A New General Multivalent Integral Operator

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Abstract — *In this paper, we define a new multivalent integral operator for certain subclass of analytic functions in the open unit disc* U. *We obtain some interesting properties for this integral operator.*

Keywords — *analytic function, multivalent function, integral operator, starlike and convex functions.*

I. INTRODUCTION, DEFINITIONS AND PRELIMINARIES

Let A_p be the class of functions $f(z)$, of the form

$$f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n, \qquad (p \in \mathbb{N}) \tag{1.1}$$

which are analytic in the unit disc $\mathrm{U} = \{z \in \mathbb{C} : |z| < 1\}$. And let $\mathrm{A} = \mathrm{A}_1$.

We denote by S^*, C, K and C^* the familiar subclasses of A consisting of functions which are respectively starlike, convex, close-to-convex and quasi-convex in U.

A function $f(z) \in \mathrm{A}_p$ is said to be $p-$ valently starlike of order $\delta\left(0 \le \delta < p\right)$ and $z \in \mathrm{U}$ denoted by $\mathrm{S}_p^*(\delta)$, if and only if

$$Re\left\{\frac{zf'(z)}{f(z)}\right\} > \delta.$$

A function $f(z) \in \mathrm{A}_p$ is said to be $p-$valently convex of order $\delta\left(0 \le \delta < p\right)$ and $z \in \mathrm{U}$ denoted by $\mathrm{C}_p(\delta)$, if and only if

$$Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \delta.$$

It is easy to see that $f(z) \in \mathrm{C}_p(\delta) \Leftrightarrow \dfrac{zf'(z)}{p} \in \mathrm{S}_p^*(\delta).$

Furthermore, $\mathrm{S}_p^*(0) = \mathrm{S}_p^*$, $\mathrm{C}_p(0) = \mathrm{C}_p$ are respectively, the classes of $p-$ valently starlike, convex functions in U. Also, let $p = 1$, the above classes reduced to $\mathrm{S}_1^* = \mathrm{S}^*$, $\mathrm{C}_1(0) = \mathrm{C}$.

A function $f(z) \in \mathrm{A}_p$ is said to be in the class $k - \mathrm{US}_p(\delta, \lambda)$ of k - uniformly $p-$valent starlike of order $\delta\left(0 \le \delta < p\right)$ in $z \in \mathrm{U}$ and satisfies

$$Re\left\{\frac{zf'(z)}{f(z)} - \delta\right\} \ge k\left|\frac{zf'(z)}{f(z)} - p\right|.$$

Further, a function $f(z) \in \mathrm{A}_p$ is said to be in the class $k - \mathrm{UC}_p(\delta, \lambda)$ of k - uniformly $p-$ valent convex of order $\delta\left(0 \le \delta < p\right)$ in $z \in \mathrm{U}$ and satisfies

$$Re\left\{1+\frac{zf''(z)}{f'(z)}-\delta\right\}\geq k\left|1+\frac{zf''(z)}{f'(z)}-p\right|.$$

In particular, when $p=1$, we obtain $k-UST(\delta)$ and $k-UCV(\delta)$, the classes of $k-$ uniformly starlike and $k-$ uniformly convex functions of order $\delta,-1<\delta<1$, respectively which were studied by various authors, example see [9].

A function $f(z)\in A_p$ is said to be in the class $S_p^*(b,\delta)$ of $p-$valently starlike of complex order $b\left(b\in\mathbb{C}-\{0\}\right)$ and type $\delta\left(0\leq\delta<p\right)$, if it satisfies the following inequality

$$Re\left\{p+\frac{1}{b}\left(\frac{zf'(z)}{f(z)}-p\right)\right\}>\delta,\qquad (z\in U). \tag{1.2}$$

A function $f(z)\in A_p$ is said to be in the class $C_p(b,\delta)$ of $p-$valently convex of complex order $b\left(b\in\mathbb{C}-\{0\}\right)$ and type $\delta\left(0\leq\delta<p\right)$, if it satisfies the following inequality

$$Re\left\{p+\frac{1}{b}\frac{zf''(z)}{f'(z)}\right\}>\delta,\qquad (z\in U). \tag{1.3}$$

