Quasi Dine sg-Open and Quasi Dine g-Closed Functions in Dine Topological Spaces

P.Karthiksankar[#1],
[#1]PG & Research Department of Mathematics, G. V. N. College, M. S. University, Kovilpatti, TamilNadu, India.

Abstract: *P.Karthiksankar introduced Dine-topological space which is a special case of generalized topological space. Functions and of course open functions stand among the most important notions in the whole of mathematical science. Many different forms of open function have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences. Aim of this paper is we introduced in Dine sg-open sets in Dine topological space.and also we studied about quasi Dine sg-open and quasi Dine g-closed functions in Dine topological spaces.*

Keywords: *Dsg-open sets, Dg-closed set, quasi Dine sg-open functions, quasi Dine g-closed functions.*

I. INTRODUCTION

I have investigated a special case of generalized topological space called Dine topological space. Functions and of course open functions stand among the most important notions in the whole of mathematical science. Many different forms of open function have been introduced over the years. Various interesting problems arise when one considers openness. Its importance is significant in various areas of mathematics and related sciences.

As a generalization of closed sets, the notion of g-closed sets were introduced and studied by Bhattacharyya and Lahiri2. In this paper, we will continue the study of related functions by involving Dg-open sets in Dine topological space. We introduce and characterize the concept of quasi Dsg-open functions and quasi Dg-closed functions fine topological space.

II. PRELIMINARIES

We recall the following definitions which are useful in the sequel.

Definition: 2.1 [10,11]

Let (X, τ) be a topological space we define, $\tau(A\alpha)= \tau\alpha =\{G\alpha(\neq X): G\alpha \cup A\alpha \neq \varphi$, for $A\alpha \in \tau$ and $A\alpha \neq X, \varphi$ for some $\alpha \in J$, where J is the index set$\}$. Now, define $\tau_d = \{\varphi, X, \cup\tau\alpha\}$. The above collection τd of subsets of X is called the dine collection of subsets of X and (X, τ, τ_d) is said to be the Dine topological space X and generated by the topology τ on X. The element of τd are called dine open sets in (X, τ, τ_d) and the complement of dine open set is called dine closed sets and it is denoted by τ_d^C.

Example: 2.2 [10, 11]

Consider a topological space X = {1, 2, 3} with the topology $\tau =\{X, \varphi, \{1\}\}$ where $A\alpha = \{1\}$. In view of Definition 2.1 we have,

$\tau\alpha =\tau(A\alpha) = \tau \{1\} = \{\{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}\}$

Then the fine collection is $\tau_d = \{ \phi, X, \cup\tau\alpha\} =\{ \phi, X, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}\}$.

We quote some important properties of dine topological spaces.

Lemma: 2.3. [10, 11]

Let (X, τ, τ_d) be a dine space then arbitrary union of dine open set in X is dine-open in X.

Lemma: 2.4. [10, 11]

The intersection of two dine-open sets again a dine-open set.

Remark: 2.5 [10, 11]

In view of Definition 2.1 of generalized topological space and above Lemmas 2.3 and 2.4 it is apparent that (X, τ, τ_d) is a special case of generalized topological space. It may be noted specifically that the topological space plays a key role while defining the fine space as it is based on the topology of X but there is no topology in the back of generalized topological space.

Definition: 2.6 [10, 11]
A subset A of a Dine space (X, τ, τ_d) is called Dine semi-open if $A \subseteq Dcl(Dint(A))$.The complement of Dine semi-open set is called Dine semi-closed. The Dine semi-closure of a subset A of Dine space X, denoted by Dscl(A), is defined to be the intersection of all Dine semi-closed sets containing A in Dine space X.

Definition: 2.8
A subset A of a dine space(X, τ, τ_d) is called Dsg-closed if Dscl (A) $\subset$ U whenever $A \subset U$ and U is dine semi-open in Dine space(X, τ, τ_d). The complement of Dsg-closed set is called Dsg-open.

