To The Study of Preliminaries on Semi Open Sets And Semi Continuous Functions

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Abstract — *"Mathematics is the supreme judge; from its decisions there is no appeal" -Tobias Danzig, Chapter 1 is devoted to the study of Preliminaries on semi open sets and semi continuous functions. Chapter 2 is devoted to the study of μ-open sets. In section one of chapter 2; we study Properties and characterization of μ-open sets and relations between regular open sets, μ-open sets and semi open sets. In section two of chapter 2, we study μ-adherent, μ-closure of a subset A of a topological space, μ-irresolute function between topological spaces and equivalence relation μ-correspondence on the set of a topologies of a set X. Chapter 3 is devoted to generalize the concept of fuzzy μ-open sets. In section one of chapter 3, preliminary results on fuzzy sets and fuzzy topological spaces that are needed for our study are collected. In Section two of chapter 3 is devoted to extend μ-open sets to fuzzy situation. We study properties and characterization of fuzzy μ-open sets.*

Keywords — *Fuzzy numbers, L - R type fuzzy numbers, trapezoidal numbers, and fuzzy μ-open*

I. INTRODUCTION

In 1963,Levine[14] introduced the concept of semiopen sets. Mashhour et al[19] introduced the concept of preopen sets. Kuratowski[12] introduced the concept of regular open sets. Cameron[5] defined regular semiopen sets. Sharma[25] has renamed regular semi open sets as μ-open sets.

Maheswari and Prasad[16] introduced the concepts of semi T_0, semi T_1, semi T_2 axioms in topological spaces using semiopen sets. Malghan and Benchalli[18] defined the concepts of rT_0 and rT_1 axioms in topological spaces using regularly open sets. Sharma[26] introduced the concepts of μT_0 ,μT_1 and μT_2 spaces in topological spaces.

The notation of a fuzzy set introduced by Zadeh[30] in 1965, has caused great interest among both 'pure' and applied mathematicians. It has also raised enthusiasum among some engineers, biologists, psychologists, economists and experts in other areas, who use mathematical ideas and methods in their research. General topology was one of the first branches of pure mathematics to which fuzzy sets have been applied systematically. In 1968, Chang[6] made the first "grating" of the notion of a fuzzy set onto general topology and defined fuzzy topological space. After the introduction of fuzzy set of Zadeh[30] and fuzzy topological space by Chang[6], several author's have worked on these concepts and developed the theory of fuzzy sets and fuzzy topological space in many directions and is applied in a wide variety of fields.

The aim of this dissertation is to generalize the concept of μ-open sets due to Sharma[25] to fuzzy topological spaces.

The dissertation is devoted to the study of the following concepts.

(1) Semi open sets and Semi continuous functions.
(2) μ-open sets and μ-axiom
(3) Fuzzy μ-open sets.

The results discussed are contained in the following articles.

(1) Semi open sets and semi continuity in topological spaces by Levine.N[14].
(2) μ-open sets by Sharma,V.K[25].
(3) μ-axiom by Sharma,V.K[26].
(4) Fuzzy topological spaces by Chang.L[6].

Chapter 1 is devoted to the study of semi open sets and semi continuous functions due to Levine [14]. Preliminaries on semi open sets and semi continuous functions that are needed for our study collected. Chapter 2 is devoted to the study of μ-open sets and μ-axiom due to Sharma [25, 26]. A subset A of a topological space (X,τ) is said to be μ-open iff there exists a regularly open set R such that $R\subseteq A\subseteq cl(R)$ In section one we study, Properties and characterization of μ-open sets and relations between regular open sets, μ-open sets and semi open sets. It is interesting to note that

(1) The component of a μ-open set is again a μ-open set.
(2) Neither the union nor the intersection of two μ-open sets is μ-open.

In section two of chapter 2, we study μ-adherant, μ-closure of a subset A of a topological space, μ-irresolute function between topological spaces and equivalence relation μ-correspondence on the set of a topologies of a set X. In section three of chapter 2 is devoted to the study of μ-axiom due to Sharma [27]. Separation axioms $\mu T_{0,}$ μT_1 and μT_2 which are generalizations of separation axioms $T_{0,}T_1$ and T_2 respectively are studied. It is interesting to note in a topological space all the three axioms $\mu T_{0,}$ μT_1 and μT_2 coincide. Therefore these axioms are referred as μ-axiom. The interrelationship between μ-axiom and various separation axioms semi $T_{2,}$ semi $T_{1,}$ semi T_0 and μT_0 are discussed. Chapter 3 is devoted to generalize the concept of μ-axiom due to Sharma [26] to fuzzy topological spaces. In section one of chapter 3, preliminary results on fuzzy sets and fuzzy topological spaces that are needed for our study are collected. Section two of chapter 3 is devoted to extend μ-open sets to fuzzy situation. We study properties and characterization of fuzzy μ-open sets. It is interesting to note that

(i) The complement of a fuzzy μ-open set is fuzzy μ-open.
(ii) If λ is a fuzzy μ-open set, then
(a) int(cl(λ))=int(λ),
(b) If μ is a fuzzy regularly open set such that μ≤λ≤cl(μ),
Then cl(μ)=cl(λ)

