzy coset $(aA)^p$ is a (Q, L)-fuzzy ideal of the ring R, for every a in R.

Proof:

Let A be a (Q, L)-fuzzy ideal of the ring R. For every x andy in R and q in Q,
we have, $((aA)^p)(x-y, q) = p(a)A(x-y, q)$
$\geq p(a) \{A(x, q) \wedge A(y, q)\}$
$= p(a)A(x, q) \wedge p(a)A(y, q)$
$= ((aA)^p)(x,q)\wedge ((aA)^p)(y,q),$

Therefore, $((aA)^p)(x-y, q) \geq ((aA)^p)(x, q)\wedge ((aA)^p) (y, q)$, for x, y in R and q in Q.
And, $((aA)^p)(xy, q) = p(a)A(xy, q)$
$\geq p(a)\{A(x, q) \wedge A(y, q)\}$

$= p(a)A(x, q) \wedge p(a)A(y, q)$

$= ((aA)^p)(x,q) \wedge ((aA)^p)(y,q).$

Therefore, $((aA)^p)(xy, q) \geq ((aA)^p)(x, q) \wedge ((aA)^p)(y, q)$, for x, y in R and q in Q.

Hence $(aA)^p$ is a (Q, L)-fuzzy ideal of the ring R.

2.15 Theorem:

Let (R, +, .) be a ring and Q be a non-empty set. If A is a (Q, L)-fuzzy ideal of R, then $x+A = y+A \Leftrightarrow A(x-y, q) = A(0, q)$, where 0 is the identity element. In that case $A(x, q) = A(y, q)$.

Proof:

Given A is a (Q, L)-fuzzy ideal of R.

Suppose that $x + A = y + A$,

$\Rightarrow (x+A)(x, q) = (y+A)(x, q),$

$\Rightarrow A(x-x, q) = A(x - y, q),$

$\Rightarrow A(0, q) = A(x- y, q).$

Conversely, assume that $A(x- y, q) = A(0, q)$,

Then, $(x+A)(z, q) = A(z-x, q)$

$= A(z-x+y-y, q)$

$\geq A(z-y, q) \wedge A(0, q)$

$= A(z-y, q)$

$= (y+A)(z, q),$

$\Rightarrow (x+A)(z, q) \geq (y+A)(z, q)$ ------------(1).

Now, $(y+A)(z, q) = A(z - y, q)$

$= A(z-y+x- x, q)$

$\geq A(z- x, q) \wedge A(0, q)$

$= A(z-x, q)$

$= (x+A)(z, q),$

$\Rightarrow (y+A)(z, q) \geq (x+A)(z, q)$ ------------(2).

From (1) and (2) we get, $x + A = y + A$.Hence the theorem proved.

III. (Q, L) -FUZZY NORMAL IDEALS OF A RING.

3.1 Definition:

Let A and B be any two (Q, L) -fuzzy subsets of sets R and H,respectively. The product of A and B, denoted by A×B, is defined as

$A\times B = \{\langle ((x, y), q), A\times B((x, y), q)\rangle\}$for all x in R and y in H and q in Q.

Where $A\times B((x, y), q) = A(x, q) \wedge B(y, q)$.

3.2 Definition:

Let A be a (Q, L) -fuzzy subset in a set S, the**strongest (Q, L) -fuzzy relation** on S, that is a (Q, L) -fuzzy relation V with respect to A given by

$V((x, y), q) = A(x, q) \wedge A(y, q)$, for all x and y in S and q in Q.

3.3 Definition:

Let (R, +, ·) be a ring and Q be a non-empty set. A (Q, L) -fuzzy idealA of R is said to be a (Q, L) -**fuzzy normal ideal** (QLFNI) of R if $A(xy,q) = A(yx, q)$, for all x and y in R and q in Q.

3.4 Definition:

A (Q, L) -fuzzy subset A of a set X is said to be **normalized** if thereexists an element x in X such that $A(x, q) = 1$.

