Fuzzy topologized domination in graphs

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Abstract— *In this paper, the new concept "fuzzy topologized domination in graphs" is introduced. This topologized domination concept may be applied to different graphs like complete graph, star graph, wheel graph, trees and paths. The basic properties of fuzzy topologized dominating set are also analysed. Finally a couple of example is also provided to illustrate fuzzy topologized domination graph.*

Keywords—*Fuzzy set, graph, topologized set, topologized domination set, fuzzy topologized domination graphs.*

I. INTRODUCTION

General topology has recently become an essential part of the mathematical field. Zedeh [15] introduced fuzzy set theory describing fuzzy mathematically for the first time. Rosenfeld [9] introduced the concept of the fuzzy graph theory by using fuzzy indicating the level of the relationship between the objects of the given set.

Chang [3] introduced the concept of the notion of fuzzy topology. Ore [7] introduced the concept of domination in theory of graphs. Heynes et al. [5] published the book in the concept of fundamental domination on graphs and they worked more concepts on domination on graphs. Haydar [4] introduced the concept of connectedness in pythagorean fuzzy topological spaces. Nagarajan et al. [6] introduced lucky edge labeling of new graphs. Vidhya et al. [11] introduced the new concept of design and develop on divisor graph

Shokry [10] introduced generating topology on graphs by operations on graph. It approaches to discerning topological properties on a connected graph by using contraction and deletion. Vimala and Kalpana [12] introduced the concept of topologized bipartite graph. Vimala and Priyanka [13] analysed topic as topologized hamiltonian and complete graph. It applied that the two types of graphs in the topologized set. Vimala and Jasmine Amala [14] defined topologized cut vertex and edge deletion. It's approach the new concept as deletion of edges and elimination of vertex in topologized set.

Revathi et al. [8] worked in connected total perfect dominating set in fuzzy graph. It accomplished the concept of total perfect domination number in fuzzy graph and matching domination in graphs introduced by Bhaskarudu [1]. This paper established the concept on domination as matching domination with some results.

Bhuvaneswari et al. [2] handled the concept of topologized domination in graphs and explained some of its properties. We extend this work and develop the concept of topologized domination into the fuzzy topologized domination in graphs. In future there has been a rapid growth of research on this area and related concepts.

II. PRELIMINARIES

In order to discuss a new concept of fuzzy topologized domination in graphs we first define some basic definitions.

2.1 Topological Space

A topological space is an ordered pair (X, τ) where X is a set, τ is a collection of subsets of X satisfying the following properties.

i. Both the empty set and X are elements of τ
ii. The union of elements of any sub collection of τ is in τ
iii. The intersection of the elements of any finite sub collection of τ is in τ .

If τ is a topology on X, then the pair (X, τ) is called a topological space. The elements of τ are called open sets in X. A subset of X is said to be closed if its complement is in τ (i.e., its complement is open). A subset of X may be open, closed, both (clopen set), or neither. The empty set and X itself are always both closed and open.

2.2 Topologized graph

A Topologized graph is a topological space X such that
i. every singleton is open or closed.
ii. $\forall$x $\in$ X, $|\partial(x)| \leq 2$. Since $\partial(x)$ is denoted by the boundary of a point x.

2.3 Dominating set

A subset S of V(G) is said to be dominating set if for every vertex v in $V(G)-S$, there is a vertex u in S such that u is adjacent to v. That is a vertex v of G is in S or is adjacent to some vertex of S.

2.4 Domination number

The number of vertices in a minimum dominating set is called domination number of the graph G. It is denoted by γ(G).

2.5 Topologized domination

A set D of vertices of G is said to be a topologized domination set D if G is a topologized graph and every vertex in $V(G)-D$ is adjacent to at least one vertex of in D.

2.6 Topologized domination number

The minimum cardinality among all the topologized dominating sets of G is called the topologized domination number of G and it is denoted by $\tau\gamma(G)$.