II. BASIC CONCEPTS OF μ-OPEN SETS AND μ-AXIOMS

In this chapter we study the concepts of **μ**-open sets and **μ**-axioms introduced by Sharma [25, 26].
In section 2.1 , we discuss properties and characterizations of **μ**-open sets.
In section 2.2, we study **μ**-adherant, **μ**-closure of a subset A of a topological space. **μ**-irresolute function between topological spaces and equivalence relation **μ**-correspondence on the set of topologies of a set X.
In section 2.3, we study separation axioms $\boldsymbol{\mu}T_{0,}$ $\boldsymbol{\mu}T_1$ and $\boldsymbol{\mu}T_2$ due to Sharma[26]. These three axioms are equivalent in topological spaces and it is called **μ**-axiom. This axiom lies between rT_i axioms of Malghan and Benchalli[18] and semi T_i axiom of Maheshwari and Prasad[16] i=0,1,2. We study properties and characterizations of **μ**-axiom.
Section: 2.1 **μ**-open sets:

Definition: 2.1.1
A subset A of a topological space (X,τ) is said to be **μ**-open iff there exist a regularly open set R such that R⊆A⊆cl(R).
Theorem: 2.1.2
A subset A of a topological space (X,τ) is said to be **μ**-open iff there exist a regularly closed set F such that int(F)⊆A⊆F.

Proof:
Assume A is a **μ**-open set.
To prove there exist a regularly closed set F such that int(F)⊆A⊆F Since A is **μ**-open, there exist a regularly open set R such that R⊆A⊆cl(R).
Since R is regularly open set, we get R=int(cl(R)) ……….(1)
Therefore cl(R) = cl(int(cl(R)))
Let F= cl(int(F)).
Hence F is regularly closed set.
Since A ⊆ cl(R) (by(1)), we get A⊆F
We know that int(F) ⊆ F.
Hence int(F)⊆A⊆F.
Hence there exist a regularly closed set F such that int(F)⊆A⊆F
Conversely assume there exist a regularly closed set F such that int(F)⊆A⊆F.
To prove A is a **μ**-open set.
Let R=int(F)
Consider int(cl(R))=int(cl(int(F)))=int(F)=R, since F is regularly closed.
Hence R is regularly open.
Since A⊆F, A⊆int(F)=R.
Since A⊆R, A⊆cl(R)
Since int(F)⊆A, R⊆A
Hence R⊆A⊆cl(R)
Hence A is **μ**-open.

Theorem: 2.1.3

A necessary and sufficient condition for a set A in a topological space (X,τ) is said to be **μ**-open is, int(cl(A))⊆A⊆cl(int(A)).

Proof :

Assume A is a **μ**-open set in (X,τ).
To prove int(cl(A))⊆A⊆cl(int(A)).
Since A is **μ**-open,there exists a regularly open set R such that R⊆A⊆cl(R).
Claim(i): int(cl(A))⊆A
Since R⊆A, int(cl(R))⊆ int(cl(A))
Since R is regularly open, R ⊆ int(cl(A)) ....(1)
Since A⊆cl(R) ⟹ cl(A) ⊆cl(R) ⟹ int(cl(A)) ⊆ int(cl(R))=R, since R is regularly open.
Hence int(cl(A))⊆R ....(2)
From (1) and (2) we get R=int(cl(A))
Hence int(cl(A))⊆A
Hence claim(i).
Claim(ii): A⊆cl(int(A))
Since R⊆A, R⊆int(A) ⟹ cl(R)⊆cl(int(A))
Since A⊆cl(R), A⊆cl(int(A))
Hence claim(ii).
Hence int(cl(A))⊆A⊆cl(int(A))
Conversely assume int(cl(A))⊆A⊆cl(int(A))
To prove A is a **μ**-open set in (X,τ)
Since A⊆cl(A) ⟹ int(A) ⊆int(cl(A)) ⟹ cl(int(A)) ⊆ cl(int(cl(A)))
Let R= int(cl(A))
Then R= int(cl(A))⊆A⊆cl(int(A)) ⊆cl(int(A))=cl(R).
Therefore we get R⊆A⊆cl(R).
Since R is a regularly open set, we have A is a **μ**-open set.

Theorem: 2.1.4

A is **μ**-open in a topological space (X,τ) iff A is semi open as well as semi closed in (X,τ).

Proof :

Assume A is a **μ**-open set in (X,τ).
To prove A is semi open as well as semi closed in (X,τ).
Since A is **μ**-open, we get int(cl(A))⊆A⊆cl(int(A)) (by theorem 2.1.3) which implies int(cl(A))⊆A and A⊆cl(int(A)).
Claim(i): A is semi open.
Since A⊆cl(int(A)), for R=cl(A), we have R⊆A⊆cl(R)
Hence A is semi open.
Claim(ii): A is semi closed.
Since int(cl(A))⊆A, for R=cl(A), we have int(R)⊆A⊆cl(A)=R
Hence A is semi closed.
Conversely assume A is semi open as well as semi closed.
To prove A is **μ**-open in (X,τ).
Since A is semi open, R⊆A⊆cl(R) for some open set R
Since R⊆A, R⊆int(A)
Hence we get cl(R)⊆cl(int(A))
Hence A⊆cl(R)⊆cl(int(A))
Since A is semi closed, int(R)⊆A⊆R for some closed set R
Since R is a closed set, A⊆R implies cl(A)⊆R
Then int(cl(A))⊆int(R)⊆A
Hence int(cl(A))⊆A
Hence int(cl(A))⊆A⊆cl(int(A))
Hence by theorem 2.1.3, we have A is **μ**-open.

Corollary :2.1.5

A set A in a topological space (X,τ) is **μ**-open iff A=s-cl-s-int(A) and A=s-int-s-cl(A).

