Total Signed And Roman Edge Dominating Functions of Corona Product Graph of A Cycle With A Complete Graph

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ABSTRACT

The theory of Graphs is one of the major areas of combinatorics that has developed into an important branch of Mathematics. The theory of domination in graphs is an emerging area of research in graph theory today. It has been studied extensively and finds applications to various branches of Science & Technology.

Frucht and Harary [13] introduced a new product on two graphs G_1 and G_2, called corona product denoted by $G_1 \odot G_2$.

In this paper, some results on total Signed and Roman edge dominating functions of corona product graph of a cycle with a complete graph are discussed.

Keywords : *Corona Product, Cycle, Complete graph, Total signed edge dominating function, Total Roman edge dominating function.*

I. INTRODUCTION

The theory of Graphs is one of the important branches of Mathematics. The major development of graph theory has occurred in recent years and inspired to a larger degree and it has become the source of interest to many researchers due to its applications to various branches of Science & Technology.

Domination in graphs has been studied extensively in recent years. It is introduced by Ore [20] and Berge [8] and has become an emerging area of research in graph theory today. Many graph theorists, Allan, R.B. and Laskar, R.[1], Cockayne and Hedetniemi [9], Rejikumar [21], Sampathkumar [23] and others have contributed significantly to the theory of dominating sets, domination numbers and other related topics. Haynes, Hedetniemi and Slater [15,16] presented a survey of articles in the wide field of domination in graphs.

Another type of domination is total domination. Total dominating sets are introduced by Cockayane, Dawes and Hedetniemi [10].

The concept of edge domination was introduced by Mitchell and Hedetniemi [19] and it is explored by many researchers. Arumugam and Velammal [7] have discussed the edge domination in graphs while the fractional edge domination in graphs is discussed in Arumugam and Jerry [6]. The complementary edge domination in graphs is studied by Kulli and Soner [18] The edge domination in graphs of cubes and Signed total domination is studied by Zelinka [24, 25].

Product of graphs occurs naturally in discrete mathematics as tools in combinatorial constructions. They give rise to an important classes of graphs and deep structural problems. Frucht and Harary [13] introduced a new product on two graphs G_1 and G_2, called corona product denoted by $G_1 \odot G_2$. This new concept enhances the study of these graphs and it is interesting to study various graph theoretic parameters of these graphs.

Recently, dominating functions in domination theory have received much attention. A purely graph – theoretic motivation is given by the fact that the dominating function problem can be seen, in a clear sense, as a proper generalization of the classical domination problem. Similarly edge dominating functions are also studied extensively.

II. CORONA PRODUCT OF C_n AND K_m

The corona product of a cycle C_n with a complete graph K_m is a graph obtained by taking one copy of a n – vertex graph C_n and n copies of K_m and then joining the i^{th} vertex of C_n to every vertex of i^{th} copy of K_m . This graph is denoted by $C_n \odot K_m$.

The vertices of C_n are denoted by $v_1, v_2, \dots, v_n$. The edges in C_n are denoted by $e_1, e_2, \dots, e_n$ where e_i is the edge joining the vertices v_i and v_{i+1}, $i \neq n$. For $i = n$, e_n is the edge joining the vertices v_n and v_1.

The vertices in the i^{th} copy of K_m are denoted by $w_{i1}, w_{i2}, \dots, w_{im}$. The edges in the i^{th} copy of K_m are denoted by $l_{ij}, j = 1,2, \dots, \frac{m(m-1)}{2}$.

There are another type of edges in G denoted by $h_{ij}, i = 1,2, \dots, n$ and $j = 1,2, \dots, m$ is the edge joining the vertex v_i of C_n to vertex w_{ij} of i^{th} copy of K_m. These edges which are in G and related to the i^{th} copy of K_m are denoted by $h_{i1}, h_{i2}, \dots, h_{im}$ and these are adjacent to each other and incident with the vertex v_i of C_n.

Some properties of corona product graph $G = C_n \odot K_m$ are studied by Anita [2] and some results on minimal edge dominating sets and functions of this graph are presented in [3]. Also some results on signed and Roman edge dominating functions of this graph are obtained by the authors [4]. Further some results on convexity of total Y - edge domination variants of corona product graph of a cycle with a complete are discussed in [5].

