

LRS Bianchi Type – II Model with Dark Matter and Holographic Dark Energy

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Abstract

In this paper we have investigated an anisotropic and spatially homogeneous LRS Bianchi type-II universe filled with minimally interacting dark matter and homographic dark energy. The exact solutions of the field equations are obtained by applying the hybrid expansion law for the average scale factor of the model. We have discussed physical and Kinematical properties of the model and the role of the anisotropic holographic dark energy in the evolution of the universe. The universe model is early decelerating and present time accelerating and the deceleration parameter has a signature flip at some finite time. We have observed that the anisotropy parameter of the universe and skewness parameter of the holographic dark energy approach to zero for large cosmic time and the universe can achieve flatness for some finite time throughout its evolution. The results are found to be consistent with the recent cosmological observations.

Keywords -- LRS Bianchi type-II universe. Dark matter. holographic dark energy. Hybrid expansion law. Anisotropic EoS parameter.

I. INTRODUCTION

Recent observations from distant type I_a supernovae as well as the combination of the anisotropies of the Cosmic Microwave Background Radiation (CMBR) and the mass-energy density estimates from galaxy clusters, weak lensing and large scale structure suggest that the universe is undergoing a phase of accelerated expansion [1, 2]. This expansion has been attributed to an exotic component, called dark energy, with negative pressure which can induce repulsive gravitational force causing the accelerated expansion. The evidence for dark energy has been indirectly verified by the measurements of Wilkinson Microwave Anisotropy Probe (WMAP) in the CMBR and the large Scale structure (LSS) observations. WMAP shows that the dark energy constitutes about 73% of the energy of our universe and dark matter occupies 23% whereas the baryon matter occupies only 4% of the total energy of the universe. Naturally a cosmological model is required to explain the accelerated expansion in the present-day universe. The dark energy is considered as a cosmic fluid with the equation of state (EoS) parameter $\omega = p/\rho$, where ρ is the density, p is the pressure and ω need not be time independent. Many cosmologists consider the Einstein's cosmological constant (Λ) as a simplest proposal for dark energy because of its weird repulsive gravity for which $\omega = -1$. But the cosmological constant problem is a long standing problem in physics as it is plagued with the fine-tuning problem. A number of other dynamically evolving scalar field proposals of dark energy such as Quintessence ($\omega > -1$), k – essence ($-1 < \omega < -\frac{1}{3}$), Tachyon ($-1 < \omega < 0$), Phantom ($\omega < -1$) etc are also considered to explain the present accelerated expansion of the universe. Bamba et al. [3] have reviewed different isotropic dark energy cosmologies with early deceleration and late-time acceleration.

Among many proposals to describe the dark cosmological sector, the holographic dark energy models have been discussed by many cosmologists. Hooft [4] first put forward the holographic principle in the context of black-hole physics according to which the number of degrees of freedom for a system within the finite region should be finite and is bounded by the area of its boundary. Holographic dark energy models have been tested and constrained by various astronomical observations (Zhang and Wu [5]; Enqvist et al [6]; Shen et al [7]; Chang et al. [8] etc.) A special class of models in which holographic dark energy is allowed to interact with dark matter has been presented by Pavon and Zimdahl [9], Wang et al [10, 11], Carvalho and Saa [12], Gong and Zhang [13], Huang and Li [14], Nojima and Odinstov [15], Hu and Ling [16], Li et al. [17], Setare [18, 19], Banerjee and Pavon [20], Kim et al. [21] Zimdahl and Pavon [22], Zimdahl [23] etc. Granda and Oliveros [24] have proposed a holographic dark energy density of the form $\rho_\Lambda \propto \alpha H^2 + \beta H$, where H is the Hubble parameter and α, β are constants which must satisfy the restrictions imposed by current observational data. This model of dark energy represents the accelerated expansion of the universe and is consistent with the current observational data.