Remark : 2.1.6

By definition of a **μ**-open set it follows that every regularly open set is a □-open set and every □-open set is semi open set,

i.e., R O (X,τ) ⊆ V O(X,τ) ⊆ S O ⊆ (X,τ)

However the converse of the above statements are not true in general as shown by the following examples.

Example : 2.1.7

Let X={a,b,c} and τ={∅,X,{a},{b},{a,b}} Then {a,c} is a □-open set but not regularly open.

Solution

Let X={a,b,c} and τ={∅,X,{a},{b},{a,b}}

Then τ^c={X,{b,c},{a,c},{c}}

To prove {a,c} is a □-open set but not regularly open.

Claim: {a,c} is not regularly open

Consider cl({a,c})=intersection of all closed sets containing{a,c}

=⋂{X,{a,c}}

={a,c}

Now consider int(cl({a,c})) =Union of all open sets contained in {a,c}

={a}

Hence int(cl({a,c})) ={a}≠{a,c}

Hence {a,c} is not regularly open

Hence claim.

By above claim {a,c} is regularly closed set.

Now to prove {a,c} is □-open.

By theorem 2.1.2, it is enough to prove there exists a regularly closed set F such that int(F) ⊆ A ⊆ F.

Since {a,c} is regularly closed set and int{a,c}⊆{a,c}⊆{a,c}

We have {a,c} is a □-open set.

Example : 2.1.8

Let X={a,b,c} & τ={∅,X,{a}} then the set {a,b} is a semi open which is not □-open.

Proof :

Let X={a,b,c} and τ={∅,X,{a}}

To prove {a,b} is semi open.

The open sets of X are =∅,X,{a}.

Then cl{∅}=∅, cl(X)=X and cl{a}=X

Therefore {a}⊆{a,b}⊆cl{a}=X

Hence {a,b} is a semi open set.

The □-open sets of X are ∅ and X.

Since □-open sets of X are X and ∅, for each pair of distinct points of X there exists no open set containing one but not the other.

Hence it is not a □-open.

Theorem :2.1.9

A □-open set A is regularly open if A⊆int(cl(A))

Proof :

Assume A is a □-open set and A⊆int(cl(A)) ....(1)

To prove A is regularly open.

Since A is □-open, there exists a regularly open set O such that O⊆A⊆cl(O)

Then cl(A)⊆cl(cl(O))=cl(O) implies int(cl(A))⊆int(cl(O))=O, Since O is a regularly open set.

Therefore int(cl(A))⊆O⊆A implies int(cl(A))⊆A ....(2)

From (1) and (2) we get A=int(cl(A))

Hence A is regularly open.

Theorem :2.1.10

A semi open set A is □-open if int(cl(A))⊆A

Proof :

Assume A is semi open and int(cl(A))⊆A

Claim: A is □-open

Since A is semi open, there exists an open set O such that O⊆A⊆cl(O)

But O⊆int(A)
Then cl(O)⊆cl(int(A))
Hence A⊆cl(O)⊆cl(int(A))
Hence A⊆cl(int(A)) ....(1)
By our assumption, int(cl(A))⊆A ....(2)
From (1) and (2),we get int(cl(A))⊆A⊆cl(int(A))
Hence by theorem 2.1.3, we get A is □-open.
Hence the claim.

Theorem :2.1.11
A semi closed set A is □-open if A⊆cl(int(A))
Proof :
Let A be a semi closed set and A⊆cl(int(A))
Claim: A is □-open
Since A is semi closed, there exists a closed set O such that int(O)⊆A⊆O ....(1)
Then cl(A)⊆cl(O), Since O is a closed set, cl(O)=O
Therefore we get cl(A)⊆O ....(2)
Hence int(cl(A))⊆int(O)⊆A (from (1) and (2))
Since A⊆cl(int(A)), we get int(cl(A))⊆A⊆cl(int(A))
By Theorem 2.1.3, we get A is □-open.
Hence the claim.

Remark : 2.1.12
Neither the union nor the intersection of two □-open sets is □-open.
Example : 2.1.13
Let X={a,b,c} and τ={∅,X,{a},{b},{a,b}} then {a,c} and {b,c} are □-open sets. But {a,c}⋂{b,c}={c} is not a □-open sets.
Proof :
Let X={a,b,c} and τ={∅,X,{a},{b},{a,b}}
Then τ^c={X, ∅,{b,c},{a,c},{c}}
The regularly open sets of X are {a},{b},X,∅.
To prove {a,c} is a □-open set.
cl({a})={a,c}, int(cl({a}))={a}
Therefore {a} is a regularly open set.
Since {a} is regularly open set and since
{a} ⊆ {a,c} ⊆ cl({a}), {a,c} is □-open.
To prove {b,c} is a □-open.
cl({b})={b,c}, int(cl({b}))={b}
Therefore {b} is regularly open set,
Since {b} is regularly open set and since b ⊆ {b,c} ⊆ cl({b}),
therefore {b,c} is □-open.
To prove {a,c}⋂{b,c}={c} is not a □-open set.
Since there exists no regularly open set R such that
R ⊆ {c} ⊆ cl(R)
Therefore {c} is not a □-open set.