We need the following Theorem which is presented in [2].

Theorem 2.1: The adjacency of an edge e in $G = C_n \odot K_m$ is given by

$$adj(e) = \begin{cases} 2m+2, \text{ if } e = e_i \in C_n, \\ 2m-2, \text{ if } e = l_{ij} \in i^{th} \text{ copy of } K_m, \\ 2m, \text{ if } e = h_{ij} \in G = C_n \odot K_m. \end{cases}$$

III. TOTAL SIGNED EDGE DOMINATING FUNCTION

The concept of Signed dominating function was introduced by Dunbar et al. [12]. There is a variety of possible applications for this variation of domination. By assigning the values -1 or $+1$ to the vertices of a graph we can model such things as networks of positive and negative electrical charges, networks of positive and negative spins of electrons and networks of people or organizations in which global decisions can be made.

Zelinka, B. [25] introduced the concept of total signed dominating function. This section contains the study of total signed edge dominating functions and minimal total signed edge dominating functions of graph $G = C_n \odot K_m$. First we recall the definitions of total signed edge dominating function of a graph.

Definition: Let $G(V, E)$ be a graph. A function $f : E \to \{-1, 1\}$ is called a **total signed edge dominating function of G** if

$$f(N(e)) = \sum_{e' \in E(G)} f(e') \geq 1 \quad \text{for each } e \in E.$$

A total signed edge dominating function f of G is called a **minimal total signed edge dominating function** (MTSEDF) if for all $g < f$, g is not a total signed edge dominating function.

Theorem 3.1: A function $f : E \to \{-1, 1\}$ defined by

$$f(e) = \begin{cases} -1, & \text{for } (m-2) \text{ edges } l_{ij} \text{ in each copy of } K_m, \\ 1, & \text{otherwise.} \end{cases}$$

is a minimal total signed edge dominating function of $G = C_n \odot K_m$.

Proof: Let f be a function defined as in the hypothesis. By the definition of the function -1 is assigned to $(m-2)$ edges l_{ij} in each copy of K_m in G and 1 is assigned to remaining edges of G.

The summation value taken over $N(e)$ of $e \in E$ is as follows.

Case 1: Let $e = e_i \in C_n$ be such that $adj(e) = 2m + 2$ in G. Then $N(e_i)$ contains two edges of C_n and $2m$ edges which are drawn from the vertices v_i and v_{i+1} respectively to the m vertices of i^{th} and $(i+1)^{th}$ copies of K_m and their functional value is 1.

$$\text{Therefore} \sum_{e \in N(e_i)} f(e) = 1 + 1 + \underbrace{[1 + 1 + \cdots + 1]}_{2m - times} = 2m + 2.$$

Case 2: Let $l_{ik} \in i^{th}$copy of $K_m$be such that $adj(l_{ik}) = 2m - 2$. By the definition of f, $(m-2)$ edges of $K_m$are assigned -1 and the remaining $(m-2)$ edges l_{ij} are assigned 1 and two edges h_{ij} that are adjacent to l_{ik} are assigned 1.

Then $\sum_{e \in N(l_{ik})} f(e) = [(m-2)(-1) + (m-2)(1)] + 1 + 1 = 2$.

Now for all other possibilities of functional values of l_{ij} that are adjacent to l_{ik} , k=1,2,…,m we could see that

$$\sum_{e \in N(l_{ij})} f(e) \geq 1 \text{ for all } e \in E(G).$$

Case 3: Let $h_{ij} \in C_n \odot K_m$ be such that $adj(h_{ij}) = 2m$.

Then $N(h_{ij})$ contains two edges of C_n , $(m-1)$ edges h_{ij} and $(m-1)$ edges l_{ij}in K_m.