For $p=1$ and $\delta=0$, the above classes reduced to the following classes:

$$S_p^*(b)=\left\{Re\left\{1+\frac{1}{b}\left(\frac{zf'(z)}{f(z)}-1\right)\right\}>0,\quad \left(b\in\mathbb{C}-\{0\}\right)\quad (z\in U)\right\}$$

which is defined by Nasr and Aouf [8] and

$$C_p(b)=\left\{Re\left\{1+\frac{1}{b}\frac{zf''(z)}{f'(z)}\right\}>\delta,\quad \left(b\in\mathbb{C}-\{0\}\right)\quad (z\in U)\right\}.$$

defined by Wiatrowski [14].

The Hadamard product of two functions $f(z)=z+\sum_{n=2}^{\infty}a_nz^n$ and $g(z)=z+\sum_{n=2}^{\infty}b_nz^n$ is given by

$$\left(f*g\right)(z)=z+\sum_{n=2}^{\infty}a_nb_nz^n.$$

For a function $f\in A_p$, we define the following operator

$$\begin{aligned}
&D^0f(z)=f(z)\\
&D^1f(z)=\frac{1}{p}zf'(z)\\
&\quad\vdots\\
&D^kf(z)=D\left(D^{k-1}f(z)\right)\qquad \left(k\in\mathbb{N}_0,z\in U\right).
\end{aligned}\tag{1.4}$$

The differential operator D^k was introduced by Shenan et al.[12]. When $p=1$, we get a familiar Salagean derivative [10].

By using the above operator, we define the following new classes:

Definition 1.1: A function $f\in A_p$ is said to be in the class $S_{p,k}^*(b,\delta)$ of $p-$ valently starlike of complex order $b\left(b\in\mathbb{C}-\{0\}\right)$ and type $\delta\left(0\leq\delta<p\right)$, if it satisfies the following inequality

$$Re\left\{p+\frac{1}{b}\left(\frac{z\left(D^kf(z)\right)'}{D^kf(z)}-p\right)\right\}>\delta,\qquad (z\in U). \tag{1.5}$$

Definition 1.2: A function $f \in A_p$ is said to be in the class $C_{p,k}(b,\delta)$ of $p-$ valently convex of complex order $b\left(b \in \mathbb{C} - \{0\}\right)$ and type $\delta\left(0 \le \mu < p\right)$, if it satisfies the following inequality

$$Re\left\{ p + \frac{1}{b}\frac{z\left(D^k f(z)\right)''}{\left(D^k f(z)\right)'} \right\} > \delta, \qquad (z \in U). \tag{1.6}$$

Definition 1.3: A function $f \in A_p$ is said to be in the class $CV_{p,k}(\lambda,\mu)$ and $\delta\left(0 \le \delta < p\right)$, if it satisfies the following inequality

$$Re\left\{ 1 + \frac{z\left(D^k f(z)\right)''}{\left(D^k f(z)\right)'} \right\} \ge \lambda \left| 1 + \frac{z\left(D^k f(z)\right)''}{\left(D^k f(z)\right)'} - p \right| + \mu, \qquad (z \in U) \tag{1.7}$$

for some $\lambda \ge 0$ and $\mu(0 \le \mu < 1)$.

The class $CV_{p,0}(\lambda,\mu)$ introduced and studied by Yang and Owa [15]. For $p = 1, \lambda = 1$, we have the class $UC(\mu)$ considered by Owa [9]. Specializing the values of the parameters p,k,δ and b, the above classes $S^*_{p,k}(b,\delta)$ and $C_{p,k}(b,\delta)$ reduces to the several well-known subclasses, which subclasses are introduced and investigated by various authors (see [4], [14], [10] and [6]).