Example : 2.1.14
Let X={a,b,c} and τ={∅,X,{a},{b},{a,b}} then {a} and {b} are □-open sets but {a}⋃{b}={a,b} is not a □-open set.
Proof :
Let X={a,b,c} and τ={∅,X,{a},{b},{a,b}}
Then τ^c={X, ∅,{b,c},{a,c},{c}}
The regularly open sets of X are {a},{b},X,∅.
To prove {a} and {b} are □-open sets.
cl({a})={a,c}, int(cl({a}))={a}
Since {a} is a regularly open set and since {a} ⊆ {a} ⊆ cl({a})
cl({b})={b,c}, int(cl({b}))={b}

Since {b} is a regularly open set and since {b} ⊆ {b} ⊆ cl({b})
Therefore {a} and {b} are -open sets.
To prove {a}∪{b}={a,b} is not a □-open set.
cl({a})={a,c}{a} ⊆ {a,b} does not contain {a,c} ⊆ cl({a})
Therefore {a,b} is not □-open.

Theorem :2.1.15
The complement of □-open set is again a □-open set.
Proof :
Assume A is □-open.
To prove the complement of A is a □-open set.
Since A is □-open, there exists a regularly open set R such that
R⊆A⊆cl(R)
Therefore we have X-R⊇X-A⊇X-cl(R)
i.e., X-cl(R)⊆X-A⊆X-R
Hence int(X-R)⊆X-A⊆X-R ....(1)
Since R is regularly open, X-R is regularly closed.
Hence by Theorem 2.1.2, and from (1), we have A is □-open.

Theorem :2.1.16
If A is a □-open set then
(a) int(cl(A))=int(A)
(b)cl(R)=cl(A), where R is a regularly open set such that R⊆A⊆cl(R).
Proof :
Assume A is □-open set.
Claim(i): int(cl(A))=int(A)
We know that int(A)⊆int(cl(A)) for any set A.
Since A is □-open, int(cl(A))⊆A, (by Theorem 2.1.3)
Therefore int(cl(A)) ⊆ int(A)
Hence int(cl(A))=int(A)
Hence claim(i)
Hence (a).
Claim(ii): cl(R)=cl(A)
Since R is a regularly open set such that R⊆A⊆cl(R),
cl(R)⊆cl(A).
Since A⊆cl(R), therefore cl(A)⊆cl(R)
Hence cl(R)=cl(A)
Hence claim(ii)
Hence (b).

Theorem :2.1.17
If A and R are regularly open sets and S is □-open such that
R⊆S⊆cl(R), then A∩S=∅, implies A∩S=∅.
Proof :
Assume A and R are regularly open sets and S is □-open such that
R⊆S⊆cl(R)
Claim: A∩R=∅ implies A∩S=∅..
Since A∩R=∅, therefore R⊆X-A.
Since A is regularly open, we get X-A is regularly closed.
Hence X-A is closed.
Therefore R⊆X-A implies cl(R)⊆X-A
Since S⊆cl(R), we get S⊆X-A.
Hence A∩S=∅.
Hence the claim.

Theorem :2.1.18
Intersection of a □-open set S and regularly open set U is a □-open set.
Proof :
Let S be a □-open set and U be a regularly open set.
To prove S∩U is a □-open set.

Since S is □-open, there exists a regularly open set R such that
R⊆S⊆cl(R).
Case(i): S∩U = ∅
Let S∩U =∅
It is obvious that then S∩U is a □-open set.
Case(ii): S∩U≠∅
Since R⊆S and S∩U ≠ ∅ , we get R∩U ≠∅ (by Theorem 2.1.17)
Since R and U are regularly open sets, we get R∩U is a regularly open set.
Since R⊆S, we get R∩U⊆ S∩U
If x∈ S∩U, then either x belongs to U and R (or) x belongs to U and S-R.
Case (a): x belongs to U and R.
Then x∈ S∩U⊆cl(U∩R)
Hence S∩U⊆cl(U∩R)
Hence U∩R⊆S∩U⊆cl(U∩R)
Therefore, S∩U is a □-open set
Case (b): x belongs to U and S-R.
Then x belongs to U and x is a limit point of R, since S⊆cl(R).
Let N be a □-neighbourhood of x.
Then N∩U is a □-neighbourhood of x and (N∩U)∩R ≠ ∅ which implies
N∩(U∩R) ≠ ∅
Therefore x is a limit point of U∩R.
Hence x belongs to cl(U∩R)
Hence R∩U⊆S∩U⊆cl(R∩U)
Hence S∩U is a □-open set.

III. FUZZY **μ** -OPEN SETS

This chapter is devoted to the study of **μ**-open set in fuzzy topological spaces. We generalize the concept of **μ**-open sets due to Sharma [25] in topological spaces to fuzzy topological spaces and extend few results on **μ**-open sets to fuzzy situations. In section one of this chapter, we collect preliminary results on fuzzy sets and fuzzy topological spaces. In section two, we introduce fuzzy **μ**-open sets and study its properties, characterizations and relations with other generalized fuzzy open sets.
Section: 3.1
Preliminaries on fuzzy topological spaces:

Definition: 3.1.1
Let X be a non-empty set and I be the unit interval [0,1]. A fuzzy set in X is a function with domain X and values in I that is an element of I^X.
Let A,B ∈ I^X. We define the following fuzzy sets,
(i) A includes B (that is B ⊂ A) by B(x)≤A(x) for every x∈X.
(ii) A ∩ B ∈ I^X by (A ∩ B)(x)=min{A(x),B(x)} for every x∈X.
(iii) A ∪ B ∈ I^X by (A ∪ B)(x)=max{A(x),B(x)} for every x∈X.
(iv) A' ∈ I^X by A'(x)=1-A(x) for every x∈X.
Let Δ be an indexing set and $\{A_\lambda / \lambda \in \Delta\}$ be a family of fuzzy sets in X. Then their union and intersection are defined as follows,
$(\cup A_\lambda)(x) = \sup \{A_\lambda(x) | \lambda \in \Delta\}$.
$(\cap A_\lambda)(x) = \inf \{A_\lambda(x) | \lambda \in \Delta\}$.