3.5 ***Definition:*** Let A be a (Q, L) -fuzzy subset of X. For α in L, a **Q-level subset** ofA corresponding to α is *the set* $a^{\alpha} = \{x \in x: a(x, q) \geqslant \alpha\}$.

Some properties:

3.6 Theorem:

Let (R, +, ·) be a ring and Q be a non-empty set. If A and B are two (Q,L) fuzzy normal ideals of R, then their intersection A∩B is a (Q,L) -fuzzy normal ideal of R.

Proof: Let C = A∩B and C = {⟨(x, q), C (x, q)⟩/ x in R and q in Q},where C(x, q) =A(x, q) ∧ B(x, q). Then, Clearly C is a (Q, L) -fuzzy ideal of R, since A and B are two (Q, L)-fuzzy ideals of R.

And, C (xy, q) = A(xy, q) ∧ B(xy, q)
= A(yx, q) ∧ B(yx, q)
= C(yx, q).

Therefore, C(xy, q) = C(yx, q), for all x and y in R and q in Q. Hence A∩B is a (Q, L) -fuzzy normal ideal of the ring R.

3.7 Theorem:

Let (R, +, ·) be a ring and Q be a non-empty set. The intersection of afamily of (Q, L) -fuzzy normal ideals of R is a (Q, L) -fuzzy normal ideal of R.

Proof:Let{ A_i } $i\in I$ be a family of (Q, L) -fuzzy normal ideals of R and A= $\bigcap_{i\in I}$ $A_{i.}$, Then for x and y in R and q in Q, clearly the intersection of a family of (Q, L) -fuzzy ideals of the ring R is a (Q, L) -fuzzy ideal of a ring R.

Now, A (xy, q) = $\inf_{i\in I} A_i$ (xy, q) = $\inf_{i\in I} A_i$ (yx,q)= A (yx, q).

Therefore, A (xy, q) = A (yx, q), for all x and yin R and q in QHence the intersection of a family of (Q, L) -fuzzy normal ideals of a ring R is a (Q, L) -fuzzy normal ideal of R.

3.8 Theorem:

A (Q, L) -fuzzy ideal A of a ring R is normalized ⇔ A (e, q)= 1, where e is the identity element of R and q in Q.

Proof:

If A is normalized, then there exists x in R such that A (x, q) = 1, but byproperties of a (Q, L) -fuzzy ideal A of R, A (x, q) ≤ A (e, q), for every x in R and q in Q.

Since A (x, q) = 1and A (x, q) ≤ A (e, q), 1 ≤ A (e, q).But 1 ≥ A (e, q). Hence A (e, q) = 1. Conversely, if A (e, q) =1,Then by the definition of normalized (Q, L) -fuzzy subset, A is normalized.

3.9 Theorem:

Let A and B be (Q, L) -fuzzy ideals of the rings R and H, respectively.If A and B are (Q, L) -fuzzy normal ideals, then A×B is a (Q, L) -fuzzy normal ideal of R×H.

Proof:

Let A and B be (Q, L) -fuzzy normal ideals of the rings R and H respectively.Clearly A×B is a (Q, L) -fuzzy ideal of R×H, since A and B are (Q, L) -fuzzy ideals R and H. Let x_1 and x_2 be in R, y_1 and y_2 be in H and q in Q. Then (x_1, y_1) and (x_2, y_2) are in R×H.

Now, A×B [(x_1, y_1) (x_2, y_2), q] = A×B ((x_1x_2, y_1y_2), q)
= A(x_1x_2, q) ∧ B(y_1y_2, q)
= A(x_2 x_1, q) ∧ B(y_2 y_1, q)
= A×B ((x_2x_1, y_2y_1), q)
= A×B [(x_2, y_2) (x_1 , y_1), q]. Therefore, A×B [(x_1, y_1) (x_2, y_2), q] = A×B [(x_2, y_2) (x_1, y_1), q].Hence A×B is a (Q,L)- fuzzy normal ideal of R×H.