Definition 1.4: Let $n \in \mathbb{N}$, $\alpha_i, \beta_i \in \mathbb{R}_+ \cup \{0\}$, $m = (m_1, m_2, \ldots, m_n) \in \mathbb{N}_0^n$, $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$, $\beta = (\beta_1, \beta_2, \ldots, \beta_n)$ and $\gamma \in \mathbb{C}$ with $Re(\gamma) > 0$. For $f_i, g_i \in A_p$ for all $i = 1,2,3,\ldots,n$, we introduced a new general integral operator $I^{\alpha_i,\beta_i}_{p,n,\gamma}(f_i,g_i) : A_p \to A_p$ by

$$I^{\alpha_i,\beta_i}_{p,n,\gamma}(z) = \left[\int_0^z \gamma p t^{\gamma p - 1} \prod_{i=1}^{n} \left(\frac{D^{m_i} f_i(t)}{t^p} \right)^{\alpha_i} \left(\frac{\left(D^{m_i} g_i(t)\right)'}{p t^{p-1}} \right)^{\beta_i} dt \right]^{\frac{1}{\gamma}}. \tag{1.8}$$

Remark 1.1: This integral operator $I^{\alpha_i,\beta_i}_{p,n,\gamma}$ generalizes the following several well-known operators introduced and studied by various authors:

- If $\beta_i = 0,\ m_i = 0$ and $\gamma = 1$, then this integral operator reduced to the operator $F_p(z)$ which was studied by Frasin [5].
- If $\beta_i = 0,\ \gamma = 1$ and $\alpha_i = \mu_i$, then this integral operator reduced to the operator

$$F_{p,n,l,\mu}(z) = \int_0^z p t^{p-1} \prod_{i=1}^{n} \left(\frac{D^{l_i} f_i(t)}{t^p} \right)^{\mu_i} dt \tag{1.9}$$

  which was studied by Saltik et al. [11].
- For $p = 1,\ \gamma = 1,\ m_i = 0$ and $\beta_i = \gamma_i$, then we obtain the operator

$$G_n(z) = \int_0^z \prod_{i=1}^{n} \left(\frac{f_i(t)}{t} \right)^{\alpha_i} \left(g_i'(t) \right)^{\gamma_i} dt \tag{1.10}$$

  introduced and studied by Stanciu and Breaz [13].
- For $p = 1,\ \gamma = 1,\ m_i = 0$ and $\beta_i = 0$, then we obtain the operator

$$F_n(z) = \int_0^z \prod_{i=1}^{n} \left(\frac{f_i(t)}{t} \right)^{\alpha_i} dt \tag{1.11}$$

  introduced and studied by D. Breaz and N. Breaz [2].

- For $p=1,\ \gamma=1,\ m_i=0,\ \beta_i=\gamma_i$ and $\alpha_i=0$, then we obtain the operator

$$F_{\gamma_1,\gamma_2,\ldots,\gamma_n}(z)=\int_0^z \left(g_1'(t)\right)^{\gamma_1}\left(g_2'(t)\right)^{\gamma_2}\cdots\left(g_n'(t)\right)^{\gamma_n}dt \tag{1.12}$$

introduced and studied by Breaz et al. [3].

- For $p=n=1,\ \gamma=1,\ m_i=0,\ \alpha_i=\alpha$ and $\beta_i=0$, then we obtain the operator

$$F_\alpha(z)=\int_0^z\left(\frac{f(t)}{t}\right)^\alpha dt \tag{1.13}$$

introduced and studied in [7]. In particular, for $\alpha=1$, we obtain Alexander integral operator

$$I(z)=\int_0^z\left(\frac{f(t)}{t}\right)dt$$ introduced in [1].

I. MAIN RESULTS

In this section, we obtain the sufficient condition for the integral operator $\mathrm{I}_{p,n,\gamma}^{\alpha_i,\beta_i}(z)$.

Theorem 2.1: Let α_i,β_i be positive real numbers $(i=1,2,3,\ldots,n)$. If $f_i\in S_{p,k}^*(b,\delta)\ (0\le\delta<1)$ and $g_i\in C_{p,k}(b,\delta),\ (i=1,2,3,\ldots,n)$ then the integral operator $\mathrm{I}_{p,n,\gamma}^{\alpha_i,\beta_i}$ defined in (1.8) is in the class $C_p(\eta)$, where

$$\eta=p+\sum_{i=1}^n\left[\alpha_i(\delta_i-p)+\beta_i(\lambda_i-p)-\beta_i\frac{Re\{b\}}{|b|^2}(p-1)\right]+(p-1)\frac{Re\{b\}}{|b|^2}.$$