Definition: 3.1.2
Let $A_1,A_2,\ldots\ldots\ldots,A_n$ be fuzzy sets in X. The product fuzzy set
$A= A_1 x A_2 x,\ldots\ldots\ldots x A_n$ in X^n is defined by
$A(x_1,x_2,\ldots\ldots,x_n)=\min(A_1(x_1),A_2(x_2),\ldots\ldots\ldots,A_n(x_n))$.
Note: 3.1.3
The ordinary subset X can be considered as fuzzy sets by identifying them with their characteristic functions.
Ordinary subsets are referred to as Crisp sets when they are considered as fuzzy sets. Ordinary topological space is referred to as Crisp topological spaces
If A ⊂ X and if we consider A is fuzzy set then we mean $\mathbf{A(x)} = \begin{cases} \mathbf{1}, & x \in A \\ \mathbf{0}, & x \notin A \end{cases}$
Notation: 3.1.4
When an ordinary set A is considered as fuzzy sets we write it as χ_A or A itself.

In view of this, empty set Φ and whole space X can be considered as fuzzy sets by identifying them with the constant functions 0 and 1 respectively.

Definition: 3.1.5

For $x \in X$ and $t \in I_0$ a fuzzy point x_t is defined by

$$x_t(Y) = \begin{cases} t, & \text{if } y = x \\ 0, & \text{otherwise} \end{cases}$$

where $I_0 = (0,1]$.

Definition: 3.1.6

Let f be a function from X to Y. Let B be a fuzzy set in Y. Then inverse image of B or preimage of B written as $f^{-1}(B)$ is a fuzzy set in X defined by $f^{-1}(B)(x)=B(f(x))$, for all $x \in X$. Conversely, let A be a fuzzy set in X. The image of A, written as f(A) is a fuzzy set in Y defined by

$$f(A)(Y) = \begin{cases} \sup_{z \in f^{-1}(y)} A(z), & f^{-1}(y) \text{ is non-empty, where } f^{-1}(y) = \{x / f(x) = y\} \\ 0, & \text{Otherwise} \end{cases}$$

for all $y \in Y$.

Note: 3.1.7

$$f(A)\, f(x) = \sup_{y \in f^{-1}(f(x))} A(y)$$

$\geq A(x)$ as $x \in f^{-1}(f(x))$

Properties: 3.1.8

Let f be a function from X to Y. Then

a) $f^{-1}(B') = \{f^{-1}(B)\}'$ for any fuzzy set B in Y.

b) $f(A') \supset \{f(A)\}'$ for any fuzzy set A in X.

c) $B_1 \subseteq B_2 \Longrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$ where B_1 and B_2 are fuzzy sets in Y.

d) $A_1 \subseteq A_2 \Longrightarrow f^{-1}(A_1) \subseteq f^{-1}(A_2)$ where A_1 and A_2 are fuzzy sets in X.

e) $B \supset f\{f^{-1}(B)\}$ for any fuzzy set B in Y.

f) $A \subset f\{f^{-1}(A)\}$ for any fuzzy set A in X.

g) Let f be a function from X to Y and g be a function from Y to Z. Then $(gof)^{-1}(c) = f^{-1}(g^{-1}(c))$ for any fuzzy set c in Z, where (gof) is the composition of g and f.

h) If f is onto then $f(f^{-1}(A))=A$.

Definition: 3.1.9

Let $\lambda, \mu \in I^X$, λ is said to be quasi-coincidence with μ, denoted by $\lambda q \mu$ if there exists $x \in X$ such that $\lambda(x)+ \mu(x)>1$. Otherwise we denote it by $\lambda \bar{q} \mu$

Definition: 3.1.10

A fuzzy topology on a set X in a collection of fuzzy sets in X satisfying the following axioms,

(i) $\emptyset, X \in \delta$.

(ii) $A,B \in \delta \Longrightarrow A \cap B \in \delta$

(iii) $A_\lambda \in \delta$ for $\lambda \in \Delta \Longrightarrow \cup A_\lambda \in \delta \quad \lambda \in \Delta$

The pair (X,δ) is referred to as fuzzy topological space. A fuzzy topological space is referred to as fts in short.

Definition: 3.1.11

If (X,δ) is a fuzzy topological space, members of δ are called open fuzzy sets. A fuzzy set A is called a closed fuzzy set iff $A' \in \delta$.

Definition: 3.1.12

Let (X,δ) be a fts. Then the closure and interior of a fuzzy set $A \in I^X$ are defined respectively as

$\bar{A} = \cap\{B | B \supset A, B' \in \delta\}$

$A^0 = \cup\{B | B \subset A, B \in \delta\}$

It is easily seen that $\bar{A}$ is the smallest closed fuzzy set larger than A and that A^0 is the largest open fuzzy set smaller than A.

Definition: 3.1.13

Let δ be a fuzzy topology on a set X. A subfamily $\mathfrak{B}$ of δ is a base for iff each member of δ can be expressed as the union member of $\mathfrak{B}$.

