Computational Analysis of MHD Flow in Trapezoidal Cavity with Sinusoidal wavy Surface Filled with Hybrid Nanofluid

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Abstract *-Two or more nanoparticles in a base fluid has been an increasing interest in modern years due to great improvement in heat transfer performance. In the present work, an analysis is carried out to study the flow characteristics and heat transfer performance on convection in a lid-driven trapezoidal cavity with sinusoidal bottom wall composed of equal quantities of Cu and* Al_2O_3 *nanoparticles dispersed in water base fluid. Further, the effect of an applied magnetic field* B_o *on the flow pattern of the cavity is also be analysed.The sinusoidal bottom wall is heated while the top side of the cavity is cooled isothermally and the left and right walls are insulated. All the walls of the cavity are kept as no-slip walls except the upper wall which is a moving wall driven by a uniform velocity* V *along the* x*-axis. The associated governing equations have been solved using finite element method. The parametric study on the implication of Richardson number Ri and solid volume fraction of nanoparticles* φ*, Prandtl number* Pr *and Hartmann number* Ha*on the flow structure and heat transfer characteristics are performed in details while the Reynolds number and Prandtl number considered fixed. The numerical results indicated that the Richardson number have significant effects on the flow and heat transfer performance. On the other hand the results also show significant effects of increasing the volume fraction of hybrid nanofluid. Moreover, it is also noticed that combination of two different nanoparticles suspension have a better performance of heat transfer. Results are presented in terms of streamlines, isotherms and average Nusselt number of the hybrid nanofluid for different values of respective parameters.*

Keywords: Hybrid Nanofluid, Finite Element Method, MHD, CFD, Cavity, Convection.

I. INTRODUCTION

Usage of nanofluid in heat transport enhances the efficiency of thermal systems like; heat exchangers, thermal storages, solar collectors, photovoltaic system, biomedical devices, nuclear reactors, engine/vehicle, cooling of electronic components, transformers etc. Researchers [1-3] are interested in conjugate heat transfer by free convection inside cavities as it has huge applications in engineering and advanced technology. In order to remain side by side developments in the extent applications and exponentially developing studies about nanofluids, many review papers have been written."Hybrid" nanofluid can be prepared by suspending more than one type of nanoparticles in base fluid. Very recently, hybrid nanofluid, is a gradually mounting research field parallelly to the incessant developments of common nanofluids [4. Compromised properties between the advantages and disadvantages of the properties of individual nanoparticles are a task of hybridization. Moreover, nanoparticles suppliers exhibit noticeable differences in prices of different nanoparticles types. For example, the price of copper nanoparticles is about 10 times greater than that of alumina nanoparticles. Hence, it is appropriate if one achieves the properties of expensive nanoparticles with minimum quantity. Indeed, the "hybrid" nanoparticles should be limited to those prepared as a single composite substance in a base fluid for which their synthetization requires more attention [5-6].

This paper showed the comal models of hybrid nanofluid properties. The experiments of Ho *et al.* [7] conducted an experiment about the mixture of particles of micro-encapsulated phase-change material and alumina nanoparticles in base fluid water. The authors found good agreement between the experimental data of the density and mass fraction and those predicted from the mixture theory. Botha *et al.* [8] performed an experiment of hybrid nanofluid based on silver-silica-oil. The authors found more deviated value of the thermal conductivity with the Maxwell relation [9] at higher solid volume fractions. The unsteady conjugate free convection in a semi-circular enclosure with a solid shell filled with water-based hybrid mixture of Al_2O_3 and Cu nanoparticles has been studied numerically by Chamkha *et al.* [10]. The authors found increasing effects of hybrid nanofluid with rising thermal conductivity of wall and Rayleigh number.

Sheikholeslami et al. [11] have performed a numerical study of MHD effects on natural convection around a horizontal circular cylinder inside a square cavity filled with nanofluid. They found that the heat transfer rate is an increasing function of nano-particle volume fractions as well as the Rayleigh number, while it is a decreasing function of the Hartmann number Ha. In addition, their results indicated that for $Ha < 20$ the enhancement in average Nusselt number at $Ra = 10^4$ is greater than at other Rayleigh numbers. In a similar work, Sheikholeslami et al. [12] studied the effects of magnetic field on ferrofluid flow and heat transfer in a semi annulus cavity. They reported that increasing magnetic number, Rayleigh number and volume fraction of the nano-particles lead to augmentation of the heat transfer rate but the average Nusselt number decreases with increase of Ha.