Proof: From (1.8), it is easy to see that

$$\mathrm{I}'_{p,n,\gamma}(z)=pz^{p-1}\prod_{i=1}^n\left(\frac{D^{m_i}f_i(z)}{z^p}\right)^{\alpha_i}\left(\frac{\left(D^{m_i}g_i(z)\right)'}{pz^{p-1}}\right)^{\beta_i}. \tag{2.1}$$

Differentiating (2.1) logarithmically with respect to z and multiply by z, we get

$$\frac{z\mathrm{I}''_{p,n,\gamma}(z)}{\mathrm{I}'_{p,n,\gamma}(z)}=(p-1)+\sum_{i=1}^n\alpha_i\left[\frac{z\left(D^{m_i}f_i(z)\right)'}{\left(D^{m_i}f_i(z)\right)}-p\right]+\sum_{i=1}^n\beta_i\left[\frac{z\left(D^{m_i}g_i(z)\right)''}{\left(D^{m_i}g_i(z)\right)'}-(p-1)\right],$$

which implies

$$\begin{aligned}p+\frac{1}{b}\frac{z\mathrm{I}''_{p,n,\gamma}(z)}{\mathrm{I}'_{p,n,\gamma}(z)}&=\frac{(p-1)}{b}+\sum_{i=1}^n\alpha_i\left[p+\frac{1}{b}\left(\frac{z\left(D^{m_i}f_i(z)\right)'}{\left(D^{m_i}f_i(z)\right)}-p\right)\right]\\&+\sum_{i=1}^n\beta_i\left[p+\frac{1}{b}\frac{z\left(D^{m_i}g_i(z)\right)''}{\left(D^{m_i}g_i(z)\right)'}\right]-\sum_{i=1}^n\beta_i\left(\frac{p-1}{b}\right)-p\sum_{i=1}^n\alpha_i-p\sum_{i=1}^n\beta_i+p.\end{aligned} \tag{2.2}$$

We calculate the real part of (2.2), we get

$$\begin{aligned}Re\left(p+\frac{1}{b}\frac{z\mathrm{I}''_{p,n,\gamma}(z)}{\mathrm{I}'_{p,n,\gamma}(z)}\right)&=p+Re\left(\frac{p-1}{b}\right)+\sum_{i=1}^n\alpha_iRe\left[p+\frac{1}{b}\left(\frac{z\left(D^{m_i}f_i(z)\right)'}{\left(D^{m_i}f_i(z)\right)}-p\right)\right]\\&+\sum_{i=1}^n\beta_iRe\left[p+\frac{1}{b}\frac{z\left(D^{m_i}g_i(z)\right)''}{\left(D^{m_i}g_i(z)\right)'}\right]-\sum_{i=1}^n\beta_iRe\left(\frac{p-1}{b}\right)-p\sum_{i=1}^n\beta_i-p\sum_{i=1}^n\alpha_i.\end{aligned} \tag{2.3}$$

Since $f_i \in S^*_{p,k}(b,\delta)\ (0 \le \delta < 1)$ and $g_i \in C_{p,k}(b,\lambda),\ (i = 1,2,3,\dots,n)$, we obtain

$$Re\left(p + \frac{1}{b}\frac{zI''_{p,n,\gamma}(z)}{I'_{p,n,\gamma}(z)}\right) > p + Re\left(\frac{p-1}{b}\right) + \sum_{i=1}^{n}\alpha_i\delta_i + \sum_{i=1}^{n}\beta_i\lambda_i$$

$$-\sum_{i=1}^{n}\beta_i Re\left(\frac{p-1}{b}\right) - p\sum_{i=1}^{n}\alpha_i - p\sum_{i=1}^{n}\beta_i \tag{2.4}$$

$$> p + \sum_{i=1}^{n}\left[\alpha_i(\delta_i - p) + \beta_i(\lambda_i - p) - \beta_i(p-1)\frac{Re(b)}{|b|^2}\right] + (p-1)\frac{Re(b)}{|b|^2}.$$