3.10 Theorem:

Let A and B be (Q, L) -fuzzy subsets of the rings R and H, respectivelyand A×B is a (Q, L) -fuzzy normal ideal of R×H. Then the following are true:

(i) if A (x, q) $\leq$ B (e$^{|}$, q), then A is a (Q, L) -fuzzy normal ideal of R.

(ii) if B (x, q) $\leq$ A (e, q), then B is a (Q, L) -fuzzy normal ideal of H.either A is a (Q, L) -fuzzy normal ideal of R or B is a (Q, L) -fuzzy normal ideal of H.

Proof:

Let A×B be a (Q, L) -fuzzy normal ideal of R×H and x, y in R and e$^{|}$in H. Then (x, e$^{|}$) and (y, e$^{|}$) are in R×H. Clearly A×B is a (Q, L) -fuzzy ideal of R×H.

Now, using the property that A (x, q) $\leq$ B (e$^{|}$, q), for all x in R, clearly A is a (Q, L) –fuzzy ideal of R.

Now, A(xy, q) = A(xy, q) ∧ μ_B (e$^{|}$e$^{|}$, q)
= A×B (((x y), (e$^{|}$ e$^{|}$)), q)
= A×B [(x, e$^{|}$)(y, e$^{|}$), q]
= A×B [(y, e$^{|}$)(x, e$^{|}$) ,q]
= A×B [((yx), (e$^{|}$e$^{|}$)), q]
= A(yx, q) ∧ B(e$^{|}$e$^{|}$, q)

$= A(yx, q)$.
Therefore, $A(xy, q) = A(yx, q)$, for all x and y in R and q in Q. Hence A is a (Q, L) -fuzzy normal ideal of R. Thus (i) is proved.

Now, using the property that $B(x, q) \leq A(e, q)$, for all x in H, let x and y in H and e in R. Then (e, x) and (e, y) are in R×H. Clearly B is a (Q, L) -fuzzy ideal of H.

Now, $B(xy, q) = B(xy, q) \wedge A(ee, q)$
$= A(ee, q) \wedge B(xy, q)$
$= A\times B\,(((ee), (xy)), q)$
$= A\times B\,[(e, x)\,(e, y), q]$
$= A\times B\,[(e, y)\,(e, x), q]$
$= A\times B\,[((ee), (yx)), q]$
$= A(ee, q) \wedge B(yx, q)$
$= B(yx, q)$.

Therefore, $B(xy, q) = B(yx, q)$, for all x and y in H and q in Q. Hence B is a (Q, L) -fuzzy normal ideal of H. Thus (ii) is proved. (iii) is clear.

3.11 Theorem:

Let A be a (Q, L) -fuzzy subset of a ring R and V be the strongest (Q,L) -fuzzy relation of R with respect to A. Then A is a (Q, L) -fuzzy normal ideal of R $\Leftrightarrow$V is a (Q, L) -fuzzy normal ideal of R×R.

Proof:

Suppose that A is a (Q, L) -fuzzy normal ideal of R. Then for any $x = (x_1, x_2)$and $y = (y_1, y_2)$ are in R×R and q in Q. Clearly V is a (Q, L) -fuzzy ideal of R×R.

We have, $V(xy, q) = V[(x_1, x_2)\,(y_1, y_2), q]$
$= V((x_1y_1, x_2y_2), q)$
$= A((x_1y_1), q) \wedge A((x_2y_2), q)$
$= A((y_1x_1), q) \wedge A((y_2x_2), q)$
$= V((y_1x_1, y_2x_2), q)$
$= V[(y_1, y_2)\,(x_1, x_2), q]$
$= V(yx, q)$.

Therefore, $V(xy, q) = V(yx, q)$, for all x and y in R×R and q in Q. This proves that V is a (Q, L) -fuzzy normal ideal of R×R.