2.7 Fuzzy sets

Let A = {(x, μ_A(x)): x ∈ X} and B = {(x, μ_B(x)): x ∈ X} be two fuzzy sets in X. Then their union A∪B, intersection A∩B and complement A^c are also fuzzy sets with the membership functions defined as follows:
i. $\mu_{A\cup B}(x) = \max\{\mu_A(x), \mu_B(x)\}, \forall x \in X$.
ii. $\mu_{A\cap B}(x) = \min\{\mu_A(x), \mu_B(x)\}, \forall x \in X$.
iii. $\mu_{A^c}(x) = 1 - \mu_A(x), \forall x \in X$.
Further,
iv. $A \subseteq B$ iff $\mu_A(x) \leq \mu_B(x), \forall x \in X$.
v. $A = B$ iff $\mu_A(x) = \mu_B(x), \forall x \in X$.

2.8 Fuzzy graph

A fuzzy graph G = (σ, μ) is a pair of membership functions or fuzzy sets σ: V→[0,1] and μ : V × V→[0,1] such that μ (u, v) ≤ σ (u)∧σ (v) for all u, v ∈ V.

2.9 Fuzzy topological space

Let A be a fuzzy subset of X. A collection δ of fuzzy subsets of A i.e $\delta \subset \mathcal{F}_A$ satisfying the following conditions:
i. $0, A \in \delta$,
ii. $U, V \in \delta \Rightarrow U\cap V \in \delta$,
iii. $\{U_i : i \in j\} \subset \delta \Rightarrow U_{i\in j} U_i \in \delta$,

is called a fuzzy topology on A. The pair (A,δ) is called a fuzzy topological space, members of δ is called a fuzzy open sets and their complements referred to A are called a fuzzy closed sets of (A,δ). The family of all fuzzy close sets in (A,δ) will be denoted by δ'_A.

III. FUZZY TOPOLOGIZED DOMINATION IN GRAPH

In this section, we explain some important concepts of fuzzy topologized domination in graphs with suitable examples.

Definition 3.1

A set D of vertices of G is said to be a fuzzy topologized dominating set D if G is a topologized graph and every vertex in V(G) – D is adjacent to atleast one vertex in D at the unit interval [0,1].

Consider the graph which is depicted in figure 3.1, $K_{1,2}$ star graph.

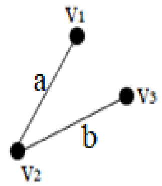

Fig. 3.1

For the above star graph $K_{1,2}$, $|\partial(V)| \leq 2$ and from the definition, it is a topologized graph. Thus the topologized dominating sets exist and are given by $D_1 = \{v_2\}$ and $D_2 = \{v_1, v_3\}$.

Theorem 3.1

Let G be a fuzzy topologized graph with at most degree two. If D is a topologized dominating set of G, then it is a fuzzy topologized dominating set of G.

Proof

Let X be a topological space with topology τ defined by $V \cup E$. Since every singleton set is open or closed and G is a fuzzy graph with unit interval [0,1] and at most degree two, the boundary of each vertices and edges are less than or equal to two. This satisfies the condition of topologized graph and implies that G is a topologized graph. Let D be a topologized dominating set. Then every vertex in $V(G) - D$ is adjacent to atleast one vertex of D thus implies that D is a fuzzy topologized dominating set of G.

IV. NUMERICAL EXAMPLE

In this section we explain the fuzzy topologized domination graph with suitable examples.

Example 4.1

Consider the graph which is depicted in figure 4.1.

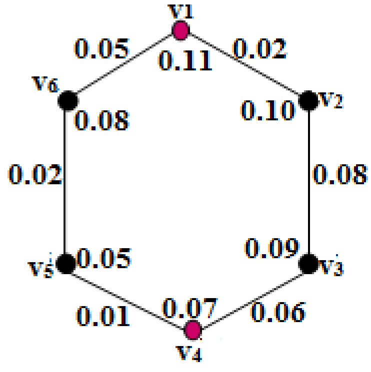

Figure 4.1

Let v_1, v_2, v_3, v_4, v_5 and v_6 denote the vertices and 0.02, 0.08, 0.06, 0.01, 0.02, 0.05 denote the edges which are labelled $f_u(0.02) = \{v_1, v_2\}$, $f_u(0.08) = \{v_2, v_3\}$, $f_u(0.06) = \{v_3, v_4\}$, $f_u(0.01) = \{v_4, v_5\}$, $f_u(0.02) = \{v_5, v_6\}$, $f_u(0.05) = \{v_6, v_1\}$.