The anisotropy plays a significant role in the study of the early stage of evolution of the universe. Bianchi types I-IX space-times are important tools for constructing spatially homogeneous and anisotropic cosmological models for describing the early stages of the evolution of universe. The anisotropy of dark energy within the frame work of Bianchi space-times has been found useful in generating arbitrary ellipsoidality of the universe and to fine-tune the observed CMBR anisotropies (Kumar and Singh [25]). This motivates cosmologist to obtain cosmological models in the presence of an anisotropic dark energy within the framework of different Bianchi space-times, Pradhan et al. [26] investigated a class of dark energy models in a locally rotationally symmetric Bianchi type-II with variable EoS parameter and constant deceleration parameter. Sarkar [27, 28, 29] has studied non-interacting holographic dark energy Bianchi type-I and V space-times using linearly varying deceleration parameter and interacting holographic dark energy models Bianchi type-II respectively. Further, Sarkar [30] studied Kantowski-Sachs space-time in the presence of minimally interacting dark matter and holographic dark energy Adhav et al. [31] studied the spatially Homogeneous and anisotropic Bianchi type-I universe with interacting dark matter and holographic dark energy. Besides, some interacting models have been studied Besides, some interacting models have been studied by several cosmologist as these models can be useful to understand the coincident problem by considering the possible interaction between dark energy and cold dark matter due to the unknown nature of dark matter and dark energy.

Motivated by the above discussions, in the present paper, we consider a locally rotationally symmetric (LRS) Bianchi type-II space-time filled with minimally interacting dark matter and holographic dark energy. The paper is organized as follows; In Sect. 2, we present the metric and Einstein's field equations. In Sect. 3, we obtain the solution of the field equations by applying the hybrid expansion law for the average scale factor of the model. The physical and kinematical features of the cosmological model are discussed in Sect. 4. Some concluding remarks are outlined in Sect. 5.

II. METRIC AND FIELD EQUATIONS

We consider the LRS Bianchi type-II space-time in the form [25].

$$ds^2 = \eta_{ij} \theta^i \theta^j, \quad \eta_{ij} = (-1, +1, +1, +1) \quad (1)$$

where the Cartan bases θ^i are given by

$$\theta^1 = A dx, \theta^2 = B (dy + x dz), \theta^3 = A dz, \theta^4 = dt \quad (2)$$

The scale factors A and B being functions of cosmic time t only.

Here we assume that the universe is filled with dark matter and a hypothetical anisotropic fluid as holographic dark energy. The Einstein's field equations (with gravitational units $8\pi G=1$ and $c=1$) are

$$R_{ij} - \frac{1}{2} g_{ij} R = -({}^m T_{ij} + {}^{(\wedge)} T_{ij}) \quad (3)$$

where all symbols have their usual meaning. The energy momentum for dark matter is given by

$${}^{(m)} T_i^j = \text{diag} [\rho_m, 0, 0, 0] \quad (4)$$

ρ_m being the dark matter density.

The simplest generalization of EoS paramter of perfect fluid may be to determine the EoS parameter separately on each spatial axis by preserving the daigonal form of the energy-momentum tensor in a consistent way with the considered metric. Therefore the energy-momentum tensor for an anisotropic holographic dark energy is taken as

$${}^{(\wedge)} T_i^j = \text{diag} [\rho_{\wedge} - \rho_{\wedge_x} - \rho_{\wedge_y}, -\rho_{\wedge_z}] \quad (5)$$

where ρ_{\wedge} is the energy density of the fluid; ρ_{\wedge_x} , ρ_{\wedge_y} and ρ_{\wedge_z} are pressures is the directions of x, y and z respectively. Allowing the anisotropy in the pressure of the holographic dark energy and in its EoS parameter, we have

$${}^{(\wedge)} T_i^j = \text{diag} [1, -\omega_x, -\omega_y, -\omega_z] \rho_{\wedge} \quad (6)$$

$$= \text{diag} [1, -(\omega + \delta), -\omega, -(\omega + \gamma)] \rho_{\wedge} \quad (7)$$

where δ and γ are skewness parameters is, deviation from ω along x and z - axes respectively. Further, since in the metric (1), $G_1^1 = G_3^3$, we have $\delta = \gamma$, and therefore

$${}^{(\wedge)}T_i^j = \text{diag} [1, -(\omega + \delta), -\omega, -(\omega + \delta)] \tag{8}$$