Suppose $f(l_{ij}) = -1$ for all $(m-2)$ edges l_{ij} that are adjacent to h_{ij}. Then

$$\sum_{e \in N(h_{ij})} f(e) = 1 + 1 + [(m-2)(-1) + (m-1)(1)] + (1)(1) = 4.$$

Suppose $f(l_{ij}) = 1$ for all $(m-1)$ edges l_{ij} that are adjacent to h_{ij}. Then

$$\sum_{e \in N(h_{ij})} f(e) = 1 + 1 + [(m-1)(1) + (m-1)(1)] = 2m.$$

Thus as in Case 2 for all other possibilities of functional values for the $(m-1)$ edges l_{ij} that are adjacent to h_{ij} we could see that

$$\sum_{e \in N(h_{ij})} f(e) > 1.$$

Therefore for all possibilities we get

$$\sum_{e \in E(G)} f(e) \geq 1 \text{ for all } e \in E(G).$$

Hence f is a total signed edge dominating function.

We now check for the minimality of f.

Define a function $g: E \to [-1,1]$ by

$$g(e) = \begin{cases} -1, & \text{for one edge } h_{ik}, \\ -1, & \text{for } (m-2) \text{ edges } l_{ij} \text{in each copy of } K_m, \\ 1, & \text{otherwise.} \end{cases}$$

Since strict inequality holds at h_{ik} , it follows that $g < f$.

Case (i): Let $e_i \in C_n$be such that $adj(e_i) = 2m + 2$ in G.

Sub case 1: Let $h_{ik} \in N(e_i)$.

Then $\sum_{e \in N(e_i)} g(e) = -1 + 1 + 1 + \underbrace{[1 + 1 + \cdots + 1]}_{(2m-1) times} = 2m$.

Sub Case 2: Let $h_{ik} \notin N(e_i)$.

$$\sum_{e \in N(e_i)} g(e) = 1 + 1 + 1 + \underbrace{[1 + 1 + \cdots + 1]}_{(2m-1) times} = 2m + 2.$$

Case (ii): Let $l_{ik} \in i^{th}$copy of K_m. Then $adj(l_{ik}) = 2m - 2$ in G.

Sub case 1: Let $h_{ik} \in N(l_{ik})$.

Then $\sum_{e \in N(l_{ik})} g(e) = [(m-2)(-1) + (m-2)(1)] + 1 + (-1) = 0$.

Sub Case 2: Let $h_{ik} \notin N(l_{ik})$. Then

$$\sum_{e \in N(l_{ik})} g(e) = [(m-2)(-1)+(m-2)(1)]+1+1 = 2.$$

Case(iii): Let $h_{ij} \in C_n \odot K_m$ be such that $adj(h_{ij}) = 2m$.

Sub case 1 : Let $h_{ik} \in N(h_{ij})$.

Suppose $g(l_{ij}) = -1$ for all $(m-2)$ edges l_{ij} that are adjacent to h_{ij}. Then

$$\sum_{e \in N(h_{ij})} g(e) = 1+1+ [(m-2)(-1)+(m-2)(1)]+(1)(1)+(1)(-1) = 2.$$

Suppose $g(l_{ij}) = 1$ for all $(m-1)$ edges l_{ij} that are adjacent to h_{ij}. Then

$$\sum_{e \in N(h_{ij})} g(e) = 1+1+ [(m-1)(1)+(m-2)(1)]+(1)(-1) = 2m-2.$$

Sub Case 2: Let $h_{ik} \notin N(h_{ij})$. Then

Suppose $g(l_{ij}) = -1$ for all $(m-2)$ edges l_{ij} that are adjacent to h_{ij}. Then

$$\sum_{e \in N(h_{ij})} g(e) = 1+1+ [(m-2)(-1)+(m-1)(1)]+(1)(1) = 4.$$

Suppose $g(l_{ij}) = 1$ for all $(m-1)$ edges l_{ij} that are adjacent to h_{ij}. Then

$$\sum_{e \in N(h_{ij})} g(e) = 1+1+ [(m-1)(1)+(m-1)(1)] = 2m.$$

Thus $\sum_{e \in E(G)} g(e) < 1$ for some $e \in E(G)$.

So g is not a total signed edge dominating function. Since g is defined arbitrarily, it follows that there exists no $g < f$ such that g is a total signed edge dominating function.