Let X = { v_1, v_2, v_3, v_4, v_5, v_6, 0.02, 0.08, 0.06, 0.01, 0.02, 0.05} be a topological space defined by the topology τ = {X, $\emptyset$, {v_1}, {v_2}, {v_3}, {v_4}, {v_5}, {v_6},{v_1, v_2}, {v_1, v_6}, {v_2, v_3},{v_3, v_4},{v_4, v_5},{v_5, v_6}, {v_1, v_2,v_3},{v_6, v_1,v_3},{v_3, v_4,v_5},{v_5, v_6,v_1},{v_6, v_1,v_3, v_4}, {v_3, v_4,v_5, v_6},…{v_1, v_2,v_3, v_4,v_5, v_6}}. Here for every {x}∈ X is open or closed.

By the definition of topologized graph, we have $|\partial(v)| \leq 2$ and $\partial(v_1) = \{v_2, v_6\}$, $\partial(v_2) = \{v_1, v_3\}$, $\partial(v_3) = \{v_2, v_4\}$, $\partial(v_4) = \{v_3, v_5\}$, $\partial(v_5) = \{v_4, v_6\}$ and $\partial(v_6) = \{v_5, v_1\}$ with $|\partial(v_i)| = 2$ where i = 1,2,…6. Thus, we have τ = {X, φ,{v_1},{v_2},{v_3},{v_4},{v_5}, {v_6}} and $\partial\{v_1, v_2, v_3, v_4, v_5, v_6\} = 2$. Here, we get $|\partial(v_i)| = 2$ where i = 1,2,…6. Hence this graph is a topologized graph. Moreover, the topologized independent dominating sets are given by D = {v_1, v_4} and the corresponding fuzzy sets for maximum are as follows.

$\mu_{v_1 \cup v_2}(x)$ = max {0.11, 0.10}= 0.11, ∀ x ∈ X, $\mu_{v_2 \cup v_3}(x)$ = max {0.10, 0.09}= 0.10, ∀ x ∈ X $\mu_{v_3 \cup v_4}(x)$ = max {0.09, 0.07}= 0.09, ∀ x ∈ X, $\mu_{v_4 \cup v_5}(x)$ = max {0.07, 0.05}= 0.07, ∀ x ∈ X and $\mu_{v_5 \cup v_6}(x)$ = max {0.05, 0.08}= 0.08, ∀ x ∈ X.

Similarly the corresponding fuzzy sets for minimum are given by $\mu_{v_1 \cap v_2}(x)$ = min {0.11, 0.10} = 0.10, ∀ x ∈ X, $\mu_{v_2 \cap v_3}(x)$ = min {0.10, 0.09}= 0.09, ∀ x ∈ X, $\mu_{v_3 \cap v_4}(x)$ = min {0.09, 0.07}= 0.07, ∀ x ∈ X, $\mu_{v_4 \cap v_5}(x)$ = min {0.07, 0.05}= 0.05, ∀ x ∈ X and $\mu_{v_5 \cap v_6}(x)$ = min {0.05, 0.08}= 0.05, ∀ x ∈ X. Hence it is clear that this graph is fuzzy topologized domination graph.

Example 4.2

Here we discuss another graph which is depicted in figure 4.2 to verify the concept of fuzzy topologized domination graph.

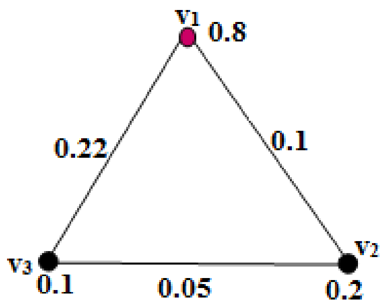

Figure 4.2

Let v_1, v_2 and v_3 denote the vertices and 0.1, 0.05, 0.22 denote the edges which are labelled $f_u(0.1) = \{v_1, v_2\}$, $f_u(0.05) = \{v_2, v_3\}$, $f_u(0.22) = \{v_3, v_1\}$.

Let X = {v_1, v_2, v_3, 0.1, 0.05, 0.22} be a topological space defined by the topology τ = {X, $\emptyset$, {v_1}, {v_2}, {v_3}, {v_1, v_2}, {v_2, v_3}, {v_3, v_1},{v_1, v_2,v_3},{v_2, v_3, v_1}}. Here for every {x} ∈ X is open or closed.

