On The Homogeneous Ternary Quadratic Diophantine Equation $5x^2+4y^2=189z^2$

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ABSTRACT:

Homogeneous ternary quadratic equation $5x^2+4y^2=189z^2$ *is analysed for its integral points on it. Employing the integral solutions of the above equation, a few interesting relations between the solutions and the special numbers are also exhibited.*

KEYWORDS: *Homogeneous, ternary, quadratic, integral solutions.*

I. INTRODUCTION:

The Diophantine equation offer an unlimited field for research due to their variety $[1-3]$. In particular, one may refer $[4-13]$ for quadratic equation with three unknowns. The communication concerns with yet another interesting equation $5x^2+4y^2=189z^2$ representing non-homogeneous quadratic equation with three unknowns for determining its infinitely many non-zero integral points. Also, a few interesting relations among the solutions are presented.

II. NOTATIONS:

1. Polygonal number of rank n with side m

$$t_{m,n}=n\left[1+\frac{(n-1)(m-2)}{2}\right]$$

2. Pronic number of rank n

$$\mathrm{Pr}_n=n(n+1)$$

3. Star number of rank n

$$S_n=6n(n-1)+1.$$

III. METHOD OF ANALYSIS:

The ternary quadratic Diophantine equation to be solved for its non- zero distinct integral solution is

$$5x^2+4y^2=189z^2 \tag{1}$$

Introduction of the linear transformations $(X\neq T\neq 0)$

$$x=X-4T,\ y=X+5T \tag{2}$$

in (1) leads to

$$X^2+20T^2=21z^2 \tag{3}$$

Different patterns of solutions of (1) are presented below:

Pattern-1:

write 20 as

$$21=\left(1+i\sqrt{20}\right)\left(1-i\sqrt{20}\right) \tag{4}$$

Assume

$$z=a^2+20b^2 \tag{5}$$

where a and b are non- zero distinct integers

using (4) and (5) in (3), we get

$$X^2+20T^2=\left(1+i\sqrt{20}\right)\left(1-i\sqrt{20}\right)\left(a^2+20b^2\right)^2$$

Employing the method of factorization the above equation is written as

$$\left(X+i\sqrt{20}T\right)\left(X-i\sqrt{20}T\right)=\left(1+i\sqrt{20}\right)\left(1-i\sqrt{20}\right)\left(a+i\sqrt{20}b\right)^2\left(a-i\sqrt{20}b\right)^2$$

Equating the positive and negative factors, the resulting equations are,

$$X+i\sqrt{20}T=\left(1+i\sqrt{20}\right)\left(a+i\sqrt{20}b\right)^2 \tag{6}$$

$$X-i\sqrt{20}T=\left(1-i\sqrt{20}\right)\left(a-i\sqrt{20}b\right)^2 \tag{7}$$

Equating real and imaginary parts in (6), we get

$$X=a^2-20b^2-40ab$$

$$T=a^2-20b^2+2ab$$

substituting the values of X and T in (2), we get

$$x=x(a,b)=-3a^2+60b^2-48ab \tag{8}$$

$$y=y(a,b)=6a^2-120b^2-30ab \tag{9}$$

Thus (8), (9) and (5) represents non- zero distinct integral solutions of (1) in two parameters.

Properties:

- $x(1,b)-60\Pr_b\equiv 0(\bmod 3)$
- $y(a,1)-6\Pr_a\equiv 0(\bmod 2)$
- $x(b,b)-z(b,b)+12t_{4,b}=0$
- $y(a,a)+144t_{4,a}=0$
- $x(1,b+1)-S_b\equiv 0(\bmod 2)$
- $z(a+1,1)-S_a-\Pr_a+t_{4,a}\equiv 0(\bmod 2)$

Pattern-2:

The equation (3) can also be written as

$$X^2-z^2=20\left(z^2-T^2\right)$$

$$(X-z)(X+z)=20(z-T)(z+T) \tag{10}$$

Equation (10) is written in the form of ratio as,

$$\frac{X+z}{5(z+T)}=\frac{4(z-T)}{X-z}=\frac{\alpha}{\beta},\beta\neq 0 \tag{11}$$

From the first and third factors of (11), we have

$$\frac{X+z}{5(z+T)}=\frac{\alpha}{\beta}$$

$$\beta X-5\alpha T+(\beta-5\alpha)z=0 \qquad (12)$$

From the second and third factors of (11), we have

$$\frac{4(Z-T)}{X-Z}=\frac{\alpha}{\beta}$$

$$-\alpha X-4\beta T+(4\beta+\alpha)z=0 \qquad (13)$$

Applying the method of cross multiplication for solving (12) and (13)

$$X=5\alpha^2+40\alpha\beta-4\beta^2$$

$$T=-5\alpha^2+2\alpha\beta+4\beta^2$$

$$z=5\alpha^2+4\beta^2$$

substituting the values of X and T in (2), we get

$$\left.\begin{aligned} x=x(\alpha,\beta)&=25\alpha^2+32\alpha\beta-20\beta^2\\ y=y(\alpha,\beta)&=-20\alpha^2+50\alpha\beta+16\beta^2\\ z=z(\alpha,\beta)&=4\beta^2+5\alpha^2 \end{aligned}\right\} \qquad (14)$$

Thus (14) represents the non-zero distinct integral solutions to (1)

Properties:

- $x(1,\beta)-32\,\mathrm{Pr}_\beta+52t_{4,\beta}\equiv 0(\mathrm{mod}\,5)$
- $y(\alpha,1)+20\,\mathrm{Pr}_\alpha\equiv 0(\mathrm{mod}\,2)$
- $x(\beta,\beta)+z(\beta,\beta)-28t_{4,\beta}=0$
- $y(\alpha,\alpha)-46t_{4,\alpha}=0$
- $x(\beta,\beta+1)+S_\beta\equiv 0(\mathrm{mod}\,2)$
- $x(\alpha+1,1)-S_\alpha\equiv 0(\mathrm{mod}\,5)$

Pattern-3:

One may write (10) in the form of ratio as

$$\frac{X+z}{4(z+T)}=\frac{5(Z-T)}{X-z}=\frac{\alpha}{\beta},\beta\neq 0 \qquad (15)$$

From the first and third factors of (15), we have

$$\frac{X+z}{4(z+T)}=\frac{\alpha}{\beta}$$

$$\beta X+(\beta-4\alpha)z-4\alpha T=0 \qquad (16)$$

From the second and third factors of (16), we have

$$\frac{5(z-T)}{X-z}=\frac{\alpha}{\beta}$$

$$-5\beta T+(5\beta+\alpha)z-\alpha X=0 \qquad (17)$$

Applying the method of cross multiplication for solving (16) and (17)

$$X=4\alpha^2+40\alpha\beta-5\beta^2$$

$$T=5\beta^2+2\alpha\beta-4\alpha^2$$

$$z=4\alpha^2+5\beta^2$$

substituting the values of X and T in (2), we get

$$\left.\begin{aligned} x = x(\alpha,\beta) &= -25\beta^2 + 32\alpha\beta + 20\alpha^2 \\ y = y(\alpha,\beta) &= 20\beta^2 + 50\alpha\beta - 16\alpha^2 \\ z = z(\alpha,\beta) &= 4\alpha^2 + 5\beta^2 \end{aligned}\right\} \quad (18)$$

Thus (18) represents the non-zero distinct integral solutions to (1)

Properties:

- $x(1,\beta) - 32\,\mathrm{Pr}_\beta + 57t_{4,\beta} \equiv 0 \pmod 2$
- $y(\alpha,1) + 16\,\mathrm{Pr}_\alpha \equiv 0 \pmod 2$
- $x(\beta,\beta) + y(\beta,\beta) - 81t_{4,\beta} = 0$
- $x(\alpha,\alpha) + z(\alpha,\alpha) - 36t_{4,\alpha} = 0$
- $y(1,\beta+1) - S_\beta \equiv 0 \pmod 5$
- $x(\alpha+1,1) - S_\alpha \equiv 0 \pmod 2$

Pattern-4:

One may write (10) in the form of ratio as

$$\frac{X - z}{4(z+T)} = \frac{5(Z-T)}{X+z} = \frac{\alpha}{\beta}, \beta \neq 0 \quad (19)$$

From the first and third factors of (19), we have

$$\frac{X - z}{4(z+T)} = \frac{\alpha}{\beta}$$

$$\beta X - (\beta + 4\alpha)z - 4\alpha T = 0 \quad (20)$$

From the second and third factors of (20), we have

$$\frac{5(z-T)}{X+z} = \frac{\alpha}{\beta}$$

$$-5\beta T + (6\beta - \alpha)z - \alpha X = 0 \quad (21)$$

Applying the method of cross multiplication for solving (20) and (21)

$$X = -4\alpha^2 + 40\alpha\beta + 5\beta^2$$

$$T = 5\beta^2 - 2\alpha\beta - 4\alpha^2$$

$$z = 4\alpha^2 + 5\beta^2$$

substituting the values of X and T in (2), we get

$$\left.\begin{aligned} x = x(\alpha,\beta) &= -15\beta^2 + 48\alpha\beta + 12\alpha^2 \\ y = y(\alpha,\beta) &= 30\beta^2 + 30\alpha\beta - 24\alpha^2 \\ z = z(\alpha,\beta) &= 4\alpha^2 + 5\beta^2 \end{aligned}\right\} \quad (22)$$

Thus (22) represents the non-zero distinct integral solutions to (1)

Properties:

- $x(1,\beta) + 15\,\mathrm{Pr}_\beta \equiv 0 \pmod 3$
- $y(\alpha,1) + 24\,\mathrm{Pr}_\alpha \equiv 0 \pmod 2$
- $x(\beta,\beta) + y(\beta,\beta) - 81t_{4,\beta} = 0$
- $x(\alpha,\alpha) + z(\alpha,\alpha) - 54t_{4,\alpha} = 0$
- $y(1,\beta+1) - S_\beta - 60\,\mathrm{Pr}_\beta + 60t_{4,\beta} \equiv 1 \pmod 2$
- $x(\alpha+1,1) - S_\alpha \equiv 0 \pmod 2$

Pattern-5:

One may write (10) in the form of ratio as

$$\frac{X-z}{4(z-T)}=\frac{5(Z+T)}{X+z}=\frac{\alpha}{\beta},\beta\neq 0 \qquad (23)$$

From the first and third factors of (23), we have

$$\frac{X-z}{4(z-T)}=\frac{\alpha}{\beta}$$

$$\beta X-(\beta+4\alpha)z+4\alpha T=0 \qquad (24)$$

From the second and third factors of (23), we have

$$\frac{5(z+T)}{X+z}=\frac{\alpha}{\beta}$$

$$5\beta T+(5\beta-\alpha)z-\alpha X=0 \qquad (25)$$

Applying the method of cross multiplication for solving (24) and (25)

$$X=4\alpha^2-40\alpha\beta-5\beta^2$$

$$T=5\beta^2-2\alpha\beta-4\alpha^2$$

$$z=-4\alpha^2-5\beta^2$$

substituting the values of X and T in (2), we get

$$\left.\begin{array}{l} x=x(\alpha,\beta)=-25\beta^2-32\alpha\beta+20\alpha^2 \\ y=y(\alpha,\beta)=20\beta^2-50\alpha\beta-16\alpha^2 \\ z=z(\alpha,\beta)=-4\alpha^2-5\beta^2 \end{array}\right\} \qquad (26)$$