Definition: 3.1.14
Let (X,**δ**),(Y,**δ**') be two fts's. A mapping f: (X,**δ**)→(Y,**δ**') is fuzzy continuous iff for each open fuzzy set V in **δ**' the inverse image f^{-1}(V) is open in **δ**.

Definition: 3.1.15
Let (X,**δ**),(Y,**δ**') be two fts's. A mapping f: (X,**δ**)→(Y,**δ**') is called fuzzy open iff for each open fuzzy set V in □ the image f(V) is open in □'.

Definition: 3.1.16
Let (X,□),(Y,□') be two fts's. A bijective mapping f: (X,□)→(Y,□') is a fuzzy homeomorphism iff it is fuzzy continuous and fuzzy open.

Proposotion: 3.1.17
Let f be fuzzy continuous (respectively, fuzzy open) mapping of a fts (X,□) into a fts (Y,□') and g be a fuzzy continuous (respectively, fuzzy open) mapping of (Y,□') into a (Z,□''). Then the composition (gof) is a fuzzy continuous (respectively, fuzzy open) mapping of (X,□) into a (Z,□'').

Definition : 3.1.18
Given two fuzzy topologies $□_1$, $□_2$ on the same set X, we say $□_1$ finer than $□_2$(and that $□_2$ is coarser than $□_1$) if the identity mapping of (X,$□_1$) into (X,$□_2$) is fuzzy continuous.

Definition : 3.1.19
Let λ be a fuzzy set of a topological space (X,□). Then λ is called (i) a fuzzy semi open set of X if there exists a such that □ ∈ □ such that
□≤λ≤cl(□) , and (ii) a fuzzy semi closed set of X if there exists a □ ∈ □ such that int(□)≤ λ≤□ .

Definition : 3.1.20
The following are equivalent :
(a) λ is a fuzzy semi closed set,
(b) λ' is a fuzzy semi open set,
(c) int(cl(λ))≤ λ, and
(d) cl(int(λ'))≥ λ'

Theorem: 3.1.21
(a) Any union of fuzzy semi open sets is a fuzzy semi open set, and
(b) Any intersection of fuzzy semi closed sets is a fuzzy semi closed set.

Definition: 3.1.22
A fuzzy set λ of a topological space (X,□) is called (i) a fuzzy regularly open set of X if int(cl(λ)) = λ , and (ii) a fuzzy regularly closed set of X if cl(int(λ)) = λ

Theorem: 3.1.
A fuzzy set λ of a topological space (X,□) is fuzzy regularly open iff λ' is fuzzy regularly closed.

Theorem: 3.1.24
(a) The intersection of two fuzzy regularly open sets is a fuzzy regularly open set, and
(b) The union of two fuzzy regularly closed sets is a fuzzy regularly closed set.

Theorem: 3.1.25
(a) The closure of a fuzzy open set is a fuzzy regular closed set, and
(b) The interior of a fuzzy closed set is a fuzzy regular open set.

Definition: 3.1.26
Let f: (X, $□_X$)→(Y,$□_Y$) be a mapping from a fuzzy topological space (X,$□_X$) to another fuzzy topological space (Y,$□_Y$) Then f is called
(i) a fuzzy continuous mapping if f^{-1}(λ) ∈ $□_X$ for each λ ∈ $□_Y$ (or) equivalently f^{-1}(□) is a fuzzy closed set of X for each fuzzy closed set □ of Y
(ii) a fuzzy open mapping if f(λ) ∈ $□_Y$ for each λ ∈ $□_X$
(iii) a fuzzy closed mapping if f(□) is a fuzzy closed set of Y, for each fuzzy closed set □ of X
(iv) a fuzzy semi continuous mapping if f^{-1}(λ) is a fuzzy semi open set of X, for each λ ∈ $□_Y$.

(v) a fuzzy semi open mapping if f(λ) is a fuzzy semi open set for each λ ∈ $□_X$
(vi) a fuzzy semi closed mapping if f(□) is a fuzzy closed set for each fuzzy closed set □ of X.
Section: 3.2
Fuzzy □-open Sets:

Definition: 3.2.1
A fuzzy set λ ∈ I^X in a fuzzy topological space (X,□) is said to be fuzzy □-open iff there exist a fuzzy regularly open set □ such that
□≤ λ ≤cl(□).

Theorem: 3.2.2
A fuzzy set λ ∈ I^X in a fuzzy topological space (X,□) is fuzzy v-open iff there exist a fuzzy regularly closed set □ such that int(□)≤ λ ≤□
Proof:
Assume is a fuzzy □-open set in (X,□).
Claim(i): There exists a fuzzy regularly closed set □ such that
int(□) ≤ λ ≤ □.
Since λ is fuzzy □-open, there exist a fuzzy regularly open set such that
□≤ λ ≤cl(□) ....(1)
Since □ is a fuzzy regularly open set, we get □=int(cl(□).
Therefore cl(□)= cl(int(cl(□)))
Let □ = cl(□)
Then we get □ = cl(int(□))
Hence □ is a fuzzy regularly closed set.
Since λ≤cl(□) (by(1)), we get λ ≤ □
Since □ is regularly open set □≤ λ, int(cl(□)) ≤ λ i.e., int(□) ≤ λ
Hence there exists a fuzzy regularly closed set □ such that int(□)≤ λ ≤ □
Hence claim(i)
Conversely assume there exists a fuzzy regularly closed set □ such that
int(□)≤ λ ≤□
Claim(ii): λ is fuzzy □-open.
Let □ =int(□).
Then int(cl(□)) = int(cl(int(□)))=int(□),since □ is a fuzzy regularly closed set.
Hence □ is a fuzzy regularly open set.
Since int(□)≤ λ, □ ≤ λ
Since λ ≤□, λ ≤cl(int(□)) implies λ ≤cl(□)
Hence □ ≤ λ ≤cl(□)
Hence λ is fuzzy □-open.
Hence claim(ii).

