An square Divisor Cordial Labelling of Graphs

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Abstract — *the Square divisor cordial labelling is a variant of cordial labelling and divisor cordial labelling. Here We the Prove that the graphs like flower F_n^l,bistar $B_{n,n}$,restricted Square Graph Of $B_{n,n}$ Shadow graph of $B_{n,n}$ as Well as Splitting graphs of star $K_{1,n}$ and bistar $B_{n,n}$ are Square divisor cordial graphs. We Show that the degree splitting graphs of $B_{n,n}$ and P_n admit square divisor cordial labelling.*

Keywords — *Square divisor cordial labelling, Divisor cordial labelling*

I. INTRODUCTION

Graph theory is one of the most important branches of Mathematics. Euler studied graph theory as a Mathematical discipline when he discussed the famous konigs-berg Bridge problem. There are several reasons for the acceleration of interest in graph theory. The fact is that graph theory serves as a mathematical model for any system involving a binary relation. There are many applications of graph theory to a wide variety of subjects. The existence of graphs for which a special set of integer values are assigned to its nodes or edges or both according to some given criteria has been investigated since the middle of the last century. Graph labelling is one of the potential area of research due to its diversified applications in computer network and it is a strong communication between number theory and structure of graph. Owing to its wide applications in diverse fields of knowledge, an enormous body of literature has grown around the theme. Nearly 650 papers on various graph labelling methods have been published in past 30 years. Interest in graph labelling problems began in the mid 1960’s. Most graph labelling methods trace their origin to one introduced by Rosa in 1967, or one The concept of cordial labelling was introduced by cahit. By combining the divisibility concept in number theory and cordial label concept in graph labelling, a new concept was introduced by varatharajan. It is called divisor cordial labelling of graph. The finite and connected and undirected graph G= (V (G), E (G)) With P vertices and edges .The Standard terminology and notations related to graph theory for any concept of number theory. The brief summary of definitions and other information which are prerequisites for the present investigations.

II. BASIC DEFINITIONS AND EXAMPLES

Definition 1.1

A graph G = (V,E) consists of a set of objects V = $\{v_1, v_2, v_3, \dots\dots\}$ of vertices and another set E = $\{e_1, e_2, \dots\dots\dots\dots\dots\}$ of edges such that each edge e_k is identified with an unordered pair. A graph G with p vertices and q edges is called a (p,q) graph.

Example 1.1.1

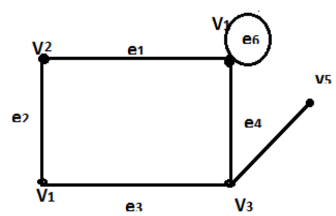


Fig.1.1

A (5,6) graph with V = { V_1,V_2,V_3,V_4,V_5 } and E = $\{e_1, e_2, e_3, e_4, e_5, e_6\}$

Definition1.2

An edge having the same vertex as both its end vertices is called a **Self-loop**.

Definition 1.3

If $e = \{u, v\}$ is an edge of G, we write $e = uv$ or $e = vu$ and say that u and v are **adjacent vertices** in G. If two vertices are adjacent, then they are said to be neighbours.

Example 1.3.1

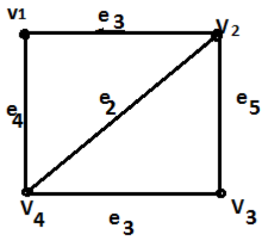


Fig.1.2

V_3 and V_4 are adjacent vertices and e_3 and e5 are adjacent edges.

Definition 1.4

A graph H is a **Sub graph** of a graph G, if all the vertices and all the edges of H, are in G, and each edge of H has the same end vertices in G.

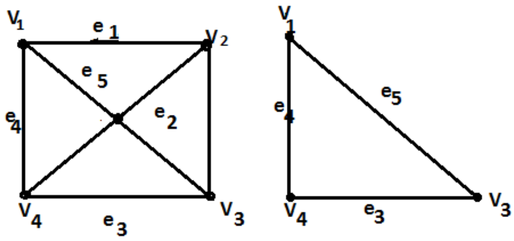
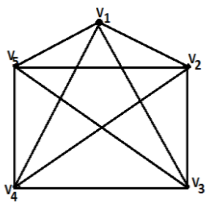


Fig.1.3

Definition 1.5

A simple graph in which each pair of distinct vertices is joined by an edge is called a **complete graph.** A complete graph with n vertices is denoted by K_n.

Example1.5



Definition 1.6

Te **degree of a vertex** of a graph is the number of edges incident to the vertex, with loops counted twice. The degree of a vertex V is denoted by deg (V)

Example 1.6.1

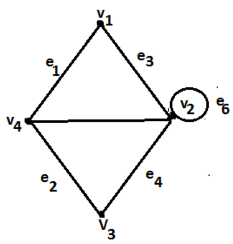


Fig 1.5

d (v_1)=2, d(v_2) =5,d(v_3) =2 and d(v_4) = 3

Definition 1.7

A **Pendant vertex** is a vertex with degree one. An isolated vertex is a vertex of degree zero.