Conversely, assume that V is a (Q, L) -fuzzy normal ideal of R×R, then for any $x = (x_1, x_2)$ and $y = (y_1, y_2)$ are in R×R,we have,

$A(x_1y_1, q) \wedge A(x_2y_2, q) = V((x_1y_1, x_2y_2), q)$
$= V[(x_1, x_2)\,(y_1, y_2), q]$
$= V(xy, q)$
$= V(yx, q)$
$= V[(y_1, y_2)(x_1, x_2), q]$
$= V((y_1x_1, y_2x_2), q)$
$= A(y_1x_1\,, q) \wedge A(y_2x_2, q)$.

If we put $x_2 = y_2 = e$, where e is the identity element of R. We get, $A(x_1y_1,q) = A(y_1x_1, q)$, for all x_1 and y_1 in R and q in Q. Hence A is a (Q, L) -fuzzy normal ideal of R.

3.12 Theorem:

Let (R, +, ·) and (R^{l}, +, ·) be any two rings and Q be a non- empty set.The homomorphic image of a (Q, L) -fuzzy normal ideal of R is a (Q, L) -fuzzy normal ideal of R^{l}.

Proof:

Let f: R→R^{l}be a homomorphism. Let A be a (Q, L) -fuzzy normal ideal ofR.

We have to prove that V is a (Q, L) -fuzzy normal ideal of f(R) = R^{l}.

Now, for f(x) and f(y) in R^{l}, we have clearly V is a (Q, L) -fuzzy ideal of a ring f(R) = R^{l}, since A is a (Q, L) -fuzzy ideal of a ring R.

Now, $V(f(x)f(y), q) = V(f(xy), q) \geq A(xy, q)$
$= A(yx, q) \leq V(f(yx),q)$
$= V(f(y)\,f(x), q)$,
$\Rightarrow V(f(x)\,f(y), q) = V(f(y)f(x), q)$. Hence V is a (Q, L) -fuzzy normal ideal of the ring R^{l}.

3.13 Theorem:

Let (R, +, ·) and (R^{l}, +, ·) be any two rings and Q be a non-empty set.The homomorphic pre-image of a (Q, L) -fuzzy normal ideal of R^{l} is a (Q, L) -fuzzy normal ideal of R.

Proof:

Let f: R→R^{l}be a homomorphism. Let V be a (Q, L) -fuzzy normal ideal of f(R)= R^{l} . We have to prove that A is a (Q, L) -fuzzy normal ideal of R.

Let x and y in R and q in Q. Then, clearly A is a (Q, L) -fuzzy ideal of the ring R, since V is a (Q, L) -fuzzy ideal of the ring R^{l}.

Now, A(xy, q) = V(f(xy), q)
= V(f(x)f(y), q)
= V(f(y)f(x), q)
= V(f(yx), q)
= A (yx, q),

⟹ A (xy, q) = A (yx, q), for x and y in R and q in Q.Hence A is a (Q, L) -fuzzy normal ideal of the ring R.

3.14 Theorem:

Let (R, +, ·) and (R^{l}, +, ·) be any two rings and Q be a non-empty set.The anti-homomorphic image of a (Q, L) -fuzzy normal ideal of R is a (Q, L) -fuzzy normal ideal of R^{l}.

Proof:

Let f: R→R^{l}be an anti-homomorphism. Let A be a (Q, L) -fuzzy normalideal of R. We have to prove that V is a (Q, L) -fuzzy normal ideal of f(R) = R^{l}.

For f(x) and f(y) in R^{l}, clearly V is a (Q, L) -fuzzy ideal of R^{l}, since A is a (Q, L) -fuzzy ideal of R.

Now, V(f(x)f(y), q) = V(f(yx), q)
≥ A(yx, q)
= A(xy, q)
≤ V(f(xy), q)
= V(f(y)f(x), q),

⟹ V(f(x)f(y), q) = V(f(y)f(x), q).

Hence V is a (Q, L) -fuzzy normal ideal of the ring R^{l}.