The parameters ω and δ are not necessarily constants and might be functions of cosmic time t

In comoving coordinate system, Einstein's field equations (3) together with (4) and (8) for the metric (1) lead to following system of non-linear equations :

$$2\frac{\ddot{A}}{A} + \frac{\dot{A}^2}{A^2} - \frac{3}{4}\frac{B^2}{A^4} = -(\omega + \delta)\rho_\wedge, \tag{9}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} + \frac{1}{4}\frac{B^2}{A^4} = -\omega\rho_\wedge \tag{10}$$

$$\frac{2\dot{A}\dot{B}}{AB} + \frac{\dot{A}^2}{A^2} - \frac{1}{4}\frac{B^2}{A^4} = \rho_m + \rho_\wedge \tag{11}$$

where an overdot denotes differentiation with respect to cosmic time t.

The spatial volume (V) and the average scale factor a for the metric (1) are given by

$$V = a^3 = A^2B \tag{12}$$

The mean Hubble parameter (H) is given by

$$H = \frac{1}{3} (H_1 + H_2 + H_3) \tag{13}$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$ and $H_3 = H_1$ are directional Hubble parameters in the directions of x, y and z -axes respectively.

The scalar expansion θ , shear scalar σ and the mean anisotropy parameter A_m are given by

$$\theta = \frac{2\dot{A}}{A} + \frac{\dot{B}}{B}, \tag{14}$$

$$\sigma^2 = \frac{1}{3} \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right)^2, \tag{15}$$

$$A_m = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H_i} \right)^2 \tag{16}$$

An important observational quantity in cosmology is the deceleration parameter q conventionally defined by

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{17}$$

The sign of q indicates where the model inflates or not. The positive sign of q corresponds to decelerating model whereas the negative sign indicates inflation.

III. COSMOLOGICAL SOLUTIONS OF THE FIELD EQUATIONS

The field equations (9) – (11) are a system of three independent equations in six unknown parameters A, B, ρ_m , ρ_\wedge , ω and δ . Three additional constants relating these parameters are required to obtain explicit solution of the system.

From (9) and (10), we obtain

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}}{A} \left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} \right) = \left(\frac{B^2}{A^4} - \delta \rho_{\wedge} \right) \tag{18}$$

which can be integrated to give

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{X}{V} + \frac{1}{V} \int \left(\frac{B^2}{A^4} - \delta \rho_{\wedge} \right) V dt \tag{19}$$

where X is an integration constants. The integral term in (19) vanishes when

$$\delta = \frac{1}{\rho_{\wedge}} \frac{B^2}{A^4} \tag{20}$$

Using (20) in (19), we get

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{X}{V}, \tag{21}$$

Differentiation of (12) leads to

$$\frac{2\dot{A}}{A} + \frac{\dot{B}}{B} = \frac{\dot{V}}{V}. \tag{22}$$

Combining (21) and (22), we obtain

$$\frac{\dot{A}}{A} = \frac{\dot{V}}{3V} + \frac{X}{3V}, \tag{23}$$

$$\frac{\dot{B}}{B} = \frac{\dot{V}}{3V} - \frac{2X}{3V} \tag{24}$$

We can determine the cosmic scale functions A and B from (23) and (24) if the function V is explicitly known, the power-law and exponential law cosmologies can be used to describe epoch based evolution of the universe because of the constancy of deceleration parameter. For instance, the cosmologies do not exhibit the transition of universe from deceleration phase to acceleration phase. Akarsu et al. [32] considered the following ansatz for the average scale factor of the universe

$$a(t) = a_1 t^n e^{mt} \tag{25}$$

where $a_1 > 0$, $m \geq 0$ and $n \geq 0$ are constants. They referred this generalized form of the scale factor as hybrid expansion law (HEL), being a mixture of power law and exponential law cosmologies. In HEL cosmology, the universe exhibits transition from deceleration to acceleration. Kumar [33] has studied the dynamics of Bianchi type V model by considering HEL law. Shri Ram and Chandel [34] have discussed the dynamics of magnetized string cosmological models of Bianchi type- V in f(R, T) gravity theory. Chandel and Shri Ram [35] investigated Bianchi type V early model decelerating and late-time accelerating cosmological model with perfect fluid and heat conduction by using HEL. Here we take $a_1 = 1$