Example 1.7.1

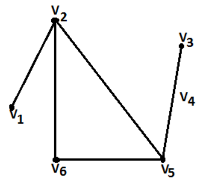


Fig.1.6

V_3 is a pendant vertex and V_4 is an isolated vertex.

Definition 1.8

A walk is a finite alternating sequence of vertices and edges beginning and ending with vertices such that each edge is incident with vertices preceding and following it.

Definition 1.9

If the terminal vertices are distinct, a walk is called **open walk.**

Definition 1.10

If the terminal vertices are the same, a walk is called **closed walk.**

Example 1.10.1

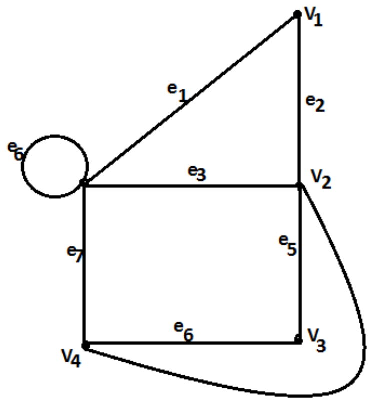


Fig.1.7

$\mathbf{V_1}e_2V_2e_3V_5e_8V_5$ is an open walk.

$V_5e_8V_5e_1V_1e_2V_2e_3V_5$ is a closed walk.

$V_2e_5V_3e_6V_4$ is a path.

$V_2e_5V_3e_6V_4e_4V_2$ is a cycle.

Definition 1.11

A **Tree** is a connected graph which contains no cycle.

Example 1.11.1

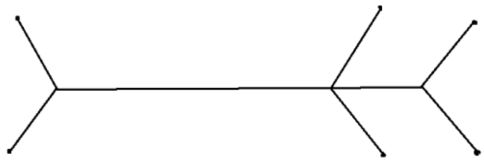


Fig.1.8

Definition 1.12

A **wheel graph** W_n is a graph with n+1 vertices, formed by connecting a single vertex to all the vertices of n cycle. If is denoted by $W_n=C_{n-1}+K_1$

Example 1.12.1

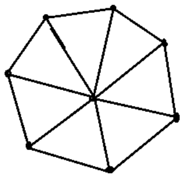


Fig.1.9

Definition 1.13

The complete bipartite graph K1, n is known as **Star graph**

Example 1.13.1

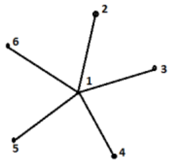


Fig.1.10

Definition1.14

A graph labelling is the assignment of unique identifiers to the edges and vertices of a graph.

Definition.1.15

Given a graph G =(V,E) such that V is the set of vertices and E is the set of edges, a **Vertex labelling** is a function from some subset of the integers to the vertices of the graph.

Example 1.15.1

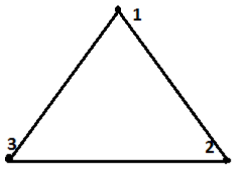


Fig.1.11

Definition 1.16

An **edge labelling** is a function from some subset of the integers to the edges of the graph.

Example 1.16.1

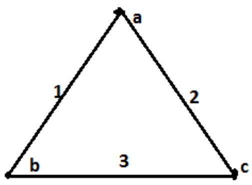


Fig .1.13

Definition 1.17

A binary vertex labelling f of a graph G is called a **Cordial labelling if** $|V_f(0)-V_f(1)|\leq|$ and $|e_f(0)-e_f(1)|\leq|$.A graph G is **Cordial** if it admits cordial

Definition 1.18

Let G=(V(G),E(G)) be a simple graph and f: ˙{1,2…..|V(G)|} be a bijection.For each edge uv,assign the label 1 if $f(u)^2|f(V)$ or $f(v)^2|f(u)$ and the label 0 otherwise is called a square divisor cordial labelling if $|e_f(0)-e_f(1)|$ 1.A graph with a Square divisor cordial labelling is called a **square divisor cordial graph.**

III. STAR AND BISTAR RELATED DIVISOR CORDIAL GRAPHS

Definition 3.1

For a graph G the splitting graph S'(G) of a graph G is obtained by adding a new vertex V' corresponding to each vertex V of G such that N(v)=N(V').