MHD mixed convection in a trapezoidal sinusoidal enclosure filled with hybrid nanofluid depending on the existing mathematical formulae needs additional attention to disclose its performance in this significant field. The objective of this paper is an effort to contribute in starting the groundwork of this research area and investigate the influence of an applied magnetic field.

II. PHYSICAL MODEL

The geometry of the interested domain consists of a two dimensional lid-driven trapezoidal enclosure of side length H. The enclosure is filled with hybrid Al_2O_3-Cu/water nanofluid. The bottom wavy wall is assigned to temperature T_h while the top wall of the enclosure is cooled at a cool temperature T_c. The condition $T_h > T_c$ is maintained, under all circumstances. The bottom wall is a wavy wall. The left and the right walls are thermally insulated. Furthermore, the top wall is assumed to slide in its upper plane at a constant speed V.

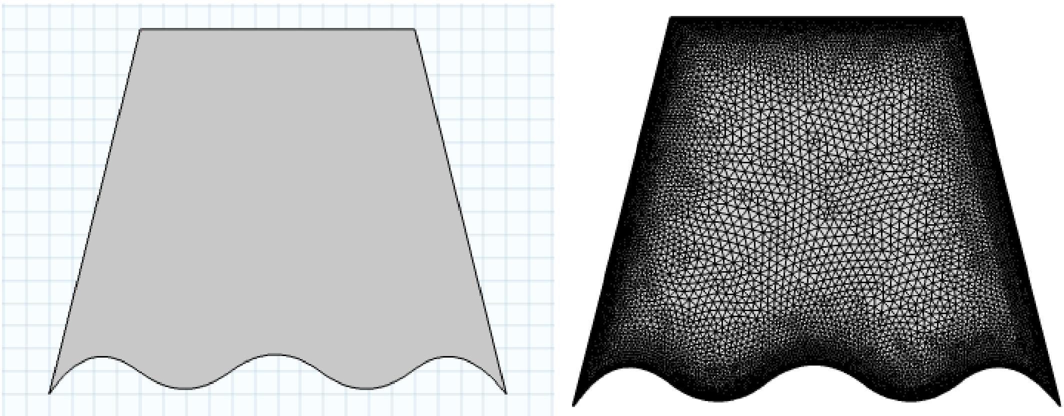


Fig. 1 Geometrical model of the interested domain and the finite element free-triangular meshing

It is also assumed that both the fluid and nanoparticles are in thermal equilibrium and there is no slip between them. The hybrid nanofluid used in the analysis considered as laminar and incompressible. It is also assumed that the gravitational acceleration g acts to the vertical downward surface. Thermophysical properties of the used nano-particles and the base fluid are given in Table 1, assumed constant except for the density variation, which is maintained on Boussinesq approximation.

Table 1

PhysicalProperties	**Fluid Phase (H_2O)**	**Nano-particle 1 (Cu)**	**Nano-particle 2 (TiO_2)**	**Nano-particle 3 (Al_2O_3)**
C_p (J/kgK)	4179	385	686.2	765
ρ (kg/m^3)	997.1	8933	4250	3970
k (W/mK)	0.6	401	8.9538	40
$\beta \times 10^{-5}$ (1/K)	21	1.67×10^{-5}	0.9×10^{-5}	0.85×10^{-5}

σ(μS/cm)	0.05	5.96×10^{-7}	6.28×10^{-5}	1×10^{-10}

III. MATHEMATICAL MODEL

The two dimensional numerical simulation has performed in steady state conditions. The non-dimensionalized governing partial differential equations for laminar flow and thermal energy equations usinghybrid nanofluidare given below:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \tag{1}$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{\rho_f}{\rho_{hnf}}\frac{\mu_{hnf}}{\mu_f}\frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{\rho_f}{\rho_{hnf}}\frac{\mu_{hnf}}{\mu_f}\frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+\frac{(\rho\beta)_{hnf}}{(\rho\beta)_f}Ri\theta-\frac{\rho_f}{\rho_{hnf}}\frac{\sigma_{hnf}}{\sigma_f}Ha^2\frac{1}{R_e}V \tag{3}$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{\alpha_{hnf}}{\alpha_f}\frac{1}{RePr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

where, $Pr=\frac{\mu_f C_p}{k_f}$, $Ri=\frac{g\beta_f\Delta TH}{U_0}$,$Ha=B_0H\sqrt{\sigma_f/\mu_f}andRe=\frac{U_0H\rho_f}{\mu_f}$be the Prandtl number, Richardson number andReynolds number respectively.