Thus f is a minimal total signed edge dominating function. ∎

IV. TOTAL ROMAN EDGE DOMINATING FUNCTION

Roman domination is suggested originally in the article Scientific American by Ian Stewart [17] and also by Revelle [22]. Later Henning et.al [14] and Cockayne et. al [11] contributed to the theory of Roman domination. In this section the concept of total Roman edge dominating function of the graph $G = C_n \odot K_m$ is studied. Also some results on minimal total Roman edge dominating function of $G = C_n \odot K_m$ are obtained.

First we define total Roman edge dominating function of a graph.

Definition: Let $G(V, E)$ be a graph. A function $f : E \rightarrow \{0,1,2\}$ is called a total **Roman edge dominating function** (TREDF) of G if

$$f(N(e)) = \sum_{e' \in N(e)} f(e') \geq 1, for\ each\ e \in E$$

and satisfying the condition that every edge e' for which $f(e') = 0$ is adjacent to at least one edge e for which $f(e) = 2$.

A total Roman edge dominating function f of G is called a **minimal total Roman edge dominating function** (MTREDF) if for all $g < f$, g is not a total Roman edge dominating function.

Theorem 4.1: A function $f : E \rightarrow \{0,1,2\}$ defined by

$$f(e) = \begin{cases} 2, \text{for } (m-1)\ edges\ h_{ij} \text{ in } C_n \odot K_m, \\ 0, \text{ otherwise.} \end{cases}$$

is a minimal total Roman edge dominating function of $G = C_n \odot K_m$.

Proof: Let f be a function defined as in the hypothesis.

Case 1: Let $e_i \in C_n$ be such that $adj(e_i) = 2m+2$ in *G*.

Then $N(e_i)$ contains m edges $h_{i1}, h_{i2}, \ldots, h_{im}$ of *G* ;

m edges $h_{(i+1)1}, h_{(i+1)2}, \dots, h_{(i+1)m}$ of G and two edges of C_n.

So $\sum_{e \in N(e_i)} f(e) = (0+0) + [(m-1)2 + (m-1)2] + 0 + 0 = 4m - 4.$

Case 2: Let $l_{ij} \in i^{th}$ copy of K_m. Then $adj(l_{ij}) = 2m - 2$ in G.

Then $N(l_{ij})$ contains $(2m-4)$ edges of K_m and two edges h_{ij} of G.

So $\sum_{e \in N(l_{ij})} f(e) = \underbrace{[0+0+\cdots+0]}_{(2m-4)times} + 2 + 2 = 4, \quad if\ f(h_{ij}) = 2,$

or

$$\sum_{e \in N(l_{ij})} f(e) = \underbrace{[0+0+\cdots+0]}_{(2m-4)times} + 2 + 0 = 2, \quad if\ f(h_{ij}) = 0.$$

Case 3: Let $h_{ij} \in C_n \odot K_m$ be such that $adj(h_{ij}) = 2m$ in G.

Then $f(h_{ij}) = 0$. $N(h_{ij})$ contains $(m-1)$ edges l_{ij} of K_m and $(m-1)$ edges h_{ij} of G and two edges of C_n.

So $\sum_{e \in N(h_{ij})} f(e) = (0+0) + [(m-1)0 + (m-1)2] = 2m - 2.$

Suppose $f(h_{ij}) = 2$. Then $N(h_{ij})$ contains $(m-2)$ edges h_{ij} whose functional values are 2, one edge h_{ij}, two edges e_i of C_n and $(m-1)$ edges l_{ij} whose functional values are 0 respectively.

Therefore $\sum_{e \in N(h_{ij})} f(e) = (0+0) + [(m-1)0 + (m-2)2] + 0 = 2m - 4.$

Therefore for all possibilities, we get

$$\sum_{e \in E(G)} f(e) > 1.$$

Let e be an edge of G such that $f(e) = 0$ and e' be another edge of G such that $e' \neq e$ and $f(e') = 2$.

Then we show that e and e' are adjacent.

Now $f(e) = 0$ implies $e = e_i \in C_n$ for some i, or $e = l_{ij}$ for some i and j.

