On k* Quasi Normal Operator of Order n

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Abstract — *In this paper, we will give new type of quasi normal operator is called K* quasi normal operator of order n and as well as, study the tensor product of these operators and some properties of this concept have been given.*

Keywords — *normal operators, quasi normal operators, n-normal operators.*

I. INTRODUCTION

H be a Hilbert space and B(H) be the algebra of all bounded linear operators acting on H . An operator T ∈ B(H) is said to be quasi normal operator if T commute with T T* this concept was given by A. Brown in 1953 [2].

II. PRELIMINARIES

LET T ∈ B(H), WHERE H IS A HIBERT SPACE, THEN T IS SAID TO BE A *K* QUASI NORMAL OPERATOR OF ORDER N* , IF SATISFY

$$(T^*)^K (T^{*n} T^n) = (T^{*n} T^n) (T^*)^K$$

WHERE, T* IS THE ADJOINT OF THE OPERATOR T AND N IS A POSITIVE NUMBER .

III. THEOREMS

3.1 THEOREM

LET T : H → K BE K* QUASI NORMAL OPERATOR OF ORDER N SUCH THAT $T^n T^{*n} = T^{*n} T^n$ THEN T* IS K* QUASI NORMAL OPERATOR OF ORDER N .

PROOF :

TO PROVE ,

$$((T^*)^*)^K ((T^*)^{*n} (T^*)^n) = ((T^*)^{*n} (T^*)^n) ((T^*)^*)^K$$

TAKE L.H.S.,

$$((T^*)^*)^K ((T^*)^{*n} (T^*)^n) = ((T^*)^K)^* ((T^{*n})^* (T^n)^*)$$

$$= ((T^*)^K)^* ((T^n)(T^{*n}))^*$$

$$= ((T^n T^{*n})(T^*)^K)^*$$

$$= ((T^{*n} T^n)(T^*)^K)^*$$

$$= ((T^*)^K (T^{*n} T^n))^*$$

[∵, T IS K* QUASI NORMAL OPERATOR OF ORDER N]

$$= (T^{*n} T^{n})^* ((T^*)^K)^*$$

$$= (T^{n} T^{*n})^* ((T^*)^K)^*$$

$$= ((T^{*n})^* (T^{n})^*) ((T^*)^K)^*$$

$$((T^*)^*)^K ((T^*)^{*n} (T^*)^n) = ((T^*)^{*n} (T^*)^n) ((T^*)^*)^K$$

∴ L.H.S = R.H.S

THEN, T* IS K* QUASI NORMAL OPERATOR OF ORDER N.

3.2 THEOREM

IF T IS K* QUASI NORMAL OPERATOR OF ORDER N AND S IS UNITARILY EQUIVALENCE TO T THEN S IS K* QUASI NORMAL OPERATOR OF ORDER N.

PROOF :

SINCE, S IS UNITARILY EQUIVALENCE TO T, THEN THERE EXIST A UNITARY OPERATOR U SUCH THAT S = U T U*

SO THAT, $S^n = U T^n U^*$ AND $S^{*n} = U T^{*n} U^*$

TO PROVE, S IS K* QUASI NORMAL OPERATOR OF ORDER N.

$$\Longrightarrow \quad (S^*)^K (S^{*n} S^n) = (S^{*n} S^n)(S^*)^K$$

TAKE L.H.S,

$$(S^*)^K (S^{*n} S^n) = (U T^{*K} U^*)(U T^{*n} U^* U T^n U^*)$$

$$= U T^{*K} U^* U T^{*n} U^* U T^n U^*$$

$$= U T^{*K} (T^{*n} T^n) U^* \quad \ldots\ldots\ldots\ldots\ldots(1)$$

TAKE R.H.S,

$$(S^{*n} S^n)(S^*)^K = (U T^{*n} U^* U T^n U^*)(U T^{*K} U^*)$$

$$= U T^{*n} U^* U T^n U^* U T^{*K} U^*$$

$$= U (T^{*n} T^n) T^{*K} U^*$$

$$= U T^{*K} (T^{*n} T^n) U^* \quad \ldots\ldots\ldots\ldots\ldots(2)$$

[∵, T IS K* QUASI NORMAL OPERATOR OF ORDER N]

∴ (1) = (2)

THAT IS,

$$(S^*)^K (S^{*n} S^n) = (S^{*n} S^n)(S^*)^K$$

THEN S IS K* QUASI NORMAL OPERATOR OF ORDER N.

3.3 PROPOSITION

IF T IS SELF ADJOINT OPERATOR THEN T IS K* QUASI NORMAL OPERATOR OF ORDER N.

PROOF :

SINCE, T IS SELF ADJOINT OPERATOR T = T* .