The above equations are non-dimensionalized by using the following dimensionless quantities

$$X=\frac{x}{H},\ Y=\frac{y}{H},U=\frac{uH}{\alpha_f},\ V=\frac{vH}{\alpha_f},\ \theta=\frac{T-T_c}{T_h-T_c},\ P=\frac{pH^2}{\rho_f\alpha_f^2} \tag{5}$$

The flow boundary conditions

Top wall:$U=U_0,\ V=0$

Bottom wall:$U=V=0$

Left wall: $U=V=0$

Right wall: $U=V=0$

The Thermal Boundary Conditions

Top wall:$\theta=0$

Bottom wall:$\theta=1$

Left wall:$\frac{\partial\theta}{\partial X}=0$

Rightwall:$\frac{\partial\theta}{\partial X}=0$

At the fluid solid wall interfaces: $\left(\frac{\partial\theta}{\partial X}\right)_{fluid}=K\left(\frac{\partial\theta}{\partial X}\right)_{solid}$, where $K=\frac{K_{hnf}}{K_f}$

The constructive equations of the effective properties of hybrid nanofluids have been chosen:

The thermal diffusivity $\propto_{hnf}={K_{hnf}}/{(\rho C_p)_{hnf}}$

The density$\rho_{hnf}=(1-\varphi)\rho_f+\varphi_1\rho_1+\varphi_2\rho_2$

The heat capacitance$(\rho C_p)_{hnf}=(1-\varphi)(\rho C_p)_f+\varphi_1(\rho C_p)_1+\varphi_2(\rho C_p)_2$

The thermal expansion coefficient,$(\rho\beta)_{hnf}=(1-\varphi)(\rho\beta)_f+\varphi_1(\rho\beta)_1+\varphi_2(\rho\beta)_2$

The specific heat$C_{phnf}=\frac{(1-\varphi)(\rho C_p)_f+\varphi_1(\rho C_p)_1+\varphi_2(\rho C_p)_2}{(1-\varphi)\rho_f+\varphi_1\rho_1+\varphi_2\rho_2}$

The viscosity of Brinkman model $\mu_{hnf}=\frac{\mu_f}{(1-\varphi_1-\varphi_2)^{2.5}}$

The thermal conductivity$K_{hnf}=K_f\frac{\frac{(\varphi_1k_1+\varphi_2k_2)}{\varphi}+2k_f+2(\varphi_1k_1+\varphi_2k_2)-2\varphi k_f}{\frac{(\varphi_1k_1+\varphi_2k_2)}{\varphi}+2k_f-(\varphi_1k_1+\varphi_2k_2)+\varphi k_f}$

Ratio of electric Conductivity $\frac{\sigma_{hnf}}{\sigma_f} = 1 + \frac{3\left\{\frac{(\varphi_1\sigma_1+\varphi_2\sigma_2)}{\sigma_f}-(\varphi_1+\varphi_2)\right\}}{\left\{\frac{(\varphi_1\sigma_1+\varphi_2\sigma_2)}{\varphi\sigma_f}+2\right\}-\left\{\frac{(\varphi_1\sigma_1+\varphi_2\sigma_2)}{\sigma_f}+(\varphi_1+\varphi_2)\right\}}$

where φ is the overall volume concentration of two different types of nanoparticles dispersed inhybrid nanofluid and is calculated as$\varphi = \varphi_1 + \varphi_2$

The Average NusseltNumber at the heated wall of the enclosure is expressed as, $Nu = -\frac{K_{hnf}}{K_f}\int_0^H \frac{\partial\theta}{\partial X} dY$, where H is the length of the heated surface.

IV.COMPUTATIONAL METHODOLOGY

The momentum and energy equations have been solved that is a set of partial differential equations by using the Galerkin weighted residual approach of the finite element method [14]. In this method, the interested domain is discretized into finite elementsusing non-uniform, unstructured, free triangular meshes. Then the nonlinear governing partial differential equations are transferred into a system of integral equations by applying Galerkin Weighted Residual approach. The basic unknowns for the above differential equations are the velocity components U, V, the temperature θ and the pressure P.the six-node triangular elements are used in this work. All six nodes are associated with velocities as well as temperature. Three corner nodes are correlated with pressure. The nonlinear algebraic equations so obtained are modified by imposition of appropriate boundary conditions. These modified nonlinear equations are transferred into linear algebraic equations by using Newton's method. Finally, these linear equations are solved by using tri-diagonal matrix algorithm (TDMA).The convergence criterion for the solution procedure is defined as $\left|\psi^{n+1}-\psi^{n}\right| \le 10^{-4}$, where n is the number of iteration and ψ is a function from$U$, V and θ.