Definition: 2.9
The union of all Dsg-open sets, each contained in a set A in a Dine space (X, τ, τ_d) is called the Dsg-interior of A and is denoted by Dsg-int (A).

Definition: 2.10
The intersection of all Dsg-closed sets containing a set A in a Dine space (X, τ, τ_d) is called the Dsg-closure of A and is denoted by Dsg-cl (A).

Definition: 2.11
A function $f: (X, \tau, \tau_d) \to (Y, \sigma, \sigma_d)$ is called
(i) Dsg-irresolute (resp. Dsg-continuous) if $f^{-1}(V)$ is Dsg-closed in Dine space X for every Dsg-closed (resp. closed) subset V of Dine space Y;
(ii) Dsg-open (resp. Dsg-closed) if f (V) is Dsg-open (resp. Dsg-closed) in Dine space Y for every Dine open (resp. Dine closed) subset of Dine space X.

Definition: 2.12
A subset A of a dine space(X, τ, τ_d) is called Dg-closed if Dcl (A) $\subset$ U whenever $A \subset U$ and U is dine open in Dine space(X, τ, τ_d). The complement of Dg-closed set is called Dg-open.

III. QUASI DINE sg-OPEN FUNCTIONS

In this section we introduce a new definition is called quasi Dsg-open function and its properties are discussed
Definition: 3.1
A function $f : (X, \tau, \tau_d) \to (Y, \sigma, \sigma_d)$ is said to be quasi Dsg-open if the image of every Dsg-open set in Dine space X is Dine open in Dine space Y.
It is evident that, the concepts of quasi Dsg-openness and Dsg-continuity coincide if the function is a bijection.

Theorem: 3.2
A function $f: (X, \tau, \tau_d) \to (Y, \sigma, \sigma_d)$ is quasi Dsg-open if and only if for every subset U of Dine space X,
$f(Dsg\text{-}int(U)) \subset Dint(f(U))$.
Proof
Let f be a quasi Dsg-open function and U be a subset of Dine space X. Now, we have $Dsg\text{-}int(U) \subset U$ and Dsg-int(U) is a Dsg-open set. Hence, we obtain that $f(Dsg\text{-}int(U)) \subset f(U)$. As f(Fsg-int(U)) is open, $f(Dsg\text{-}int(U)) \subset Dint(f(U))$. Conversely, assume that U is a Dsg-open set in Dine space X.
Then, $f(U) = f(Dsg\text{-}int(U)) \subset Dint(f(U))$ but $Dint(f(U)) \subset f(U)$.
Consequently, $f(U) = Dint(f(U))$ and f(U) is Fine open in Fine space Y. Hence f is quasi Dsg-open.

Lemma: 3.3
If a function $f : (X, \tau, \tau_d) \to (Y, \sigma, \sigma_d)$ is quasi Dsg-open, then $Dsg\text{-}int(f^{-1}(G)) \subset f^{-1}(Dint(G))$ for every subset G of Dine space Y.
Proof
Let G be any arbitrary subset of Dine space Y. Then, $Dsg\text{-}int(f^{-1}(G))$ is a Dsg-open set in Dine space X and since f is quasi Dsg-open, then $f(Dsg\text{-}int(f^{-1}(G))) = Dint(f(f^{-1}(G))) \subset Dint(G)$. Thus, $Dsg\text{-}int(f^{-1}(G)) \subset f^{-1}(Dint(G))$.
Recall that a subset S is called a Dsg-neighbourhood of a point x of Dine space X if there exists a Dsg-open set U such that $x \subset U \subset S$.

Theorem: 3.4
For a function $f: (X, \tau, \tau_d) \to (Y, \sigma, \sigma_d)$ the following are equivalent:
(i) f is quasi Dsg-open;
(ii) For each subset U of Dine space X, $f(Dsg\text{-}int(U)) \subset Dint(f(U))$;
(iii) For each $x \in$ Dine space X and each Dsg-neighbourhood U of x in Dine space X, there exists a neighbourhood f(U) of f(x) in Dine space Y such that $f(V) \subset f(U)$.