3.15 Theorem:

Let (R, +, ·) and (R^{l}, +, ·) be any two rings and Q be a non- empty set.The anti-homomorphic pre-image of a (Q, L) -fuzzy normal ideal of R^{l} is a (Q, L) - fuzzy normal ideal of R.

Proof:

Let f: R→R^{l}be anti-homomorphism. Let V be a (Q, L) -fuzzy normal idealof f(R) = R^{l} . We have to prove that A is a (Q, L) -fuzzy normal ideal of R.

Let x and y in R and q in Q, we have clearly A is a (Q, L) - fuzzy ideal of R, since V is a (Q, L) - fuzzy ideal of R^{l}.

Now, A(xy, q) = V(f(xy), q)
= V(f(y)f(x), q)
= V(f(x)f(y), q)
= V(f(yx), q)
= A(yx, q),

⟹ A (xy, q) = A (yx, q), for x and y in R and q in Q.

Hence A is a (Q, L) -fuzzy normal ideal of the ring R.

3.16 Theorem:

Let A be a (Q, L) -fuzzy normal ideal of a ring H and f is aisomorphism from a ring R onto H. Then A◦f is a (Q, L) -fuzzy normal ideal of R.

Proof:

Let x and y in R and A be a (Q, L) -fuzzy normal ideal of a ring H. Thenclearly (A◦f) is a (Q, L) -fuzzy ideal of the ring R.

Then we have, (A◦f) (xy, q) = A(f(xy), q)
= A(f(x)f(y), q)
= A(f(y)f(x), q)
= A(f(yx), q)
= (A◦f) (yx, q),

⟹ (A◦f) (xy, q) = (A◦f) (yx, q), for x and y in R and q in Q.

Hence A◦f is a (Q, L) -fuzzy normal ideal of the ring R.

3.17 Theorem:
Let A be a (Q, L) -fuzzy normal ideal of a ring H and f is an anti-isomorphism from a ring R onto H. Then A∘f is a (Q, L) -fuzzy normal ideal of R.
Proof:
Let x and y in R and A be a (Q, L) -fuzzy normal ideal of a ring H. Thenclearly (A∘f) is a (Q, L) -fuzzy ideal of the ring R.
Then we have, (A∘f) (xy, q) = A(f (xy), q)
= A(f(y)f(x), q)
= A(f(x)f(y), q)
= A(f(yx), q)
= (A∘f) (yx, q), ⟹ (A∘f) (xy, q) = (A∘f) (yx, q), for x and y in R and q in Q.Hence A∘f is (Q, L) -fuzzy normal ideal of the ring R.

3.18 Theorem:
Let A be a (Q, L) -fuzzy normal ideal of a ring R, then the pseudo(Q,L) -fuzzy coset $(aA)^p$ is a (Q, L) -fuzzy normal ideal of the ring R, for a in R.
Proof:
Let A be a (Q, L) -fuzzy normal ideal of a ring R. For every x and y in R and qin Q, we have, clearly $(aA)^p$ is a (Q, L) -fuzzy ideal of the ring R
And, $((aA)^p)(xy, q) = p(a)A(xy, q)$
$= p(a)A(yx, q)$
$= ((aA)^p)(yx, q)$.
Therefore, $((aA)^p)(xy, q) = ((aA)^p)(yx, q)$, for x and y in R and q in Q.
Hence $(aA)^p$ is a (Q, L) -fuzzy normal ideal of the ring R.

3.19 Theorem: Let A be a (Q, L) -fuzzy ideal of a ring R. Then for α in L such that $\alpha \leq A(e, q)$, A^{α} is a ideal of R.

Proof: For all x and y in A^{α}, we have, $A(x, q) \geq \alpha$ and $A(y, q) \geq \alpha$
Now, $A(x-y, q) \geq A(x, q) \wedge A(y, q)$
$\geq \alpha \wedge \alpha$
$= \alpha$,
⟹, $A(x-y, q) \geq \alpha$
And, $A(xy, q) \geq A(x, q) \vee A(y, q)$
$\geq \alpha \vee \alpha = \alpha$,
⟹, $A(xy, q) \geq \alpha$
Therefore, $A(x-y, q) \geq \alpha$, $A(xy, q) \geq \alpha$, we get x-y and xy in A^{α}.Hence A^{α} is a ideal of R.