By the definition of topologized graph $|\partial(v)| \leq 2$ we have $\partial(v_1) = \{v_2, v_3\}$, $\partial(v_2) = \{v_1, v_3\}$ and $\partial(v_3) = \{v_2, v_1\}$ with $|\partial(v_i)| = 2$ where i = 1,2,3. Thus, we have τ = {X, φ, {v_1}, {v_2}, {v_3}} and $\partial\{v_1, v_2, v_3\} = 2$. Here, we get $|\partial(v_i)| = 2$ where i = 1,2,3. Hence this graph is topologized graph.

Moreover, the topologized independent dominating sets are given by D = {v_1} and {v_2, v_3} and the corresponding fuzzy sets for maximum are given by $\mu_{v_1 \cup v_2}(x)$ = max {0.8, 0.2} = 0.8, ∀ x ∈ X, $\mu_{v_2 \cup v_3}(x)$ = max {0.2, 0.1} = 0.2, ∀ x ∈ X and $\mu_{v_3 \cup v_1}(x)$ = max {0.1, 0.8} = 0.8, ∀ x ∈ X.

Similarly the corresponding fuzzy sets for minimum are given by $\mu_{v_1 \cap v_2}$(x) = min {0.8, 0.2}= 0.2, ∀ x ∈ X, $\mu_{v_2 \cap v_3}$(x) = min {0.2, 0.1}= 0.1, ∀ x ∈ X and $\mu_{v_3 \cap v_1}$(x) = min {0.1, 0.8}= 0.1, ∀ x ∈ X. Hence it is clear that this graph is fuzzy topologized domination graph.

Example 4.3

Consider the graph which is depicted in figure 4.3.

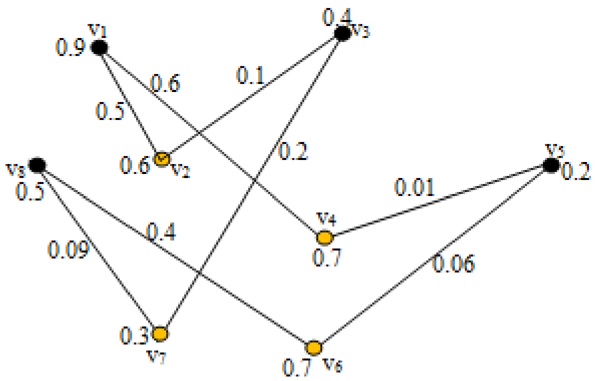

Figure 4.3

Let v_1, v_2, v_3, v_4, v_5, v_6, v_7 and v_8 denote the vertices and 0.6,0.5, 0.1, 0.2, 0.01, 0.06, 0.4, 0.09 denote the edges which are labelled f_u(0.5) = {v_1, v_2}, f_u(0.6) = {v_1, v_4}, f_u(0.1) = {v_2, v_3}, f_u(0.2) = {v_3, v_7}, f_u(0.01) = {v_4, v_5}, f_u(0.06) = {v_5, v_6}, f_u(0.4) = {v_6, v_8}, f_u(0.09) = {v_7, v_8}.

Let X = {v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, 0.6, 0.5, 0.1, 0.2, 0.01, 0.06, 0.4, 0.09 } be a topological space defined by the topology τ = {X, ∅, {v_1}, {v_2}, {v_3}, {v_4}, {v_5}, {v_6}, {v_7}, {v_8},{v_1, v_2}, {v_1, v_4}, {v_3, v_7},{v_4, v_5}, {v_2, v_3},{v_5, v_6}, {v_6, v_8}, {v_7, v_8}, {v_1, v_2,v_3}, {v_6, v_5,v_4},{v_3, v_4,v_5}, {v_5, v_6,v_7},{v_2, v_3,v_4, v_5}, {v_3, v_4,v_5, v_6},…, {v_1, v_2, v_3,v_7, v_8,v_6, v_5, v_4}}. Here for every {x}∈ X is open or closed.