Thus (26) represents the non-zero distinct integral solutions to (1)

Properties:

- $x(1,\beta)+32\,\mathrm{Pr}_\beta-7t_{4,\beta}\equiv 0(\mathrm{mod}\,2)$
- $y(\alpha,1)+16\,\mathrm{Pr}_\alpha\equiv 0(\mathrm{mod}\,2)$
- $x(\beta,\beta)-y(\beta,\beta)-9t_{4,\beta}=0$
- $y(\alpha,\alpha)-z(\alpha,\alpha)+37t_{4,\alpha}=0$
- $z(1,\beta+1)+S_\beta\equiv 0(\mathrm{mod}\,5)$
- $x(\alpha+1,1)-S_\alpha\equiv 0(\mathrm{mod}\,2)$

Pattern-6:

One may write (10)in the form of ratio as

$$\frac{X+z}{2(z+T)}=\frac{10(Z-T)}{X-z}=\frac{\alpha}{\beta},\beta\neq 0 \qquad (27)$$

From the first and third factors of (27), we have

$$\frac{X+z}{2(z+T)}=\frac{\alpha}{\beta}$$

$$\beta X+(\beta-2\alpha)z-2\alpha T=0 \qquad (28)$$

From the second and third factors of (2.27), we have

$$\frac{10(z-T)}{X-z}=\frac{\alpha}{\beta}$$

$$-10\beta T+(10\beta+\alpha)z-\alpha X=0 \tag{29}$$

Applying the method of cross multiplication for solving (28) and (29)

$$X=-10\beta^2+40\alpha\beta+2\alpha^2$$
$$T=10\beta^2+2\alpha\beta-2\alpha^2$$
$$z=2\alpha^2+10\beta^2$$

substituting the values of X and T in (2), we get

$$\left.\begin{aligned} x=x(\alpha,\beta)&=-50\beta^2+32\alpha\beta+10\alpha^2\\ y=y(\alpha,\beta)&=40\beta^2+50\alpha\beta-8\alpha^2\\ z=z(\alpha,\beta)&=2\alpha^2+10\beta^2\end{aligned}\right\} \tag{30}$$

Thus (30) represents the non-zero distinct integral solutions to (1)

Properties:

- $x(1,\beta)+50\Pr_\beta\equiv 0(\mathrm{mod}\,2)$
- $y(\alpha,1)+8\Pr_\alpha\equiv 0(\mathrm{mod}\,2)$
- $x(\beta,\beta)-y(\beta,\beta)+90t_{4,\beta}=0$
- $y(\alpha,\alpha)-z(\alpha,\alpha)-70t_{4,\beta}=0$
- $z(1,\beta+1)-S_\beta-10\Pr_\beta+10t_{4,\beta}\equiv 1(\mathrm{mod}\,2)$
- $x(\alpha+1,1)-S_\alpha\equiv 0(\mathrm{mod}\,7)$

Pattern-7:

One may write (10) in the form of ratio as

$$\frac{X+z}{10(z-T)}=\frac{2(Z+T)}{X-z}=\frac{\alpha}{\beta},\beta\neq 0 \tag{31}$$

From the first and third factors of (31), we have

$$\frac{X+z}{10(z-T)}=\frac{\alpha}{\beta}$$
$$\beta X+(\beta-10\alpha)z+10\alpha T=0 \tag{32}$$

From the second and third factors of (31), we have

$$\frac{2(z+T)}{X-z}=\frac{\alpha}{\beta}$$
$$2\beta T+(2\beta+\alpha)z-\alpha X=0 \tag{33}$$