3.20 Theorem:

Let A be a (Q, L) -fuzzy ideal of a ring R. Then two Q-level ideals A^{α_1}, A^{α_2} and α_1 and α_2 in L and $\alpha_1 \leq A(e, q)$, $\alpha_2 \leq A(e, q)$ with $\alpha_2 < \alpha_1$ of A are equal ⟺there is no x in R such that $\alpha_1 > A(x, q) > \alpha_2$.
Proof: Assume that $A^{\alpha_1} = A^{\alpha_2}$.Suppose there exists an x∈R such that $\alpha_1 > A(x, q) > \alpha_2$. Then $A^{\alpha_1} \subseteq A^{\alpha_2}$, ⟹ x belongs to A^{α_2}, but not in A^{α_1}. This is a contradiction to $A^{\alpha_1} = A^{\alpha_2}$. Therefore, there is no x∈R such that $\alpha_1 > A(x, q) > \alpha_2$. Conversely, if there is no x∈R such that $\alpha_1 > A(x, q) > \alpha_2$, then $A^{\alpha_1} = A^{\alpha_2}$.

***3.21 Theorem*:**
Let A be a (Q, L) -fuzzy ideal of a ring R. The intersection of two Q-level ideals of A in R is also a Q-level ideal of A in R.
Proof:
For α_1 and α_2 in L, $\alpha_1 \leq A(e, q)$ and $\alpha_2 \leq A(e, q)$.
Case (i):If $\alpha_1 < A(x, q) < \alpha_2$, then $A^{\alpha_2} \subseteq A^{\alpha_1}$.
Therefore, $A^{\alpha_1} \cap A^{\alpha_2} = A^{\alpha_2}$, but A^{α_2} is a Q-level ideal of A.

Case (ii):If α_1> A(x, q) >α_2, then $A^{\alpha_1} \subseteq A^{\alpha_2}$.

Therefore, $A^{\alpha_1} \cap A^{\alpha_2} = A^{\alpha_1}$, but A^{α_1} is a Q- level ideal of A.

Case (iii): If $\alpha_1 = \alpha_2$, then $A^{\alpha_1} = A^{\alpha_2}$.

In all cases, intersection of any two Q-level ideals is also a Q-level ideal of A.

3.22 Theorem:

Let A be a (Q, L) -fuzzy ideal of a ring R. The union of two Q-level ideals of A in R is also a Q-level ideal of A in R.

Proof:

Let α_1 and α_2 be in L, $\alpha_1 \leq A(e, q)$ and $\alpha_2 \leq A(e, q)$.

Case (i): If α_1< A(x, q)< α_2, then $A^{\alpha_2} \subseteq A^{\alpha_1}$.

Therefore, $A^{\alpha_1} \cup A^{\alpha_2} = A^{\alpha_1}$, but A^{α_1} is a Q-level ideal of A.

Case (ii):If α_1> A (x, q) >α_2, then $A^{\alpha_1} \subseteq A^{\alpha_2}$.

Therefore, $A^{\alpha_1} \cup A^{\alpha_2} = A^{\alpha_2}$, but A^{α_2} is a Q-level ideal of A.

Case (iii): If $\alpha_1 = \alpha_2$, then $A^{\alpha_1} = A^{\alpha_2}$. In all cases, union of any two Q-level ideal is also a Q-level ideal of

3.23 Theorem:

The homomorphic image of a Q-level ideal of a (Q, L) -fuzzy ideal ofthe ring R is a Q-level ideal of a (Q, L) -fuzzy ideal of the ring R^{l}.