By the definition of topologized graph, we have $|\partial(v)| \le 2$ and $\partial(v_1)$ = {v_4, v_2}, $\partial(v_2)$ = {v_1, v_3}, $\partial(v_3)$ = {v_2, v_7}, $\partial(v_4)$ = {v_1, v_5}, $\partial(v_5)$ = {v_4, v_6}, $\partial(v_6)$ = {v_5, v_8}, $\partial(v_7)$ = {v_3, v_8} and $\partial(v_8)$ = {v_7, v_6} with $|\partial(v_i)| = 2$ where i = 1,2,…8. Thus, we have τ = {X, φ, {v_1}, {v_2}, {v_3}, {v_4}, {v_5}, {v_6}, {v_7}, {v_8}} and ∂{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8} = 2. Here, we get $|\partial(v_i)| = 2$ where i = 1,2,…8. Hence this graph is a topologized graph. Moreover, the topologized independent dominating sets are given by D = {v_2, v_4, v_6, v_7} and the corresponding fuzzy sets for maximum are as follows.

$\mu_{v_1 \cup v_2}$(x) = max {0.9, 0.6}= 0.9, ∀ x ∈ X, $\mu_{v_1 \cup v_4}$(x) = max {0.9, 0.7}= 0.9, ∀ x ∈ X, $\mu_{v_2 \cup v_3}$(x) = max {0.6, 0.4}= 0.6, ∀ x ∈ X, $\mu_{v_3 \cup v_7}$(x) = max {0.4, 0.3}= 0.4, ∀ x ∈ X, $\mu_{v_4 \cup v_5}$(x) = max {0.7, 0.2}= 0.7, ∀ x ∈ X, $\mu_{v_5 \cup v_6}$(x) = max {0.2, 0.7}= 0.7, ∀ x ∈ X, $\mu_{v_6 \cup v_8}$(x) = max {0.7, 0.5}= 0.7, ∀ x ∈ X and $\mu_{v_7 \cup v_8}$(x) = max{0.3, 0.5}= 0.5, ∀ x ∈ X.

Similarly the corresponding fuzzy sets for minimum are given by $\mu_{v_1 \cap v_2}$(x) = min{0.9, 0.6}= 0.6, ∀ x ∈ X, $\mu_{v_1 \cap v_4}$(x) = min {0.9, 0.7}= 0.7, ∀ x ∈ X, $\mu_{v_2 \cap v_3}$(x) = min {0.6, 0.4}= 0.4, ∀ x ∈ X, $\mu_{v_3 \cap v_7}$(x) = min {0.4, 0.3}= 0.3, ∀ x ∈ X, $\mu_{v_4 \cap v_5}$(x) = min {0.7, 0.2}= 0.2, ∀ x ∈ X, $\mu_{v_5 \cap v_6}$(x) = min {0.2, 0.7}= 0.2, ∀ x ∈ X, $\mu_{v_6 \cap v_8}$(x) = min {0.7, 0.5}= 0.5, ∀ x ∈ X and $\mu_{v_7 \cap v_8}$(x) = min {0.3, 0.5}= 0.3, ∀ x ∈ X.

Hence it is clear that this graph is fuzzy topologized domination graph.

Preposition 4.1

For a path P_p with p vertices, $\gamma(P_p) = [p/3]$.

Proof

Let G be a path P_n which is depicted in figure 4.4 with 6 vertices. Here, $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $E = \{a, b, c, d, e\}$.

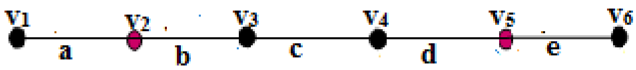

Figure 4.4

Let $f_u(a) = \{v_1, v_2\}$, $f_u(b) = \{v_2, v_3\}$, $f_u(c) = \{v_3, v_4\}$, $f_u(d) = \{v_4, v_5\}$, $f_u(e) = \{v_5, v_6\}$. From the above graph we get $|\partial(V)| \leq 2$. Hence G is a topologized domination graph and $\tau\gamma_{ids}(P_n) = n/3 = 2$.

V. CONCLUSION

The concept of domination in graphs is very rich both in theoretical developments and applications. In this paper, we obtained the existence of topologized domination graph into fuzzy topologized domination graph and also established some general results. Finally, a couple of example was provided to illustrate fuzzy topologized domination graph. In future, this can be extended to other domination parameters.

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