Proof

(i) ⇒ (ii): It follows from Theorem 3.2.

(ii) ⇒ (iii): Let $x \in$ Dine space X and U be an arbitrary Dsg-neighbourhood of x in Fine space X. Then there exists a Dsg-open set V in Dine space X such that $x \in V \subset U$. Then by (ii), we have

$f(V) = f(Dsg\text{-}int(V)) \subset Dint(f(V))$ and hence $f(V) = Dint(f(V))$. Therefore, it follows that f(V) is open in Y such that $f(x) \in f(V) \subset f(U)$.

(iii) ⇒ (i): Let U be an arbitrary Dsg-open set in Dine space X such that $x \in U$. Then for each $f(x) = y \in f(U)$, by (iii) there exists a neighbourhood Vy of y in Dine space Y such that $Vy \in f(U)$. As Vy is a neighbourhood of y, there exists an Dine open set Wy in Dine space Y such that $y \in Wy \subset Vy$. Thus $f(U) = \cup\{Wy : y \in f(U)\}$ which is an Dine open set in Dine space Y. This implies that f is quasi Dsg-open function.

Theorem: 3.5

A function $f: (X, \tau, \tau_d) \to (Y, \sigma, \sigma_d)$ is quasi Dsg-open if and only if for any subset B of Dine space Y and for any Dsg-closed set H of Dine space X containing $f^{-1}(B)$, there exists a Dine closed set G of Fine space Y containing B such that $f^{-1}(G) \subset H$.

Proof

Suppose f is quasi Dsg-open. Let $B \subset Y$ and H be a Dsg-closed set of Dine space X containing $f^{-1}(B)$. Now, put $G = Y – f(X – H)$. It is clear that $f^{-1}(B) \subset H$ implies $B \subset G$. Since f is quasi Dsg-open, we obtain G is a Dine closed set of Dine space Y. Moreover, we have $f^{-1}(G) \subset H$.

Conversely, let U be a Dsg-open set of Dine space X and put $B = Y\backslash f(U)$. Then $X\backslash U$ is a Dsg-closed set in Dine space X containing $f^{-1}(B)$. By hypothesis, there exists a Dine closed set H of Dine space Y such that $B \subset H$ and $f^{-1}(H) \subset X\backslash U$. Hence, we obtain $f(U) \subset Y\backslash H$. On the other hand, it follows that $B \subset H$, $Y\backslash H \subset Y\backslash B = f(U)$. Thus, we obtain $f(U) = Y\backslash H$ which is Dine open and hence f is a quasi Dsg-open function.

Theorem: 3.6

A function $f: (X, \tau, \tau_d) \to (Y, \sigma, \sigma_d)$ is quasi Fsg-open if and only if $f^{-1}(cl(B)) \subset Dsg\text{-}cl(f^{-1}(B))$ for every subset B of Dine space Y.

Proof

Suppose that f is quasi Dsg-open. For any subset B of Dine space Y, $f^{-1}(B) \subset Fsg\text{-}cl(f^{-1}(B))$. Therefore by Theorem 3.5, there exists a Dine closed set H in Y such that $B \subset H$ and $f^{-1}(H) \subset Dsg\text{-}cl(f^{-1}(B))$. Therefore, we obtain $f^{-1}(Dcl(B)) \subset f^{-1}(H) \subset Fsg\text{-}cl(f\text{-}1(B))$.

Conversely, let $B \subset Y$ and H be a Dsg-closed of Dine space X containing $f^{-1}(B)$. Put $W = DclY(B)$, then we have $B \subset W$ and W is Dine closed in Dine space Y and $f^{-1}(W) \subset Dsg\text{-}cl(f^{-1}(B)) \subset H$. Then, by Theorem 3.5, f is quasi Dsg-open.