Proof:

Let f: R→R^{l}be a homomorphism. Let V = f (A), where A is a (Q, L) -fuzzyideal of the ring R. Clearly V is a (Q, L) -fuzzy ideal of the ring R^{l}.

Let x and y in R and q in Q, implies f(x) and f(y) in R^{l}. Let A^{α} is a Q-level ideal of A. That is, A(x, q) ≥ α and A(y, q) ≥ α; A(x-y, q) ≥ α, A(xy, q) ≥ α. We have to prove that f(A^{α}) is a Q-level ideal of V.

Now, V(f(x), q) ≥ A(x, q)

$\geq \alpha$,

⟹ V(f(x), q) ≥ α and

V(f(y), q) ≥ A(y, q)

$\geq \alpha$,

⟹ V(f(y), q) ≥ α and ,

V(f(x)-f(y), q) = V(f(x-y), q)

≥ A(x-y, q)

$\geq \alpha$,

⟹ V(f(x)-f(y), q) ≥ α.

Also, V(f (x)f(y), q) = V(f(xy), q)

≥ A (xy, q)

$\geq \alpha$,

⟹ V(f(x)f(y), q) ≥ α. Therefore, V(f(x)-f(y), q) ≥ α and μ_V(f(x)f(y), q) ≥ α. Hence f(A^{α}) is a Q-level ideal of a (Q, L) -fuzzy ideal V of the ring R^{l}.

3.24 Theorem:

The homomorphic pre- image of a Q-level ideal of a (Q, L) -fuzzyideal of the ring R^{l} is a Q-level ideal of a (Q, L) -fuzzy ideal of the ring R.

Proof:

Let f: R→R^{l}be a homomorphism. Let V = f(A), where V is a (Q, L) -fuzzyideal of the ring R^{l}. Clearly A is a (Q, L) -fuzzy ideal of the ring R.

Let f(x) and f(y) in R^{l}, implies x and y in R and q in Q.

Let f(A^{α}) is a Q-level ideal of V. That is, V(f(x), q) ≥ α and V(f(y), q) ≥ α,

$V(f(x)\text{-}f(y), q) \geq \alpha$, $V(f(x)f(y), q) \geq \alpha$.

We have to prove that A^{α} is a Q-level ideal of A.

Now, $A(x, q) = V(f(x), q)$

$\geq \alpha$,

$\Rightarrow A(x, q) \geq \alpha$,

$A(y, q) = V(f(y), q)$

$\geq \alpha$,

$\Rightarrow A(y, q) \geq \alpha$

And, $A(x\text{-}y, q) = V(f(x\text{-}y), q)$

$= V(f(x)\text{-}f(y), q$

$\geq \alpha$,

$\Rightarrow A(x\text{-}y, q) \geq \alpha$.

Also, $A(xy, q) = V(f(xy), q)$

$= V(f(x)f(y), q)$

$\geq \alpha$,

$\Rightarrow A(xy, q) \geq \alpha$. Therefore, $A(x\text{-}y, q) \geq \alpha$, $A(xy, q) \geq \alpha$. Hence, A^{α} is a Q-level ideal of a (Q, L) -fuzzy ideal A of R.

3.25 Theorem:

The anti-homomorphic image of a Q-level ideal of a (Q, L) -fuzzyideal of a ring R is a Q-level ideal of a (Q, L) -fuzzy ideal of a ring R^{l}.

Proof:

Let f: $R \rightarrow R^{l}$ be an anti- homomorphism. Let V=f(A), where A is a (Q, L) -fuzzy ideal of R.

Clearly V is a (Q, L) -fuzzy ideal of R^{l}.

Let x and y in R and q in Q, implies f (x) and f (y) in R^{l}.

Let A^{α} is a Q-level ideal of A.

That is, $A(x, q) \geq \alpha$ and $A(y, q) \geq \alpha$. $A(y\text{-}x, q) \geq \alpha$, $A(yx, q) \geq \alpha$. We have to prove that f (A^{α}) is a Q-level ideal of V.

