Special Pythagorean Triangle In Relation With Pronic Numbers

S. Vidhyalakshmi[1], T. Mahalakshmi[2], M.A. Gopalan[3]
[1,2,3](Department of Mathematics, Shrimati Indra Gandhi College, India)

Abstract:

This paper illustrates Pythagorean triangles, where, in each Pythagorean triangle, the ratio $\frac{2*Area}{Perimeter}$ *is a Pronic number.*

Keywords: *Pythagorean triangles,Primitive Pythagorean triangle, Non primitive Pythagorean triangle, Pronic numbers.*

Introduction:

It is well known that there is a one-to-one correspondence between the polygonal numbers and the sides of polygon. In addition to polygon numbers, there are other patterns of numbers namely Nasty numbers, Harshad numbers, Dhuruva numbers, Sphenic numbers, Jarasandha numbers, Armstrong numbers and so on. In particular, refer [1-17] for Pythagorean triangles in connection with each of the above special number patterns. The above results motivated us for searching Pythagorean triangles in connection with a new number pattern. This paper illustrates Pythagorean triangles, where, in each Pythagorean triangle, the ratio $\frac{2*Area}{Perimeter}$ is a Pronic number.

Method of Analysis:

Let $T(x, y, z)$ be a Pythagorean triangle, where

$$x = 2pq\ ,\quad y = p^2 - q^2\ ,\quad z = p^2 + q^2\ ,\quad p > q > 0 \tag{1}$$

Denote the area and perimeter of $T(x, y, z)$ by A and P respectively.

The mathematical statement of the problem is

$$\frac{2A}{P} = n(n+1)\ \text{, pronic number of rank n} \tag{2}$$

$$\Rightarrow q(p-q) = n(n+1) \tag{3}$$

It is observed that (3) is satisfied by the following two sets of values of p and q: Set 1: $p = 2n+1,\quad q = n$

Set 2: $p = 2n+1,\quad q = n+1$

However, there are other choices for p and q that satisfy (3). To obtain them, treating (2) as a quadratic in q and solving for q, it is seen that

$$q = \frac{1}{2}\left[p + \sqrt{p^2 - 4n(n+1)}\right] \tag{4}$$

To eliminate the square root on the R.H.S of (4), assume

$$\alpha^2 = p^2 - 4n(n+1) \quad (5)$$

which is equivalent to the system of double equations as presented in Table 1 below:

Table 1: System of double equations

System	1	2	3	4	5
$p+q$	$4(n+1)$	$(n+1)$	$2(n+1)$	$n(n+1)$	$2n(n+1)$
$p-q$	n	$4n$	$2n$	4	2

Solving each of the above systems in Table1,the values of p, q are obtained. In view of (1), the corresponding sides of the Pythagorean triangles satisfying (2) are obtained. For the sake of clear understanding, the values of p, q along with the sides of the respective Pythagorean triangles are given in Table 2 below

Table 2: Pythagorean triangle

q	p	(x, y, z)	$\frac{2*A}{P}$	Pythagorean Triangle
$4k+2$	$5k+2$	$(40k^2+36k+8, 9k^2+4k, 41k^2+36k+8)$	$2k(2k+1)$	Primitive when k is odd
$k+1$	$5k+3$	$(10k^2+16k+6, 24k^2+28k+8, 26k^2+32k+10)$	$2k+1(2k+2)$	Non primitive
$n+1$	$2n+1$	$(4n^2+6n+2, 3n^2+2n, 5n^2+6n+2)$	$n(n+1)$	Primitive when n is odd
$2k^2+k$	$2k^2+k+2$	$(8k^4+8k^3+10k^2+4k, 8k^2+4k+4, 8k^4+8k^3+10k^2+4k+4)$	$2k(2k+1)$	Non primitive
n^2+n	n^2+n+1	$(2n^4+4n^3+4n^2+2n, 2n^2+2n+1, 2n^4+4n^3+4n^2+2n+1)$	$n(n+1)$	All primitive

It is noted that one may generate Diophantine 3- tuples from the generators of the above Pythagorean triangle with suitable property.

Illustration:

Consider, $q = 4k+2$, $p = 5k+2$

Note that $p*q + (16k^2+6k) = (6k+2)^2$

$\therefore$ $(4k+2, 5k+2)$ represents Diophantine 2-tuples with property $D(16k^2+6k)$.

Let c be any non-zero integer such that

$$ac + 16k^2 + 6k = \alpha^2 \quad (6)$$

$$bc + 16k^2 + 6k = \beta^2 \quad (7)$$

Eliminating c between (6) and (7), we have

$$(b-a)(16k^2+6k)=b\alpha^2-a\beta^2 \tag{8}$$

Take $\alpha = X + aT$, $\beta = X + bT$ (9)

From (8) and (9), on simplification, we have

$$X^2 = abT^2 + 16k^2 + 6k$$

which is satisfied by $T_0 = 1$, $X_0 = 6k + 2$

Using the above values in (9), we have

$$\alpha = 10k + 4\,, \qquad \beta = 11k + 4$$

Substituting the above value of α in (6) and performing some calculation, we get

$$c = 21k + 8$$

Thus, it is observed that the triple $(4k+2\,,5k+2\,,21k+8)$ represents Diophantine 3-Tuple with property $D(16k^2+6k)$

By repeating the above process, a sequence of Diophantine triples with property $D(16k^2+6k)$ is generated.