IV. QUASI Dg-CLOSED FUNCTIONS

In this section we introduce a new definition is called quasi Dg-closed function and its properties are discussed

Definition: 4.1

A function $f: (X, \tau, \tau_d) \to (Y, \sigma, \sigma_d)$ is said to be quasi Dg-closed if the image of each Dg-closed set in Dine space X is Dine closed in Dine space Y.

Clearly, every quasi Dg-closed function is Fine closed as well as Dg-closed.

Lemma: 4.2

If a function $f : (X, \tau, \tau_d) \to (Y, \sigma, \sigma_d)$ is quasi Dg-closed, then $f^{-1}(Dcl(B)) \subset Dg\text{-}cl(f^{-1}(B))$ for every subset B of Dine space Y.

Proof

Let G be any arbitrary subset of Dine space Y. Then, $Dg\text{-}int(f^{-1}(G))$ is a Dg-open set in Dine space X and since f is quasi Dg-closed, then $f(Dg\text{-}int(f^{-1}(G))) = Dint(f(f^{-1}(G))) \subset Dint(G)$. Thus, $Dg\text{-}int(f^{-1}(G)) \subset f^{-1}(Dint(G))$.

Theorem: 4.3

A function $f: (X, \tau, \tau_d) \to (Y, \sigma, \sigma_d)$ is quasi Dg-closed if and only if for any subset B of Dine space Y and for any Dg-open set G of Dine space X containing $f^{-1}(B)$, there exists an Dine open set U of Dine space Y containing B such that $f^{-1}(U) \subset G$.

Proof

This proof is similar to that of Theorem 3.5.

Definition: 4.4

A function $f: (X, \tau, \tau_d) \to (Y, \sigma, \sigma_d)$ is called Dg*-closed if the image of every Dg-closed subset of Dine space X is Dg-closed in Dine space Y.

Theorem: 4.5

If f: $(X, \tau, \tau_d) \rightarrow (Y, \sigma, \sigma_d)$ and g : $(Y, \sigma, \sigma_d) \rightarrow (Z, \rho, \rho_d)$ are two quasi Dg-closed functions, then g o f : $(X, \tau, \tau_d) \rightarrow (Z, \rho, \rho_d)$ is a quasi Dg-closed function.

Theorem: 4.6

Let f: $(X, \tau, \tau_d) \rightarrow (Y, \sigma, \sigma_d)$ and g : $(Y, \sigma, \sigma_d) \rightarrow (Z, \rho, \rho_d)$ be any two functions. Then

(i) If f is Dg-closed and g is quasi Dg-closed, then g o f is Dine closed;

(ii) If f is quasi Dg-closed and g is Dg-closed, then g o f is Dg*- closed;

(iii) If f is Dg* -closed and g is quasi Dg-closed, then g o f is quasi Dg-closed.

Theorem: 4.7

Let (X, τ, τ_d) and (Y, σ, σ_d) be Dine topological spaces. Then the function g: $(X, \tau, \tau_d) \rightarrow (Y, \sigma, \sigma_d)$ is a quasi Dg-closed if and only if g(X) is Dine closed in Dine space Y and g(V) \ g(X\V) is Dine open in g(X) whenever V is Dg-open in Dine space X.

Proof

Necessity: Suppose g: $(X, \tau, \tau_d) \rightarrow (Y, \sigma, \sigma_d)$ is a quasi Dg-closed function. Since Dine space X is Dg-closed, g(X) is Dine closed in Dine space Y and g(V) \ g(X\V)=g(V) ∩ g(X)\g(X\V) is Fine open in g(X) when V is Dg-open in Dine space X.

Sufficiency: Suppose g(X) is Dine closed in Dine space Y, g(V) \ g(X\V) is Dine open in g(X) when V is Dg-open in X, and let C be Dine closed in Dine space X. Then g(C) = g(X) \ (g(X\C) \ g(C)) is Dine closed in g(X) and hence, Dine closed in Dine space Y .