Theorem: 3.2.3
A necessary and sufficient condition for a fuzzy set λ in a fuzzy topological space (X,□) to be fuzzy □-open is,
int(cl(λ)) ≤ λ ≤cl(int(λ)).
Proof:
Assume λ is a fuzzy □-open set in a fuzzy topological space (X,□)
To prove int(cl(λ)) ≤ λ ≤cl(int(λ)).
Since λ is a fuzzy □-open, there exists a fuzzy regularly closed set □ such that □ ≤ λ ≤ cl(□).
Claim(i): int(cl(λ)) ≤ λ.
Since □ ≤ λ, int(cl(□)) ≤ int(cl(λ))
Since □ is fuzzy regularly open, □=int(cl(□)) ≤ int(cl(λ)) ......(1)
Since λ≤cl(□), cl(λ)≤cl(□) implies int(cl(λ)) ≤ int(cl(□)) = □, Since □ is fuzzyregularlyopen set .....(2)
From (1) and (2) □ = int(cl(λ)).
Hence int(cl(λ)) ≤ λ.
Hence claim(i).
Claim(ii): λ≤cl(int(λ))
Since □≤ λ , we get □≤int(λ) implies cl(□) ≤ cl(int(λ)).
Since λ≤cl(□), λ≤cl(int(λ))
Hence claim(ii).
Conversely assume int(cl(λ)) ≤ λ ≤ cl(int(λ))
To prove λ is fuzzy □-open.

Since λ ≤ cl(λ), int(λ) ≤ int(cl(λ)) implies cl(int(λ)) ≤ cl(int(cl(λ)))
Let □ = int(cl(λ))
Then □ is a fuzzy regularly open set, λ is fuzzy □-open.

Theorem: 3.2.4
λ is fuzzy □-open iff λ is fuzzy semiopen as well as fuzzy semi closed in (X,□).
Proof:
Assume λ is fuzzy □-open in a fuzzy topological space (X,□).
To prove λ is fuzzy semi open as well as fuzzy semi closed.
Since λ is fuzzy □-open, by Theorem 3.2.3, we get
int(cl(λ)) ≤ λ ≤ cl(int(λ)) , which implies int(cl(λ)) ≤ λ and λ ≤ cl(int(λ))
Claim(i): λ is fuzzy semi open .
Since λ ≤ cl(int(λ)), for □ = int(λ), we have □ ≤ λ ≤cl(□)
Hence λ is fuzzy semi open.
Hence claim (i).
Claim(ii): λ is fuzzy semi closed.
Since int(cl(λ)) ≤ λ, for □ = int(λ), we have
int(□)≤ λ ≤cl(λ)= □
Hence λ is fuzzy semi closed.
Hence claim (ii).
Conversely assume λ is fuzzy semi open as well as fuzzy semi closed
To prove λ is fuzzy □-open.
Since λ is fuzzy semi open, □ ≤ λ ≤ cl(□) for some fuzzy open set □.
Since □ ≤ λ , □ ≤ int(λ) and cl(□) ≤ cl(int(λ)).
Hence λ ≤ cl(□) ≤ cl(int(λ)) ......(1)
Since λ is fuzzy semi closed, int(□)≤ λ ≤ □ for a fuzzy closed set □
Since □ is a fuzzy closed set , λ ≤ □ implies cl(λ) ≤ □
Hence int(cl(λ)) ≤ int(□)≤ λ
Hence int(cl(λ)) ≤ λ ....(2)
From (1) and (2), we have int(cl(λ)) ≤ λ ≤ cl(int(λ))
Hence by Theorem 3.2.3, we get λ is fuzzy □-open.
Corollary: 3.2.5
A fuzzy set λ in (X,□) is a fuzzy □-open set iff λ =s-cl(s-int(λ)) or scl(sint(λ)) and λ =s-int(s-cl(λ)) or sint(scl(λ)).
Theorem: 3.2.6
A fuzzy □-open set λ is fuzzy regularly open if λ ≤ int(cl(λ))
Proof:
Assume λ is a fuzzy □-open set and λ ≤ int(cl(λ)) .....(1)
Claim: λ is fuzzy regularly open.
Since λ is fuzzy □-open, by Theorem 3.2.3,
int(cl(λ)) ≤ λ .....(2)
From (1) and (2), we get λ = int(cl(λ))
Hence λ is fuzzy regularly open.
Hence the claim.
Theorem: 3.2.7
A fuzzy semi open set λ fuzzy □-open if int(cl(λ)) ≤ λ.
Proof:
Assume λ is fuzzy semi open set and int(cl(λ)) ≤ λ.
Claim: λ is fuzzy □-open.
Since λ is fuzzy semi open, there exists a fuzzy open set □ such that
□≤ λ ≤cl(□).
But □ ≤ int(λ).
Hence cl(□) ≤ cl(int(λ)).
Hence λ ≤cl(□) ≤ cl(int(λ)).
Hence λ ≤ cl(int(λ)). .....(1)
By our assumption, int(cl(λ)) ≤ λ .....(2)
From (1) and (2), we get int(cl(λ)) ≤ λ ≤ cl(int(λ))
Hence by Theorem 3.2.3. we get λ is fuzzy □-open.
Hence the claim.