Now, $V(f(x), q) \geq A(x, q)$

$\geq \alpha$,

$\Rightarrow V(f(x), q) \geq \alpha$;

$V(f(y), q) \geq A(y, q)$

$\geq \alpha$,

$\Rightarrow V(f(y), q) \geq \alpha$.

Now, $V(f(x)\text{-}f(y), q) = V(f(x)\text{-}f(y), q)$

$= V(f(y\text{-}x), q)$

$\geq A(y\text{-}x, q)$

$\geq \alpha$,

$\Rightarrow V(f(x)\text{-}f(y), q) \geq \alpha$.

Also, $V(f(x)f(y), q) = V(f(yx), q)$

$\geq A(yx, q)$

$\geq \alpha$,

$\Rightarrow V(f(x)f(y), q) \geq \alpha$.Therefore, $V(f(x)\text{-}f(y), q) \geq \alpha$ and $V(f(x)f(y), q) \geq \alpha$. Hence $f(A^{\alpha})$ is a Q-level ideal of a (Q, L) -fuzzy ideal V of R^{l}.

3.26 Theorem:

The anti-homomorphism pre-image of a Q-level ideal of a (Q, L) -fuzzyideal of a ring R^{I} is a Q-level ideal of a (Q, L) -fuzzy ideal of a ring R.

Proof:

Let f: $R \rightarrow R^{I}$be an anti-homomorphism. Let V= f(A), where V is a (Q, L) -fuzzy ideal of the ring R^{I}. Clearly A is a (Q, L) -fuzzy ideal of the ring R.

Let f(x) and f(y) in R^{I}, implies x and y in R and q in Q. Let $f(A^{\alpha})$ is a Q-level ideal of V.

That is, V(f (x), q) $\geq \alpha$ and V(f(y), q) $\geq \alpha$; V(f(y)-f(x), q) $\geq \alpha$, V(f(y)f(x), q) $\geq \alpha$. We have to prove that A^{α} is a Q-level ideal of A.

Now, A(x, q) = V(f(x), q)

$\geq \alpha$,

$\Rightarrow$ A(x, q) $\geq \alpha$;

A(y, q) = V(f(y), q)

$\geq \alpha$,

$\Rightarrow$ A(y, q) $\geq \alpha$.

Now, A(x-y, q) = V(f(x-y), q)

= V(f(y)-f (x), q)

$\geq \alpha$,

$\Rightarrow$ A(x-y, q) $\geq \alpha$.

Also, A(xy, q) = V(f(xy), q)

= V(f(y)f(x), q)

$\geq \alpha$,

$\Rightarrow$ A(xy, q) $\geq \alpha$. Therefore, A(x-y, q) $\geq \alpha$ and A(xy, q) $\geq \alpha$. Hence A^{α} is a Q-level ideal of a (Q, L) -fuzzy ideal A of R.

3.27 Theorem: Let A be a (Q, L) -fuzzy ideal of a ring R. Then$a+A_{\alpha}=(a+A)_{\alpha}$,for every a in R, α in L.

Proof:

Let A be a (Q, L) -fuzzy ideal of a ring R and let x in R.

Now, x $\in (a+A)_{\alpha}$

$\Leftrightarrow$ (a+A) (x, q) $\geq \alpha$

$\Leftrightarrow$ A(x-a, q) $\geq \alpha$

$\Leftrightarrow$ x-a $\in A_{\alpha}$

$\Leftrightarrow$ x $\in$ a+ A_{α} .

Therefore, a+ $A_{\alpha} = (a + A)_{\alpha}$, for every x in R.

IV. CONCLUSIONS

This dissertation provides various types of results on (Q, L) fuzzy ideals of ring , & (Q, L) fuzzy normal ideals of ring so that it will be easier to understand the concepts in the material, we have given the list of references from where we have collected the details for this dissertation. I hope that whatever the things that are discussed in the dissertation will give be clear to the reader.

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