Substituting (12), (24) into (23), (24) and integrating the results, we obtain the scale factor A and B of the form.

$$A(t) = k_1 \left(t^n e^{mt} \right) \exp \left[\frac{-X}{3} (3m)^{3n-1} \Gamma(1-3n, 3mt) \right], \tag{26}$$

$$B(t) = k_2 \left(t^n e^{mt} \right) \exp \left[\frac{2X}{3} (3m)^{3n-1} \Gamma(1-3n, 3mt) \right] \tag{27}$$

where k_1 and k_2 are integration constants and $\Gamma(r, s)$ is the lower incomplete lower gamma function without loss of generality we can take $k_1 = k_2 = 1$.

The directional Hubble parameters, the average Hubble parameter, scalar expansion, shear scalar and the anisotropic parameters have the following values.

$$H_1 = H_3 = \left(\frac{n}{t} + m\right) + \frac{X}{3t^{3n}e^{3mt}}, \tag{28}$$

$$H_2 = \left(\frac{n}{t} + m\right) - \frac{2X}{3t^{3n}e^{3mt}}, \tag{29}$$

$$H = \frac{n}{t} + m, \tag{30}$$

$$\theta = 3H = 3\left(\frac{n}{t} + m\right), \tag{31}$$

$$\sigma^2 = \frac{X^2}{3t^{6n}e^{6mt}}, \tag{32}$$

$$A_m = \frac{2X^2}{qt^{6n}e^{6mt}\left(\frac{n}{t} + m\right)^2}, \tag{33}$$

For $M_p^{-2} = 8\pi G = 1$, the holographic dark energy density is given by

$$\rho_\Lambda = 3\left(\alpha H^2 + \beta \dot{H}\right) \tag{34}$$

where α, β are constants and H is the average Hubble parameter (Granda and Oliveros [24]).

Using (28) in (34), we obtain

$$\rho_\Lambda = 3\left[\alpha\left(\frac{n}{t} + m\right)^2 - \frac{\beta n}{t^2}\right], \tag{35}$$

Substitution of (26), (27) and (35) into (11), we find the matter density as

$$\rho_m = 3\left(\frac{n}{t} + m\right)^2 - \frac{X^2}{3t^{6n}e^{6mt}} - \frac{1}{4t^{2n}e^{2mt}}\left\{\frac{8X}{3}(3m)^{2n-1}\Gamma(1-3n, 3mt)\right\} - 3\left\{\alpha\left(\frac{n}{t} + m\right)^2 - \frac{3n}{t^2}\right\} \tag{36}$$

From (10), (20) and (35) we can write the dark energy EoS parameter, skewness parameter and anisotropic parameter as

$$\omega = -\frac{\frac{-2n}{t^2} + 3\left(\frac{n}{t} + m\right)^2 + \frac{X^2}{3t^{6n}e^{6mt}} + \frac{1}{4t^{2n}e^{2mt}}\exp\left\{\frac{8X}{3}(3m)^{3n-1}\Gamma(1-3n, 3mt)\right\}}{3\left\{\alpha\left(\frac{n}{t} + m\right)^2 - \frac{\beta n}{t^2}\right\}}, \tag{37}$$

$$\delta = \frac{\exp\left\{\frac{8X}{3} (3m)^{3n-1} \Gamma(1-3n, 3mt)\right\}}{3t^{2n} e^{2mt} \left\{\alpha \left(\frac{n}{t} + m\right)^2 - \frac{\beta n}{t^2}\right\}}, \quad (38)$$

$$A_m = \frac{2X^2}{q t^{6n} e^{6mt} \left(\frac{n}{t} + m\right)^2}. \quad (39)$$

The deceleration parameter has the value

$$q = -1 + \frac{n}{(n + mt)^2}, \quad (40)$$

IV. DISCUSSION

We now discuss the physical and kinematical features of the holographic dark energy model as follows :