Definition 3.2

Let $G = (V(G),E(G))$ be a graph with $V = S_1 \cup S_2 \cup S_3 \cup, \dots \dots \dots, S_i \cup T$ where each S_i is a set of vertices having atleast two vertices of the same degree and $T = V \backslash \cup S_i$.
The degree splitting graph of G denoted by DS(G) is obtained from G by adding vertices $W_1, W_2, W_3, \dots\dots, W_t$ and joining to each vertex of S_i for $1 \leq i \leq t$.

Definition 3.3

The shadow graph $D_2(G)$ of a connected graph G is constructed by taking two copies of G say G' and G". Join each vertex u' in G' to the neighbors of the corresponding vertex V' in G".

Definition3.4

For a simple connected graph G the square of graph G is denoted by G^2 and defined as the graph with the same vertex set as of G and two vertices are adjacent in G^2 if they are at a distance 1 or 2 a part in G.

Theorem 3.1

$S'(B_{n,n})$ is a divisor cordial graph.

Proof

Consider $B_{n,n}$ with vertex.

Set $\{u,v,u_i,v_i, 1 \leq i \leq n\}$, where u_i, v_i are pendant vertices.

In order to obtain $S'(B_{n,n})$, and u', v', u_i', v_i' vertices corresponding to u,v,u_i,v_i where$1 \leq i \leq n.$
If $G = S'(B_{n,n})$.
Then $|V(G)|=4(n+1)$ and $|E(G)| = 6n+3$.

Define vertex labeling $f:V(G) \to \{1,2,\dots..,4(n+1)\}$ as follows
Let p_1 be the highest prime number $<4(n+1)$.
$f(u) = 2$,
$f(u')=1$,
$f(v)=4$,
$f(V')=p_1$,
$f(u_i)= 6+2(i-1); 1 \leq i \leq n$
$f(u_i') =f(u_n)+2i; 1 \leq i \leq n$
For the vertices $V_1, V_2, V_3, \dots\dots, V_n$ and $V_1', V_2', \dots.., V_n'$, assign distinct odd number (except)p_1. In view of the above labeling pattern,
$e_f(0) = 3n+1$ and $e_f(1) = 3n+2$. Thus, $|e_f(0)-e_f(1)| \leq 1$. Hence, $S'(B_{n,n})$ is a divisor cordial graph.

Example 3.1.1

Divisor cordial labelling of the graph $S'(B_{6,6})$ is shown in figure 4.1

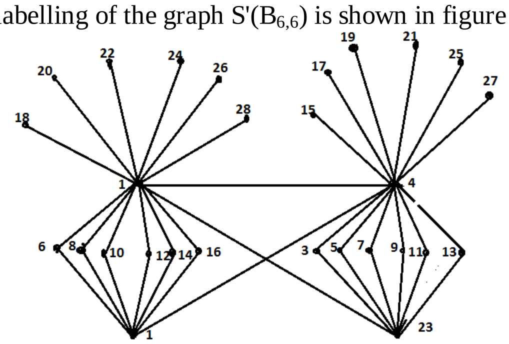


Fig.3.1

Theorem 3.2

$DS(B_{n,n})$ is a divisor cordial graph.

Proof

Consider $B_{n,n}$with$V(B_{n,n}) = \{u,v,u_i,v_i:1 \leq i \leq n\}$ where u_i, v_i are pendent vertices.

Here, $V(B_{n,n}) = v_1 \cup v_2$, where $V_1 = \{u_i, v_i : 1 \leq i \leq n\}$ and $v_2 = \{u,v\}$.
Now in order to obtain $DS(B_{n,n})$ from G, add W_1, W_2 corresponding to v_1, v_2. Then, $|V(DS(B_{n,n}))|=2n+4$ and

$E(DS(B_{n,n})) = \{uv, uw_2, vw_2\} \cup \{uu_i, vv_i, w_1u_i, wv_i : 1 \le i \le n\}$

So, $|E(Ds(B_{n,n}))| = 4n+3$

Define vertex labeling $f : V(DS(B_{n,n})) \to \{1,2,\ldots,2n+4\}$ as follows.

$f(u) = 4$,

$f(v) = 2n+3$,

$f(w_1) = 1$,

$f(w_2) = 2$,

$f(u_i) = 3+2(i-1);\ 1 \le i \le n$.

$f(v_i) = 6+2(i-1);\ 1 \le i \le n$

In view of the above defined labeling pattern. $e_f(0) = 2n+2$ and $e_f(1) = 2n+1$. Thus, $|e_f(0)-e_f(1)| \le 1$. Hence, DS $(B_{n,n})$ is a divisor cordial graph.

Example 3.2.1

Divisor cordial labelling of the graph $DS(B_{5,5})$ is shown in figure 3.2

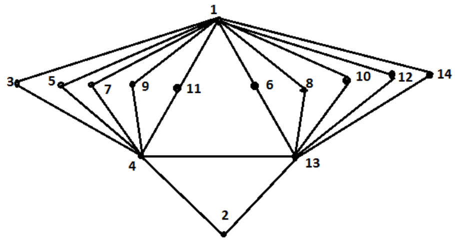


Fig.3.2

Theorem 3.3

$D_2(B_{n,n})$ is a divisor cordial graph.