V. RESULT AND DISCUSSION

The results are represented in terms of streamlines and isotherm patterns. The variations of average Nusselt numbers are also highlighted. The solid fluid thermal conductivity ratio $K = 7.0$ have considered throughout the simulation.

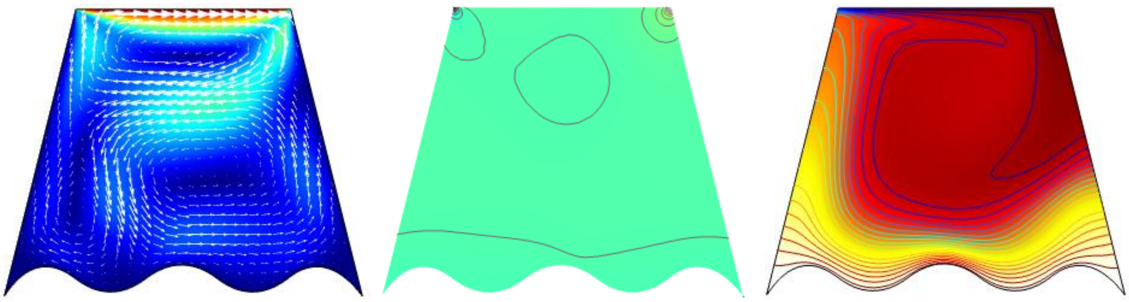
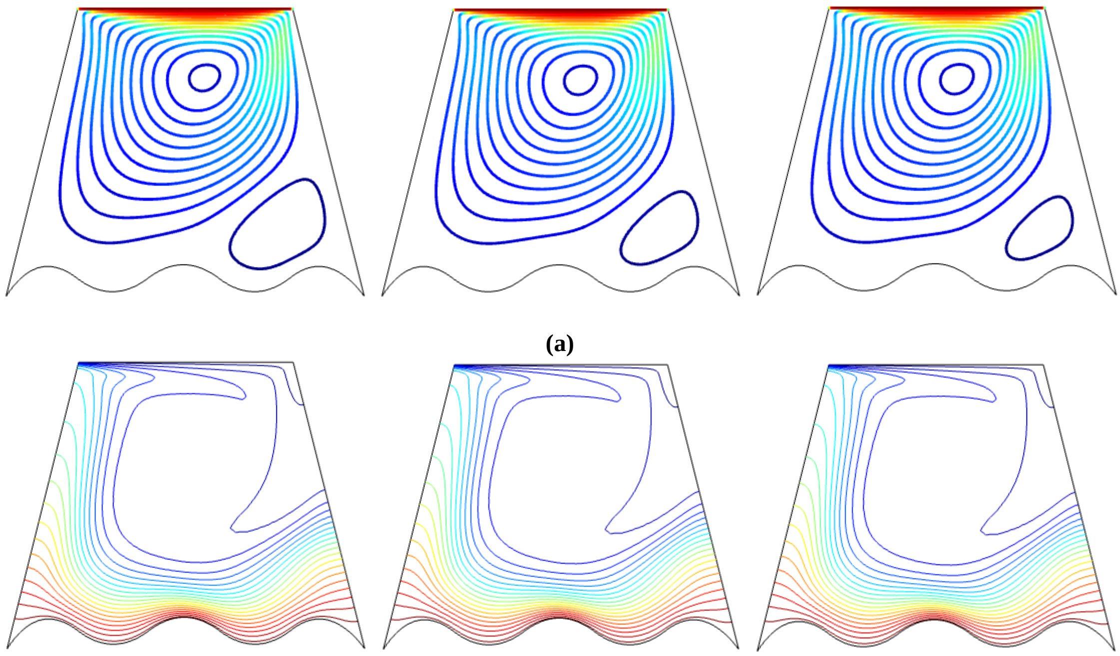