Corollary: 4.8

Let (X, τ, τ_d) and (Y, σ, σ_d) be Dine topological spaces. Then a surjective function g: $(X, \tau, \tau_d) \rightarrow (Y, \sigma, \sigma_d)$ is quasi Dg-closed if and only if g(V) \ g(X\V) is open in Dine space Y whenever V is Dg-open in Dine space X.

Definition: 4.9

A Dine topological space (X, τ, τ_d) is said to be Dg*-normal if for any pair of disjoint Dg-closed subsets F1 and F2 of Dine space X, there exist disjoint Dine open sets U and V such that F1 ⊂ U and F2 ⊂ V.

Theorem 4.10

Let (X, τ, τ_d) and (Y, σ, σ_d) be Dine topological spaces with Dine space X is Dg*- normal. If g: $(X, \tau, \tau_d) \rightarrow (Y, \sigma, \sigma_d)$ is a Dg-continuous quasi Dg-closed surjective function, then Fine space Y is normal.

Proof

Let K and M be disjoint Dine closed subsets of Dine space Y. Then $g^{-1}(K)$ and $g^{-1}(M)$ are disjoint Dg-closed subsets of Dine space X. Since Dine space X is Dg*-normal, there exist disjoint open sets V and W such that $g^{-1}(K) \subset V$ and $g^{-1}(M) \subset W$. Then K ⊂ g(V)\g(X\V) and M ⊂ g(W)\g(X\W). Further by Corollary 4.8, g(V)\g(X\V) and g(W)\g(X\W) are Fine open sets in Fine space Y and clearly (g(V) \ g(X\V)) ∩ (g (W) \ g(X\W)) = ∅. This shows that Fine space Y is normal.

REFERENCES

[1] D.Andrijevic, "Semi – pre open sets", Mat.Vesnik, 38(1), 24 – 32 (1986).

[2] P. Bhattacharyya, and B. K. Lahiri, Semi Generalized Closed Sets in Topology, Indian J.Pure Appl.Math., Vol. 29, Pp. 375-382 (1987).

[3] S.Chandrasekar,M.Suresh,T.Rajesh kannan Nano sg-interior and Nano sg-closure in nano topological spaces. International Journal. Mathematical Archive, 8(4), 94-100 (2017).

[4] S.Chandrasekar, A Atkinswestly, M ,Sathyabama, g#b-closed sets in Topological paces. International Journal of Advanced Research (IJAR) 5(7) , 2722-2730

[5] S Chandrasekar, M Sathyabama, A Atkinswestley g# b-Interior and g #b-Closure in Topological Spaces. International Journal of Mathematics and its Applications,552(B)7(2017).

[6] S.Chandrasekar, M.Suresh and T.Rajesh Kannan,On Quasi Soft sg-open and Quasi Soft sg-closed Functions in Soft Topological Spaces. International Journal of Mathematics and its Applications (239-246), Volume 5, Issue 2–B, 239–246 (2017).

[7] S.Chandrasekar, T Rajesh Kannan, M Suresh,,δωα-Closed Sets in Topological Spaces,Global Journal of Mathematical Sciences: Theory and Practical.9 (2), 103-116(2017).

[8] S.Chandrasekar,T Rajesh Kannan,R.Selvaraj δωα closed functions and δωα Closed Functions in Topological Spaces, International Journal of Mathematical Archive 8 (11) (2017).

[9] N.Levine, Generalized closed sets in Topological Spaces, Rend. Circ. Mat. Palermo, Vol 19, Pp.89-96 (1970).

[10] P.L. Powar and Pratibha Dubey A Concise form of Continuity in Fine Topological Space Advances in Computational Sciences and Technology Vol. 10, N. 6, pp.1785-1805 (2017),

[11] P.L. Powar and K. Rajak, Fine irresolute mapping, Journal of Advanced Studied in Topology, 3(4) 125–139 (2012).

[12] O.Ravi, S. Ganesan and S. Chandrasekar On Totally sg-Continuity, Strongly sg- Continuity and Contra sg-Continuity Genral Mathematical notes. Notes, Vol.7,No 1, pp.13-24 Nov (2011).