Proof

Consider two copies of Bn, n.

Let $\{u, v, u_i, v_i, 1 \le i \le n\}$ and $\{u', v', u_i', v_i', 1 \le i \le n\}$ be the corresponding vertex sets of each copy of $B_{n,n}$.

Let G be $D_2(B_{n,n})$ then $|V(G)| = 4(n+1)$ and $|E(G)| = 4(2n+1)$.

Define vertex labeling $f : V(G) \to \{1,2,\ldots,4(n+1)\}$ as follows.

Let p_1 be the highest prime number and p_2 be the second highest prime number such that $p_2 < p_1 < 4(n+1)$.

$f(u) = 2$, $f(u') = 1$, $f(v) = p_1$, $f(v') = p_2$,

$f(v_1) = 4$,

$f(u_i(= 6+2(i-1)\ \ 1 \le i \le n$

$f(u_i') = f(u_n)+2i;\ 1 \le i \le n$

For the vertices $V_2, V_3, \ldots, V_n$ and $V_1', V_2', \ldots, V_n'$ we assign distinct odd number (except p_1 and p_2).

In view of the above defined labeling pattern $e_f(0) = 4n+2 = e_f(1)$

Thus, $|e_f(0) - e_f(10| \le 1$.

Hence, D $(B_{n,n})$ is a divisor cordial.

Example 3.3.1

Divisor cordial labelling of graph $D_2(B_{5,5})$ is shown in figure 3.3

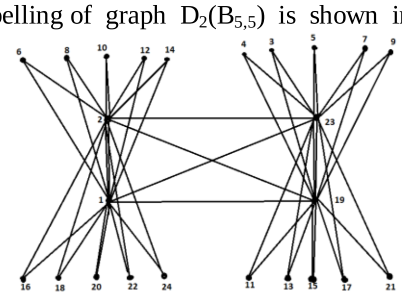


Fig 3.3

Theorem 3.4

$B^2_{n,n}$ is a divisor cordial graph.

Proof

consider $B_{n,n}$ with vertex set $\{u,v,u_i,v_i, 1 \le i \le n\}$ where u_i,v_i are pendent vertices. Let G be $B^2_{n,n}$. Then $|V(G)| = 2n+2$ and $|E(G)| = 4n+1$

Define Vertex labeling $f:V(G \rightarrow \{1,2,\ldots,2n+2\}$ as follows.

Let p_1 be the highest prime number $<2n+2$.

$f(u) = 1, f(v) = p_1, f(u_1) = 2$,

$f(v_1) = 4+2(i-1); \; 1 \le i \le n.$

For the vertices $u_2,u_3,\ldots,u_n$ we assign distinct odd numbers (except p_1). In view of the above defined labelling pattern we have, $e_f(0) = 2n$ and $e_f(1) = 2n+1$.

Thus, $|e_f(0) - e_f(1)| \le 1$.

Hence, $B^2_{n,n}$ is a divisor cordial graph.

Example 3.4.1

Divisor cordial labelling of the graph $B^2_{7,7}$ is shown in figure 3.4.

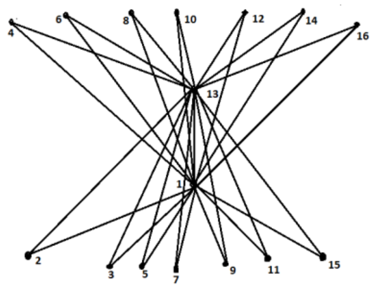


Fig.3.4

IV. CONCLUSIONS

Graph theory is one of the most important branches of mathematics. Graph labelling is one of the potential areas of research due to its diversified applications in computer network and it is a strong communication between number theory and structure of graph. The divisor cordial labelling is a variant of cordial labelling. It is very interesting to investigate to investigate graph or graph families which are divisor cordial as all the graphs do not admit divisor cordial labelling. Initially, the divisor cordial labelling of some standard graphs such as path, cycle, wheel, star, complete bipartite graphs, wheel, $G*K_{2,n}$, $G*K_{3,n}$, $<k^{(1)}_{1,n}, K^{(2)}_{1,n}>$ and $<k^{(1)}_{1,n}, K^{(2)}_{1,n}, K^{(3)}_{1,n}>$ are studied. The properties of some special classes of graphs are also studied. Finally, the star and bistar related graphs like the splitting graphs of $k_{1,n}$ and $B_{n,n}$, the degree splitting graph of $B_{n,n}$, shadow graph of $B_{n,n}$, and square graph of $B_{n,n}$ are studied.

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