Theorem: 3.2.8
A fuzzy semi closed set λ fuzzy $\square$-open if $\lambda \leq cl(int(\lambda))$.
Proof:
Assume λ is a fuzzy semi closed set and $\lambda \leq cl(int(\lambda))$.
To prove λ is fuzzy $\square$-open
Since λ is fuzzy semi closed set , there exists a fuzzy closed set $\square$ such that $int(\square) \leq \lambda \leq \square$ ....(1)
Then $cl(\lambda) \leq cl(\square)$, since $\lambda \leq \square$.
Since $\square$ is fuzzy closed set $cl(\square) \leq \square$.
Therefore $cl(\lambda) \leq \square$ ....(2)
Hence $int(cl(\lambda)) \leq int(\square) \leq \lambda$ (from (1) and (2))
Since $\lambda \leq cl(int(\lambda))$, we get $int(cl(\lambda)) \leq \lambda \leq cl(int(\lambda))$
Hence by Theorem 3.2.3, we get λ is fuzzy $\square$-open.

Theorem: 3.2.9
The complement of fuzzy $\square$-open set is again a fuzzy $\square$-open set .
Proof:
Assume λ is fuzzy $\square$-open set.
To prove the complement of λ is fuzzy $\square$-open.
Since λ is fuzzy $\square$-open, there is a fuzzy regularly open set $\square$ such that
$\square \leq \lambda \leq cl(\square)$.
Therefore we get $1-\square \geq 1-\lambda \geq 1-cl(\square)$.
i.e., $1-cl(\square) \leq 1-\lambda \leq 1-\square$.
Hence $int(1-\square) \leq 1-\lambda \leq 1-\square$.
Since $\square$ is fuzzy regularly open, $1-\square$ is fuzzy regularly closed. Hence by Theorem 3.2.2, we get $1-\lambda$ is a fuzzy $\square$-open set.

Theorem: 3.2.10
If λ is a fuzzy $\square$-open set then
a) $int(cl(\lambda)) = int(\lambda)$
(b) $cl(\square) = cl(\lambda)$, where $\square$ is a fuzzy regularly open set
Such that $\square \leq \lambda \leq cl(\square)$.
Proof:
Assume λ is a fuzzy $\square$-open set.
To prove (a)
Claim (i): $int(cl(\lambda)) = int(\lambda)$
We know that $int(\lambda) \leq int(cl(\lambda))$ for any fuzzy set λ.
Since λ is fuzzy v-open, $int(cl(\lambda)) \leq \lambda$, (by Theorem 3.2.3)
Therefore we get $int(cl(\lambda)) \leq int(\lambda)$.
Hence claim (i)
Hence (a)
To prove (b)
Claim (ii): $cl(\square) = cl(\lambda)$ where $\square$ is a regularly open set such that
$\square \leq \lambda \leq cl(\square)$.
Since $\square \leq \lambda$, $cl(\square) \leq cl(\lambda)$.
Since $\lambda \leq cl(\square)$, $cl(\lambda) \leq cl(\square)$.
Hence we get $cl(\square) = cl(\lambda)$.
Hence claim (ii)
Hence (b).

Theorem: 3.2.11
If $\square$ is a fuzzy set in a fuzzy topological space $(X, \square)$ such that
$\lambda \leq \square \leq cl(\lambda)$. Then $\square$ is a fuzzy $\square$-open set if λ is fuzzy $\square$-open.
Proof:
Let $\square$ be a fuzzy set in a fuzzy topological space $(X, \square)$ such that
$\lambda \leq \square \leq cl(\lambda)$.
Assume λ is fuzzy $\square$-open
Claim : $\square$ is a fuzzy $\square$-open set.
Since λ is a fuzzy $\square$-open set, there exists a fuzzy regularly open set $\square$ such that $\square \leq \lambda \leq cl(\square)$.
Since $\lambda \leq \square$ and $\leq \lambda$, $\square \leq \square$ .....(1)
Since $\lambda \leq cl(\square)$, $cl(\lambda) \leq cl(\square)$

Since □ ≤ cl(λ), □ ≤ cl(□) .....(2)
From (1) and (2), we have □ ≤ □≤ cl(□) for a fuzzy regularly open set □.
Hence is a fuzzy □-open set.
Hence the claim.

IV. CONCLUSIONS

This dissertation is an attempt to generalize **μ**-open sets due to Sharma[25] to fuzzy topological spaces. In chapter 1, semi open sets and semi continuous functions due to Levine[14] are analysed. In chapter 2, **μ**-open sets and **μ**-axiom due to Sharma[25,26] are studied. In section one of chapter 2, **μ**-open sets, its properties and characterizations are studied. In section two of chapter 2, **μ**-adherent, **μ**-closure of a subset A of a topological space, **μ**-irresolute function between topological spaces and the relation **μ**-correspondence on the set of the topologies on a set X are studied. In section three of chapter 2, separation axioms μT_0, μT_1 and μT_2 and their equivalence in topological spaces are discussed. Properties and characterizations of **μ**-spaces are analysed. In chapter 3, some of the results discussed on **μ**-open sets in chapter 2 are generalized to fuzzy situation. In section one of chapter 3, fundamentals on fuzzy sets and fuzzy topological spaces are collected. In section two of chapter 3, **μ**-open sets, its properties and characterizations are generalized to fuzzy topological spaces.

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