Hence $I^{\alpha_i,\beta_i}_{p,n,\gamma} \in C_p(\eta)$, where

$$\eta = p + \sum_{i=1}^{n}\left[\alpha_i(\delta_i - p) + \beta_i(\lambda_i - p) - \beta_i(p-1)\frac{Re(b)}{|b|^2}\right] + (p-1)\frac{Re(b)}{|b|^2}.$$

□

Remark 2.1: For the choices of the parameters γ, β and α, we get the following results for the various authors:

- Letting $p = 1,\ \gamma = 1,\ m_i = 0$ and $\beta_i = \gamma_i$, in Theorem 2.1, we obtain Theorem 2.1 in [13].
- If $\beta_i = 0,\ \gamma = 1$ and $\alpha_i = \mu_i$, in Theorem 2.1, we obtain the result in Theorem 2.1 Saltik et al. [11].

Theorem 2.2: Let α_i, β_i be positive real numbers $(i = 1,2,3,\dots,n)$. We suppose that the functions f_i are starlike functions by order $\frac{1}{\alpha_i}$, that is $f_i \in S^*_{p,k}\left(1, \frac{1}{\alpha_i}\right)$ and $g_i \in CV_{p,\lambda_i}(\mu_i),\ 0 \le \mu_i < 1,\ i = 1,2,\dots,n$. If

$$\sum_{i=1}^{n}\left[\beta_i(p - \mu_i) + p\alpha_i\right] - n < p,$$

then the integral operator $I^{\alpha_i,\beta_i}_{p,n,\gamma}$ defined in (1.8}) is in the class $C(\nu)$, where

$$\nu = p + n + \sum_{i=1}^{n}\left[\beta_i(\mu_i - p) - p\alpha_i\right].$$

Proof: Using a similar argument in Theorem 2.1, we have

$$\frac{zI''_{p,n,\gamma}(z)}{I'_{p,n,\gamma}(z)} = (p-1) + \sum_{i=1}^{n}\left[\alpha_i\left(\frac{z(D^{m_i}f_i(z))'}{(D^{m_i}f_i(z))} - p\right) + \beta_i\left(\frac{z(D^{m_i}g_i(z))''}{(D^{m_i}g_i(z))'} - (p-1)\right)\right]$$

$$= (p-1) + \sum_{i=1}^{n}\left[\alpha_i\left(\frac{z(D^{m_i}f_i(z))'}{(D^{m_i}f_i(z))}\right) - \alpha_i p + \beta_i\left(\frac{z(D^{m_i}g_i(z))''}{(D^{m_i}g_i(z))'} - (p-1)\right)\right].$$

(2.5)

From (2.5), we have

$$1 + \frac{zI''_{p,n,\gamma}(z)}{I'_{p,n,\gamma}(z)} = p + \sum_{i=1}^{n}\left[\alpha_i\left(\frac{z(D^{m_i}f_i(z))'}{(D^{m_i}f_i(z))}\right) - \alpha_i p + \beta_i\left(\frac{z(D^{m_i}g_i(z))''}{(D^{m_i}g_i(z))'} - (p-1)\right)\right] \tag{2.6}$$

Taking the real part of the above expression, we obtain

$$\begin{aligned} Re\left(1+\frac{zI''_{p,n,\gamma}(z)}{I'_{p,n,\gamma}(z)}\right) &= p+\sum_{i=1}^{n}\alpha_i Re\left(\frac{z\left(D^{m_i}f_i(z)\right)'}{\left(D^{m_i}f_i(z)\right)}\right)-p\sum_{i=1}^{n}\alpha_i \\ &+\sum_{i=1}^{n}\beta_i Re\left(\frac{z\left(D^{m_i}g_i(z)\right)''}{\left(D^{m_i}g_i(z)\right)'}+1\right)-p\sum_{i=1}^{n}\beta_i \\ &= p+\sum_{i=1}^{n}\alpha_i Re\left(\frac{z\left(D^{m_i}f_i(z)\right)'}{\left(D^{m_i}f_i(z)\right)}\right)-p\sum_{i=1}^{n}\alpha_i \\ &+\sum_{i=1}^{n}\beta_i Re\left(\frac{z\left(D^{m_i}g_i(z)\right)''}{\left(D^{m_i}g_i(z)\right)'}+1\right)-p\sum_{i=1}^{n}\beta_i. \end{aligned} \tag{2.7}$$