Now $f(e') = 2$ implies $e' = h_{ij}$ for some i and j. But $f(h_{ij}) = 2$ for all i and j.

Suppose $e = e_i \in C_n$. Then obviously e_i and h_{ij} are adjacent. That is e and e' are adjacent.

Suppose $e = l_{ij}$ for some $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, m$. Then also e and e' are adjacent.

This implies that f is a total Roman edge dominating function.

Now we check for the minimality of f.

Define a function $g : E \to \{0, 1, 2\}$ by

$$g(e) = \begin{cases} 1, \text{for one edge } h_{ik} \text{ in } C_n \odot K_m, \\ 2, \text{for all edges } h_{ij} \text{ in } C_n \odot K_m, j \neq k \\ 0, \text{otherwise.} \end{cases}$$

Since strict inequality holds at an edges h_{ik} it follows that $g < f$.

Case (i): Let $e_i \in C_n$ be such that $adj(e_i) = 2m + 2$ in G.

Sub Case 1: Let $h_{ik} \in N(e_i)$. Then

$$\sum_{e \in N(e_i)} g(e) = (0+0) + [(m-1)2 + (m-2)2 + (1)(1)] + 0 + 0 = 4m - 3.$$

Sub Case 2: Let $h_{ik} \notin N(e_i)$. Then

$$\sum_{e \in N(e_i)} g(e) = (0+0) + [(m-1)2 + (m-1)2] + 0 + 0 = 4m - 4.$$

Case (ii): Let $l_{ij} \in i^{th}$ copy of K_m. Then $adj(l_{ij}) = 2m - 2$ in G.

Sub Case 1: Let $h_{ik} \in N(\, l_{ij}\,)$. Then

$$\sum_{e \in N(\, l_{ij}\,)} g(e) = \underbrace{[0+0+\cdots+0]}_{(2m-4)times} + 2 + 1 = 3, \quad if\ f\,(h_{ij}) = 2\,,$$

or

$$\sum_{e \in N(\, l_{ij}\,)} g(e) = \underbrace{[0+0+\cdots+0]}_{(2m-4)times} + 1 + 0 = 1, \quad if\ f\,(h_{ij}) = 0.$$

Sub Case 2: Let $h_{ik} \notin N(l_{ij})$. Then

$$\sum_{e \in N(\, l_{ij}\,)} g(e) = \underbrace{[0+0+\cdots+0]}_{(2m-4)times} + 2 + 2 = 4, \quad if\ f\,(h_{ij}) = 2,$$

or

$$\sum_{e \in N(\, l_{ij}\,)} g(e) = \underbrace{[0+0+\cdots+0]}_{(2m-4)times} + 2 + 0 = 2, \quad if\ f\,(h_{ij}) = 0.$$

Case (iii): Let $h_{ij} \in C_n \odot K_m$ be such that $adj(\, h_{ij}\,) = 2m$ in G.

Sub Case 1: Let $h_{ik} \in N(\, h_{ij}\,)$. Then

$$\sum_{e \in N(\, h_{ij}\,)} g(e) = (0+0) + [(m-1)0 + (m-2)2] + (1)(1) = 2m-3,$$

or

$$\sum_{e \in N(\, h_{ij}\,)} g(e) = (0+0) + [(m-1)0 + (m-3)2] + (0+1) = 2m-5.$$

Sub Case 2: Let $h_{ik} \notin N(h_{ij})$. Then

$$\sum_{e \in N(\, h_{ij}\,)} g(e) = (0+0) + [(m-1)0 + (m-1)2] = 2m-2,$$

or

$$\sum_{e \in N(\, h_{ij}\,)} g(e) = (0+0) + [(m-1)0 + (m-2)2] + 0 = 2m-4.$$

Hence for all possibilities, we get

$$\sum_{e \in E(G)} g(e) > 1\ \text{, for some } e \in E(G).$$

i.e. g is a total edge dominating function. But g is not a total Roman edge dominating function, since the REDF definition fails in the i^{th} copy of K_m in G.