- (I) We observe that the spatial volume of the model is zero when $t = 0$. As this epoch the parameters θ , σ , ρ_m and ρ_Λ are all infinite and therefore the model has a point singularity at $t = 0$. As time increases, these parameters decrease. The expansion scalar θ tends to a constant $(3m)$ while the shear scalar tends to zero at late time. The matter energy density ρ_m and the holographic dark energy density are both constants for large time.
- (II) From (40) we observe that the present universe evolves with variable deceleration parameter which is positive for $t < \frac{\sqrt{n} - n}{m}$ and negative for $t > \frac{\sqrt{n} - n}{m}$, the transition from deceleration to acceleration takes place at the instant $t = \frac{\sqrt{n} - n}{m}$, where n should be in the range $0 < n < 1$. Thus the deceleration parameter has signature flip at this time. As $t \rightarrow \infty$, $q = -1$ which shows the inflationary behaviour of the universe. This further indicates that the present day universe is undergoing through a phase of an accelerated expansion due to the dominance of the dark energy at late time.
- (III) From (31) we see that the anisotropy parameter of expansion A_m is infinite at the initial time of the evolution of the universe which decreases as time increases and ultimately tends to zero at late time. Therefore the model approaches to isotropy after some finite time which matches with the recent observation as the universe is isotropic on large scale. For sufficiently large time we find that $H \sim m$ which again shows that the universe expands forever with constant expansion state with the dominance of dark energy.
- (IV) The EoS parameter ω , given by (37), is a function of time which attains the value $\omega = -\frac{1}{\alpha}$ after some finite time. We observe that for sufficiently large time the present model approaches to (i) a phantom dark energy model ($\omega < -1$) for $0 < \alpha < 1$, (ii) a Λ CDM model ($\omega = -1$) for $\alpha = 1$ and (iii) a quintessence scalar field model ($\omega > -1$) for $\alpha > 1$.
- (V) The skewness parameter δ , given by (38), is infinite at the initial stage of evolution of the universe which tends to zero for large cosmic time. Thus, for large time, the anisotropy of the holographic dark energy model dies out and the model becomes isotropic.
- (VI) The recent observations demand that the ratio of two energy densities $M = \rho_\Lambda / \rho_m$ is, the coincident parameter stays constant or varies very slowly around the present time with respect to the expansion of the universe for the present model, we find that

$$M = \frac{3 \left\{ \alpha \left(\frac{n}{t} + m \right)^2 - \frac{\beta n}{t^2} \right\}}{3 \left(\frac{n}{t} + m \right)^2 - \frac{X^2}{3t^{6n} e^{6mt}} - \frac{1}{4t^{2n} e^{2mt}} \exp \left\{ \frac{8X}{3} (3m)^{3n-1} \Gamma(1-3n, 3mt) \right\}}. \quad (41)$$

It is observed that M converges to a constant value α after some finite time and remains constant throughout further evolution of the universe.

(VII) The matter density parameter (Ω_m) and the holography dark energy parameter (Ω_Λ) are defined by

$$\Omega_m = \frac{\rho_m}{3H^2} \text{ and } \Omega_\Lambda = \frac{\rho_\Lambda}{3H^2}, \quad (42)$$

$$\Omega_m = 3 \left(\frac{n}{t} + m \right)^2 - \frac{1}{3} \frac{X^2}{t^{6n} e^{6mt}} - \frac{1}{4t^{2n} e^{2mt}} \exp \left\{ \frac{8X}{3} (3m)^{3n-1} \Gamma(1-3n, 3mt) \right\}, \quad (43)$$

$$\Omega_\Lambda = \alpha - \frac{\beta n}{(n + mt)^2}. \quad (44)$$

Using (43) and (44) we get the overall density parameter $\Omega_m + \Omega_\Lambda$ which approaches close to 1, for sufficiently large time. This means that the present universe approaches towards a flat spatially homogeneous and isotropic universe but the flatness of the universe will occur for some particular moment throughout the entire history of the universe.