Fig. 2 (a) Velocity field with arrow surface, (b) Pressure surface with contour plots and (c) Temperature field with isothermal contours without an imposition of magnetic field

Richardson number is the ratio that measures the effect of natural to forced convection modes. Figure. 2 (a) and (b) provides the information about the sensitivity of the stream line and isotherms pattern due to the variations of Richardson number for $Re = 100$,$Pr = 6.2$, $\varphi = 0.05$ and $K = 7$.From the streamlines contours it is found that for forced convection (Ri=1) there exist one primary clockwise circulation cell with two secondary egg shape cores in the cavity. This is due to the dominance of forced convection exerted by the movement of the inclined walls of the cavity. At $Ri = 1.0$ (combined convection regime) it can be seen that shape of the primary cell become large in size.It ia also noticed that at this step the secondary two core reduces to one single core near the heated circular cylinder. On the other hand when $Ri = 5.0$ and $Ri = 10$ the clockwise vortex spread in wavy form with egg shape core near the heated moving wall. In this case the strength of the natural convection dominant the flow regime. This is reasonable because increasing the parameter Ri assists buoyancy flow, hence the natural convection mode.

The effective influence of Ri on temperature field is plotted in Figure 2 (b). From the figure it is noticed that for $Ri = 1$ the isotherms line depart from the middle section and try to crowed on the left moving lid due to the influenceof forced convection. In this case, less energy are noticed to carry away from the sliding left wall to the cavity and, subsequently the conduction heat transfer regime has become the dominant mode of energy transfer in the cavity with vertical isotherms near the heated wall. With further increase of $Ri = 1$ steeper temperature gradient near the heated wall are evident and the isotherms get the parabolic shape near the inclined cold wall. In the considered regime, the buoyancy effect balanced the effect of sliding top wall and hence, the total heat transfer in the cavity is controlled by the combined mechanisms of forced and free convection. Escalating the convective parameter *Ri* up to 10, a thin boundary layer is developed near the top corner of the inclined wall. From the figure it is also ascertain that increasing in the buoyancy force causes the isotherms to deform due to the dominating influence of the convective flow.

Ri=1.0 *Ri=5* *Ri=10*



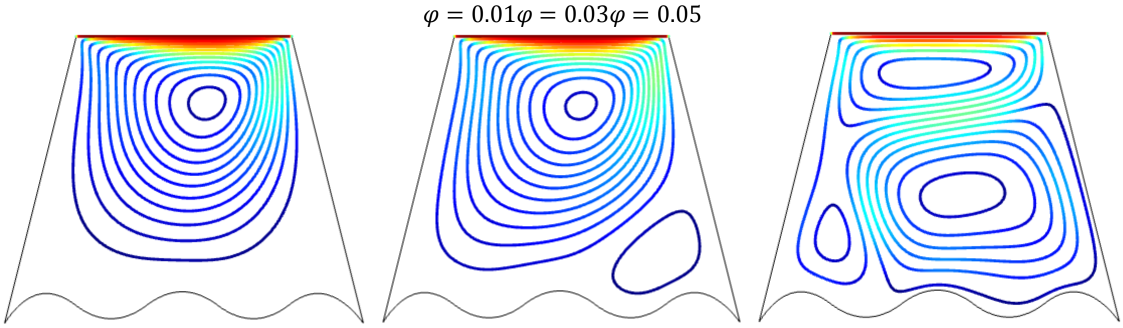
(a)

(b)

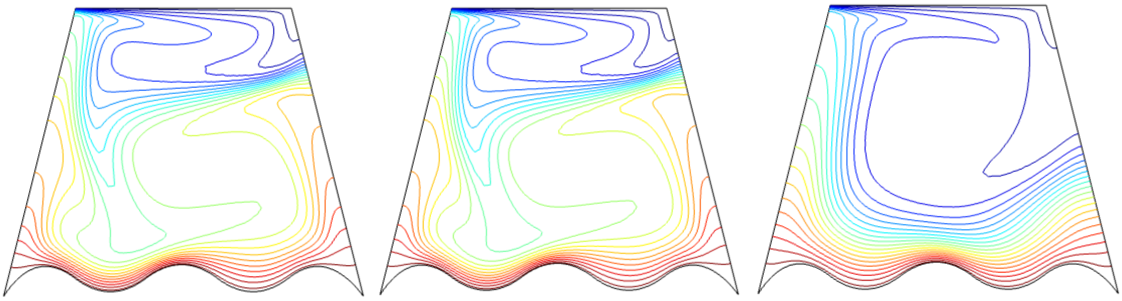
Fig. 2 (a) Stream lines and (b) Isotherms for different values of *Ri* at *Re = 100, Pr = 6.2,* $\varphi = 0.05$, $K = 7$