Applying the method of cross multiplication for solving (32) and (33)

$$X=-10\alpha^2-40\alpha\beta+2\beta^2$$
$$T=2\beta^2+2\alpha\beta-10\alpha^2$$
$$z=-10\alpha^2-2\beta^2$$

substituting the values of X and T in (2), we get

$$\left.\begin{aligned} x=x(\alpha,\beta)&=-6\beta^2-48\alpha\beta+30\alpha^2\\ y=y(\alpha,\beta)&=12\beta^2-30\alpha\beta-60\alpha^2\\ z=z(\alpha,\beta)&=-10\alpha^2-2\beta^2\end{aligned}\right\} \tag{34}$$

Thus (34) represents the non-zero distinct integral solutions to (1)

Properties:

- $x(1,\beta)+6\Pr_{\beta} \equiv 0(\bmod 2)$
- $y(\alpha,1)+60\Pr_{\alpha} \equiv 0(\bmod 2)$
- $x(\beta,\beta)+z(\beta,\beta)+36t_{4,\beta} = 0$
- $y(\alpha,\alpha)+z(\alpha,\alpha)+90t_{4,\alpha} = 0$
- $x(1,\beta+1)-S_{\beta}+18\Pr_{\beta}-18t_{4,\beta} \equiv 0(\bmod 7)$
- $z(\alpha+1,1)+S_{\alpha}+10\Pr_{\alpha}-10t_{4,\alpha} \equiv 1(\bmod 2)$

Pattern-8:

One may write (10) in the form of ratio as

$$\frac{X-z}{2(z+T)}=\frac{10(Z-T)}{X+z}=\frac{\alpha}{\beta},\beta\neq 0 \qquad (35)$$

From the first and third factors of (35), we have

$$\frac{X-z}{2(z+T)}=\frac{\alpha}{\beta}$$

$$\beta X-(\beta+2\alpha)z-2\alpha T=0 \qquad (36)$$

From the second and third factors of (35), we have

$$\frac{10(z-T)}{X+z}=\frac{\alpha}{\beta}$$

$$-10\beta T+(10\beta-\alpha)z-\alpha X=0 \qquad (37)$$

Applying the method of cross multiplication for solving (36) and (37)

$$X=-2\alpha^2+40\alpha\beta+10\beta^2$$

$$T=10\beta^2-2\alpha\beta-2\alpha^2$$

$$z=2\alpha^2+10\beta^2$$

substituting the values of X and T in (2), we get

$$\left.\begin{aligned} x&=x(\alpha,\beta)=-30\beta^2+48\alpha\beta+6\alpha^2\\ y&=y(\alpha,\beta)=60\beta^2+30\alpha\beta-12\alpha^2\\ z&=z(\alpha,\beta)=2\alpha^2+10\beta^2\end{aligned}\right\} \qquad (38)$$

Thus (38) represents the non-zero distinct integral solutions to (1)

Properties:

- $x(1,\beta)+30\Pr_{\beta} \equiv 0(\bmod 2)$
- $y(\alpha,1)+12\Pr_{\alpha} \equiv 0(\bmod 2)$
- $x(\beta,\beta)+z(\beta,\beta)-36t_{4,\beta} = 0$
- $y(\alpha,\alpha)+z(\alpha,\alpha)-90t_{4,\alpha} = 0$
- $y(1,\beta+1)-S_{\beta}-90\Pr_{\beta}+90t_{4,\beta} \equiv 1(\bmod 2)$
- $z(\alpha+1,1)-S_{\alpha}-2\Pr_{\alpha}+2t_{4,\alpha} \equiv 1(\bmod 2)$

Pattern-9:

One may write (10) in the form of ratio as

$$\frac{X-z}{10(z-T)}=\frac{2(Z+T)}{X+z}=\frac{\alpha}{\beta},\beta\neq 0 \tag{39}$$