TO PROVE, $(T^*)^K (T^{*n} T^n) = (T^{*n} T^n)(T^*)^K$

TAKE L.H.S.,

$$\Rightarrow (T^*)^K (T^{*n} T^n) = (T^*)^K (T^* T)^n$$
$$= T^K (T T)^n$$
$$= T^K T^{2n}$$

$$(T^*)^K (T^{*n} T^n) = T^{K+2n} \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

TAKE R.H.S.,

$$\Rightarrow (T^{*n} T^n)(T^*)^K = (T^* T)^n (T^*)^K$$
$$= (T T)^n T^K$$
$$= T^{2n} T^K$$
$$= T^{2n+K}$$
$$= T^{K+2n} \quad \ldots\ldots\ldots\ldots\ldots\ldots(2)$$

$\therefore$ (1) = (2)

THEN T IS K* QUASI NORMAL OPERATOR OF ORDER N .

3.4 PROPOSITION

LET $T_1, T_2, \ldots\ldots, T_n$ ARE K* QUASI NORMAL OPERATORS OF ORDER N FROM A HILBERT SPACE H THEN $T_1 \otimes T_2 \otimes \ldots\ldots\ldots\otimes T_n$ IS ALSO K* QUASI NORMAL OPERATOR N .

PROOF :

TO PROVE ,

$$((T_1\otimes T_2\otimes\ldots\otimes T_n)^*)^K ((T_1\otimes T_2\otimes\ldots\otimes T_n)^{*n} (T_1\otimes T_2\otimes\ldots\otimes T_n)^n)$$
$$= ((T_1\otimes T_2\otimes\ldots\otimes T_n)^{*n}(T_1\otimes T_2\otimes\ldots\otimes T_n)^n)((T_1\otimes T_2\otimes\ldots\otimes T_n)^*)^K$$

TAKE L.H.S., $\Rightarrow$

$$((T_1\otimes T_2\otimes\ldots\otimes T_n)^*)^K ((T_1\otimes T_2\otimes\ldots\otimes T_n)^{*n} (T_1\otimes T_2\otimes\ldots\otimes T_n)^n)$$
$$= (T_1^*\otimes T_2^*\otimes\ldots\otimes T_n^*)^K ((T_1^{*n}\otimes T_2^{*n}\otimes\ldots\otimes T_n^{*n})(T_1^n\otimes T_2^n\otimes\ldots\otimes T_n^n))$$
$$= ((T_1^*)^K\otimes(T_2^*)^K\otimes\ldots\otimes(T_n^*)^K)(T_1^{*n}T_1^n\otimes T_2^{*n}T_2^n\otimes\ldots\otimes T_n^{*n}T_n^n)$$
$$= ((T_1^*)^K(T_1^{*n} T_1^n)\otimes(T_2^*)^K(T_2^{*n}T_2^n)\otimes\ldots\otimes(T_n^*)^K(T_n^{*n}T_n^n))$$
$$= ((T_1^{*n}T_1^n)(T_1^*)^K\otimes(T_2^{*n}T_2^n)(T_2^*)^K\otimes\ldots\otimes(T_n^{*n} T_n^n)(T_n^*)^K)$$

[$\because$, $T_1, T_2, \ldots\ldots, T_n$ ARE K* QUASI NORMAL OPERATORS OF ORDER N]

$$= (T_1^{*n} T_1^n\otimes T_2^{*n} T_2^n\otimes\ldots\otimes T_n^{*n} T_n^n)((T_1^*)^K\otimes(T_2^*)^K\otimes\ldots\otimes(T_n^*)^K)$$

$$= \left(\left(T_1^{*n} \otimes T_2^{*n} \otimes \ldots\ldots \otimes T_n^{*n} \right) \left(T_1^{n} \otimes T_2^{n} \otimes \ldots\ldots \otimes T_n^{n} \right) \right) \left(\left(T_1^{*} \right)^K \otimes \left(T_2^{*} \right)^K \otimes \ldots\ldots \otimes \left(T_n^{*} \right)^K \right)$$

$$= \left(\left(T_1 \otimes T_2 \otimes \ldots \otimes T_n \right)^{*n} \left(T_1 \otimes T_2 \ldots . \otimes T_n \right)^n \right) \left(T_1^{*} \otimes T_2^{*} \otimes \ldots . \otimes T_n^{*} \right)^K$$

$$= \left(\left(T_1 \otimes T_2 \otimes \ldots \otimes T_n \right)^{*n} \left(T_1 \otimes T_2 \ldots . \otimes T_n \right)^n \right) \left(\left(T_1 \otimes T_2 \otimes \ldots . \otimes T_n \right)^* \right)^K$$

∴ L.H.S = R.H.S

THEN $T_1 \otimes T_2 \otimes \ldots\ldots\ldots \otimes T_n$ IS K* QUASI NORMAL OPERATORS OF ORDER N .

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