For simplicity, in Table 3 below, we present a few sequences of other Diophantine Triples with suitable properties.

Table 3: Examples

Properties	Sequences of Diophantine Triples			
$D(16k^2+6k)$	$(4k+2,5k+2,21k+8)$	$(5k+2,21k+8,48k+18)$	$(5k+2,48k+18,85k+32)$	$(48k+18,85k+32,261k+98)$
$D(11k^2+8k+1)$	$(k+1,5k+3,14k+8)$	$(5k+3,14k+8,37k+21)$	$(5k+3,37k+21,70k+40)$	$(37k+21,70k+40,209k+119)$
$D(2n^2+5n+3)$	$(n+1,2n+1,7n+6)$	$(2n+1,7n+6,17n+13)$	$(2n+1,17n+13,31n+22)$	$(17n+13,31n+22,94n+69)$
$D(1)$	$(2k^2+k,2k^2+k+2,8k^2+4k+4)$	$(2k^2+k+2,8k^2+4k+4,18k^2+9k+12)$	$(2k^2+k+2,18k^2+9k+12,32k^2+16k+24)$	$(2k^2+k,2k^2+k+2,8k^2+4k+4)$
$D(n^2+n+1)$	$(n^2+n,n^2+n+1,4n^2+4n+3)$	$(n^2+n+1,4n^2+4n+3,9n^2+9n+8)$	$(n^2+n+1,9n^2+9n+8,16n^2+16n+15)$	$(9n^2+9n+8,16n^2+16n+15,49n^2+49n+45)$

To conclude, one may search for Pythagorean triangles with other characterizations in relation with special patterns of numbers.

References:

[1] W. Sierpinski, Pythagorean triangles, Dover publications, INC, Newyork, 2003.
[2] M.A. Gopalan, A. Gnanam and G. Janaki, A Remarkable Pythagorean problem, Acta Ciencia Indica, Vol.XXXIII M, No.4, 2007, pp 1429-1434 .
[3] M.A. Gopalan and G. Janaki, Pythagorean triangle with Area Perimeter as a special number, Bulletin of pure and Applied sciences, Vol 27(2), 2008, pp 393-402.
[4] M.A. Gopalan and G. Janaki, Pythagorean triangle with nasty number as a leg, Journal of Applied Analysis and Applications, Vol 4, No 1-2, 2008, pp 13-17.
[5] M.A. Gopalan and A. Vijaysankar, Observations on a Pythagorean problem, Acta Ciencia Indica, Vol.XXXVI M. No.4, 2010, pp 517-520.
[6] M.A. Gopalan and A. Gnanam, Pythagorean triangles and Polygonal numbers, International Journal of Mathematical Sciences, Vol 9, No.1-2, 2010, pp 211-215.
[7] G. Janaki and R. Radha, Special Pythagorean triangle and six digit Harshad numbers, IJIRSET, Vol. 5, Issue 3, March 2016, pp 3931-3933.
[8] G. Janaki and P. Saranya, Special pairs of Pythagorean triangles and Narcissistic numbers, IJMRD, Vol. 3, Issue. 4, April 2016, pp 106-108.
[9] G. Janaki and P. Saranya, Pythagorean Triangle with Area/Perimeter as a Jarasandha numbers of orders 2 and 4, IRJET, Volume.3 , Issue.7, July 2016, pp 1259-1264.
[10] G. Janaki and R. Radha, Special pairs of Pythagorean triangle and Harshad numbers, Asian Journal of Science and Technology, Volume.7, Issue. 8, August 2016, pp 3397-3399.
[11] G. Janaki and P. Saranya, Special Pythagorean triangle in connection with triangles Narcissistic Numbers of order 3 and 4, AIJRSTEM 16-177, Volume 14, Issue.2, 2016, pp 150-153.
[12] G. Janaki and R. Radha, Pythagorean Triangle with Area/Perimeter as a Harshad number of digits 4,5 and 6, IJRASET, Volume. 5, Issue.12, December 2017, pp 1754-1762.
[13] S. Mallika, Pythagorean triangle with 2A/P+H-Leg as a Narcisstic number of orders 3,4 and 5, GJESR, Volume 6, Issue 3, March 2019, pp 1-4.
[14] S. Mallika, A connection between Pythagorean triangle and Sphenic Numbers, IJRASET, Volume 7, Issue III, March 2019, pp 63-66.
[15] A. Kavitha, A connection between Pythagorean triangle and Harshad Numbers, IJRASET, Volume 7, Issue III, March 2019, pp 91-101.
[16] S.Vidhyalakshmi, M.A. Gopalan, T. Mahalakshmi, Pythagorean triangle with $\frac{2*A}{P}$ as Gopa-Vidh Number, IJRAR, Volume 6, Issue 2, April-June 2019, pp 59-63.
[17] S.Vidhyalakshmi, M.A. Gopalan, T. Mahalakshmi, Special characterization of Pythagorean triangles in connection with Trimorphic Numbers, accepted for publication in IJSTR.