From the first and third factors of (39), we have

$$\frac{X-z}{10(z-T)}=\frac{\alpha}{\beta}$$

$$\beta X-(\beta+10\alpha)z+10\alpha T=0 \tag{40}$$

From the second and third factors of (39), we have

$$\frac{2(z+T)}{X+z}=\frac{\alpha}{\beta}$$

$$2\beta T+(2\beta-\alpha)z-\alpha X=0 \tag{41}$$

Applying the method of cross multiplication for solving (40) and (41)

$$X=10\alpha^2-40\alpha\beta-2\beta^2$$

$$T=2\beta^2-2\alpha\beta-10\alpha^2$$

$$z=-10\alpha^2-2\beta^2$$

substituting the values of X and T in (2), we get

$$\left.\begin{aligned} x&=x(\alpha,\beta)=-10\beta^2-32\alpha\beta+50\alpha^2\\ y&=y(\alpha,\beta)=8\beta^2-50\alpha\beta-40\alpha^2\\ z&=z(\alpha,\beta)=-10\alpha^2-2\beta^2 \end{aligned}\right\} \tag{42}$$

Thus (42) represents the non-zero distinct integral solutions to (1)

Properties:

- $x(1,\beta)+10\,\mathrm{Pr}_\beta\equiv 0(\mathrm{mod}\,2)$
- $y(\alpha,1)+40\,\mathrm{Pr}_\alpha\equiv 0(\mathrm{mod}\,2)$
- $x(\beta,\beta)+z(\beta,\beta)+4t_{4,\beta}=0$
- $z(\alpha,\alpha)+12t_{4,\alpha}=0$
- $x(1,\beta+1)+S_\beta\equiv 0(\mathrm{mod}\,7)$
- $y(\alpha+1,1)+S_\alpha+90\,\mathrm{Pr}_\alpha-90t_{4,\alpha}\equiv 0(\mathrm{mod}\,2)$

Pattern-10:

One may write (10) in the form of ratio as

$$\frac{X+z}{20(z+T)}=\frac{1(z-T)}{X-z}=\frac{\alpha}{\beta},\beta\neq 0 \tag{43}$$

From the first and third factors of (43), we have

$$\frac{X+z}{20(z+T)}=\frac{\alpha}{\beta}$$

$$\beta X+(\beta-20\alpha)z-20\alpha T=0 \tag{44}$$

From the second and third factors of (43), we have

$$\frac{(z-T)}{X-z}=\frac{\alpha}{\beta}$$

$$-\beta T+(\beta+\alpha)z-\alpha X=0 \tag{45}$$

Applying the method of cross multiplication for solving (44) and (45)

$$X=20\alpha^2+40\alpha\beta-\beta^2$$
$$T=\beta^2+2\alpha\beta-20\alpha^2$$
$$z=20\alpha^2+\beta^2$$

substituting the values of X and T in (2), we get

$$\left.\begin{array}{l} x=x(\alpha,\beta)=-5\beta^2+32\alpha\beta+100\alpha^2 \\ y=y(\alpha,\beta)=4\beta^2+50\alpha\beta-80\alpha^2 \\ z=z(\alpha,\beta)=20\alpha^2+\beta^2 \end{array}\right\} \tag{46}$$

Thus (46) represents the non-zero distinct integral solutions to (1)

Properties:

- $x(1,\beta)-32\,\mathrm{Pr}_\beta+37t_{4,\beta}\equiv 0(\mathrm{mod}\,2)$
- $y(\alpha,1)+80\,\mathrm{Pr}_\alpha\equiv 0(\mathrm{mod}\,2)$
- $x(\beta,\beta)+z(\beta,\beta)-148t_{4,\beta}=0$
- $z(\alpha,\alpha)-21t_{4,\alpha}=0$
- $x(1,\beta+1)+S_\beta\equiv 0(\mathrm{mod}\,3)$
- $y(\alpha+1,1)+S_\alpha\equiv 0(\mathrm{mod}\,3)$

IV. CONCLUSION

In this paper, we have made an attempt to obtain all integer solutions to ternary quadratic equation $5x^2+4y^2=189z^2$ .One may search for other patterns of solutions and their corresponding properties.

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