V. CONCLUSION

In this paper, we have studied an anisotropic LRS Bianchi type-II early decelerating and late-time accelerating universe filled with minimally interacting dark matter and holographic dark energy components. We have obtained exact solution of Einstein's field equations by using hybrid expansion law for the average scale factor of the model. We have found that the anisotropy of expansion and the skewness parameter of the holographic dark energy vanish for large cosmic time leading to an isotropic universe. The EoS parameter of the dark energy is a function of time and assumes a constant negative value for large time and consequently the present and future holographic dark energy model behaves like a phantom model, Λ CDM model and quintessence model for specific values of the parameter (α).

The flatness of the universe can be achieved for some particular time throughout the entire history of the universe. Since the Hubble parameter tends to a constant and the deceleration parameter approaches to -1 for large time the universe is accelerating forever due to the dominance of the dark energy. The results obtained in this paper are consistent with recent cosmological observations.

REFERENCE

- [1] Riess, A.G. et al : Astron. J. 116, 1009 (1998)
- [2] Perlmutter, S, et al. : Nature 391, 51 (1998)
- [3] Bamba, K; et al.; Astrophys Space Sci. 342, 155 (2012)
- [4] Hooft, G; gr. qc/9310026 (1995)
- [5] Zhang, X; Wu, F.Q.; Phys. Rev. D. 72, 043524 (2005)
- [6] Enguist, K; et al.; J. Cosmol. Astropart Phys. 0502, 004 (2005)
- [7] Shen, J; et al : Phys. Lett. B 609, 200 (2005)
- [8] Chang, Z; et al. : Phys. Lett. B 633, 14 (2006)
- [9] Pavon, D; Zimdahl, W : Phys Lett B 628, 206 (2005)
- [10] Wang, B; et al; Phys. Lett. B 624, 141 (2005)
- [11] Wang, B; et al.; Phys Lett. B 637, 357 (2006)
- [12] Carvalho, F.C., Saa, A. : Phys. Rev. D 70, 087302 (2004)

- [13] Gong, Y.G; Zhang, Y.Z; class. Quantum gravity 22, 4895 (2005)
- [14] Huang, Q.G; Li, M; J, Cosmol, Astropart Phys, 0408, 013 (2004)
- [15] Nojima, S; Odinstov, S.D; Phys. Lett. B 565 (2003)
- [16] Hu, B., Ling, Y. : Phys. Rev. D 73, 123510 (2006)
- [17] Li, H; et al; Int. J. Mod. Phys. D 15, 869 (2006)
- [18] Setare, M.R; Phys. Lett. B 642, 1. (2006)
- [19] Setare, M.R; Phys. Lett. B 644, 99 (2007)
- [20] Banerjee, N; Pavon, D; Phys. Lett. B. 647, 477 (2007)
- [21] Kim, H. et al; Phys. Lett. B 632, 605 (2006)
- [22] Zimdahl, W; Pavon, D. : Gen. Rel. Gravit. 36, 1483 (2004)
- [23] Zimdahl, W; Int. J. Mod. Phys. D 17, 651 (2008)
- [24] Granda, L.N., Oliveros, A : Phys. (2011) Lett. 669, 275 (2008).
- [25] Kumar, S; Singh, C.P. : Gen. Relativ. Gravit 43, 1427 (2011).
- [26] Pradhan, A. et al. : Astrophys Space Sci. 334 (2011).
- [27] Sarkar, S. : Astrophys Space Sci. 349, 985 (2014)
- [28] Sarkar, S. : Astrophys, Space Sci. (2014) doi : 10.1007/s 10509-014-1920.0
- [29] Sarkar, S. : Astrophys, Space Sci. 350, 821 (2014)
- [30] Sarkar, S. : Astrophys, Space Sci. 351, 361 (2014)
- [31] Adhav, K.S; et al. : Astrophys. Space Sci. Doi 10.1007/s 10509-014-2015-7
- [32] Akarsu, A., et al. : (2014). ar xIV : 1307. 4911
- [33] Kumar, S. : Gravit. Cosmol. 19, 284 (2013)
- [34] Shri Ram, Chandel, S. : Astrophys – Space Sci. 355, 2160 (2014)
- [35] Chandel, S., Shri Ram : Candian J. Phys. 93, 632 (2015)