Let the edge $l_{ij} \in i^{th}$ copy of K_m. Then $g(\, l_{ij}\,) = 0$. We know that every edge l_{ij} in K_m is adjacent to two edges $h_{ij}, j = 1, 2, \ldots, m$. The condition of Roman dominating function fails for the edge l_{ij} which is adjacent to h_{ik} and h_{ij} as $g(\, h_{ik}\,) = 1$ and $g(\, h_{ij}) = 0$.

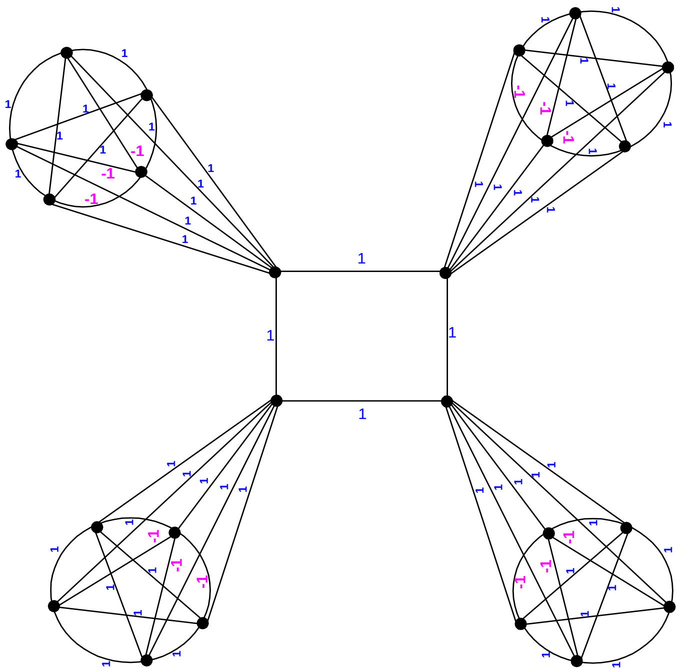
Thus f is a minimal total Roman edge dominating function. ■

V. ILLUSTRATIONS

5.1 MINIMAL TOTAL SIGNED EDGE DOMINATIONG FUNCITION

Theorem 3.1

The functional values are given at each edge of the graph G.

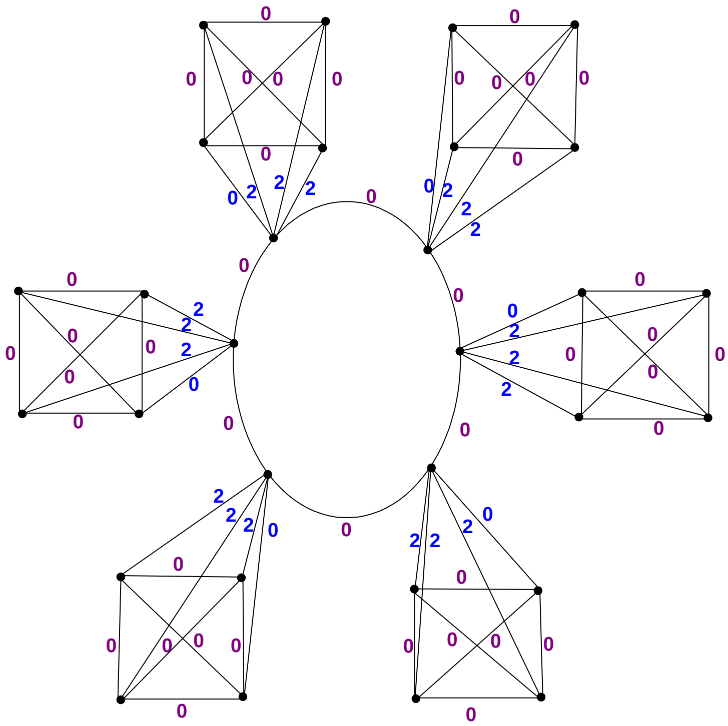


$G = C_4 \odot K_5$

5.2 MINIMAL TOTAL ROMAN EDGE DOMINATING FUNCTION

Theorem 4.1

The functional values are given at each edge of the graph G.



$G = C_6 \odot K_4$

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