But $f_i \in S^*_{p,k}\left(1,\frac{1}{\alpha i}\right)$, so $Re(\frac{z\left(D^{m_i}f_i(z)\right)'}{\left(D^{m_i}f_i(z)\right)})>\frac{1}{\alpha_i}$ and since $g_i \in CV_{p,\lambda_i}(\mu_i)$ for $\mu_i \geq 0$ and $0\leq\lambda_i<1,\ i=1,2,\ldots,n$, from (2.7),

$$\begin{aligned} Re\left(1+\frac{zI''_{p,n,\gamma}(z)}{I'_{p,n,\gamma}(z)}\right) &> p+n-p\sum_{i=1}^{n}\alpha_i+\sum_{i=1}^{n}\beta_i\left(\lambda_i\left|1+\frac{z\left(D^{m_i}g_i(z)\right)''}{\left(D^{m_i}g_i(z)\right)'}-p\right|+\mu_i\right)-p\sum_{i=1}^{n}\beta_i \\ &> p+n-p\sum_{i=1}^{n}\alpha_i+\sum_{i=1}^{n}\beta_i\lambda_i\left|1+\frac{z\left(D^{m_i}g_i(z)\right)''}{\left(D^{m_i}g_i(z)\right)'}-p\right|+\sum_{i=1}^{n}\beta_i\left(\mu_i-p\right) \end{aligned} \tag{2.8}$$

Since $\beta_i\lambda_i\left|1+\frac{z\left(D^{m_i}g_i(z)\right)''}{\left(D^{m_i}g_i(z)\right)'}-p\right|>0$, we obtain

$$\begin{aligned} Re\left(1+\frac{zI''_{p,n,\gamma}(z)}{I'_{p,n,\gamma}(z)}\right) &> p+n-p\sum_{i=1}^{n}\alpha_i+\sum_{i=1}^{n}\beta_i\left(\mu_i-p\right) \\ &> p+n+\sum_{i=1}^{n}\left[\beta_i\left(\mu_i-p\right)-p\alpha_i\right]. \end{aligned} \tag{2.9}$$

Using the hypothesis $\sum_{i=1}^{n}\left[\beta_i\left(p-\mu_i\right)+p\alpha_i\right]-n<p$ in (2.9), we obtain that the integral operator $I^{\alpha_i,\beta_i}_{p,n,\gamma}$ is in the class $C(\nu)$, where

$$\nu = p+n+\sum_{i=1}^{n}\left[\beta_i\left(\mu_i-p\right)-p\alpha_i\right]. \qquad \square$$

Taking $n=1$ in Theorem 2.2, we obtain the following corollary:

Corollary 2.1: Let α,β be positive real numbers. We suppose that the functions f is a starlike functions by order $\frac{1}{\alpha}$, that is $f\in S^*_{p,k}\left(1,\frac{1}{\alpha}\right)$ and $g_i \in CV_{p,\lambda}(\mu)$. If

$$\beta(p-\mu)+p\alpha<1+p,$$

then the integral operator $I_{p,\gamma}^{\alpha,\beta}$ defined in (1.8}) is in the class $C(\nu)$, where

$$\nu=p+1+\beta(\mu-p)-p\alpha.$$

Letting $p=1$, for the choices of ν and β the above Corollary 2.1 reduce to the following result, which was proved earlier by [13].

Corollary 2.2: Let α,β be positive real numbers. We suppose that the functions f is a starlike functions by order $\frac{1}{\alpha}$, that is $f\in S_{1,k}^{*}\left(1,\frac{1}{\alpha}\right)$ and $g_i\in CV_{1,\lambda}(\mu)$. If

$$\beta(1-\mu)+\alpha<2,$$

then the integral operator $I_{1,\gamma}^{\alpha,\beta}$ defined in (1.8}) is in the class $C(\nu)$, where

$$\nu=2+\beta(\mu-1)-\alpha.$$

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