As mentioned before the volume fraction of hybrid nanofluid comprises of equal quantities of two different Cu and Al_2O_3 nanoparticles. From the Figure 3(a), When the nanoparticles volume fraction is increased from $\phi = 0.0$ to $\phi = 0.05$ for $Re = 100, Ri = 1, Pr = 6.2, K = 7$, the vertical double eye behavior of streamlines associated with pure water $(\phi = 0)$ is transformed into single eye behavior with the increase in the nanoparticle volume fraction $\varphi \geq 0.03$and the intensification of the fluid flow behavior decrease when the fluid particle volume fraction drop out. This is due to the enhancement of viscosity of the hybrid nanofluid. Since with the augment of ϕ, the thermal conductivity of the nanofluid increases, hence the buoyancy flow, these interns improved the heat transfer rate.

$\varphi = 0.01\varphi = 0.03\varphi = 0.05$



(a)



(b)

Fig. 3 (a) Stream lines and (b) Isotherms for different values of φat *Ri*=1.0, *Re =100, Pr=6.2, K = 7*

However, it is also obvious from figure 3(b), that the temperature gradients are generally not affected on escalating the solid volume fraction of nanoparticle with any fraction except with the increase of φ the isotherms close to the heated wall begin to be more scattered. Theenrichment of the thermal conductivity produces denser isotherms which is the indication of development nanofluid convection. We can conclude that the mixing advantage of Copper and Alumina nanoparticles may blow up the convective heat transfer. Hence, this results encourage us to examine other combination of different types of nanoparticles, with different ratio of nanoparticles, and to work with innovative models on behalf of dissimilar thermo-physical properties.

From the Figure 4(a), it is also obvious that with the incrase of the value of Ri heat transfer rate increase. Because with the augmentation of Ri buoyancy effect enhance, so large amount of heat is transferred from the heated wall to the enclosure. It is also perceived that the rate of heat transfer is much higher for hybrid nanofluid compared to the base fluid. This is a decent discovery to use hybrid nanofluid.

However, the distribution of Nusselt number for different solid volume fraction of nanoparticles presented in Figure 4(b) indicates that the intensifying of viscous and inertia forces dominant over the enhanced thermal conductivity, which effects both buoyancy and share rate in addition. Consequently, the enhancement in the Nusselt number is recorded for a range of φ. It is also perceptible that rate of increasing heat transfer is much higher while the value of solid volume fraction is small than for the superior value of φ. So, it is very reasonable to conclude that the impact of the hybrid nanofluid becomes active when the natural convection is tiny.

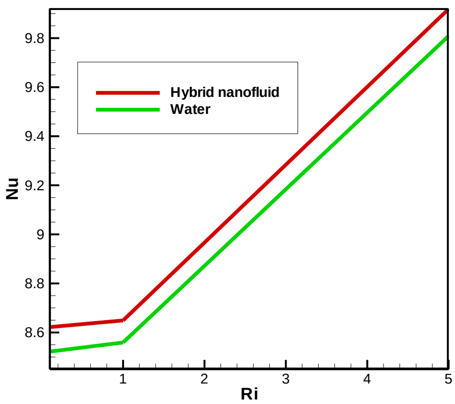


Fig. 4(a): Average Nusselt number at various Richardson numbers

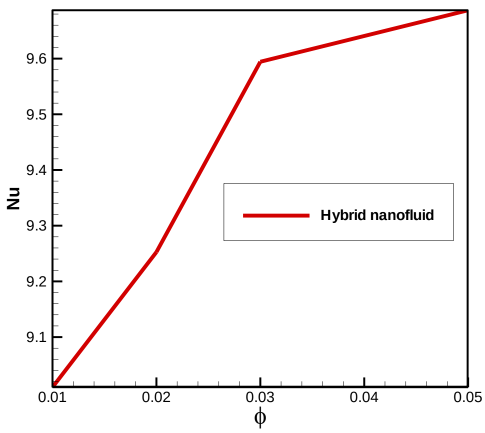


Fig. 4(b): Average Nusselt number at various volume concentrations of nanoparticles

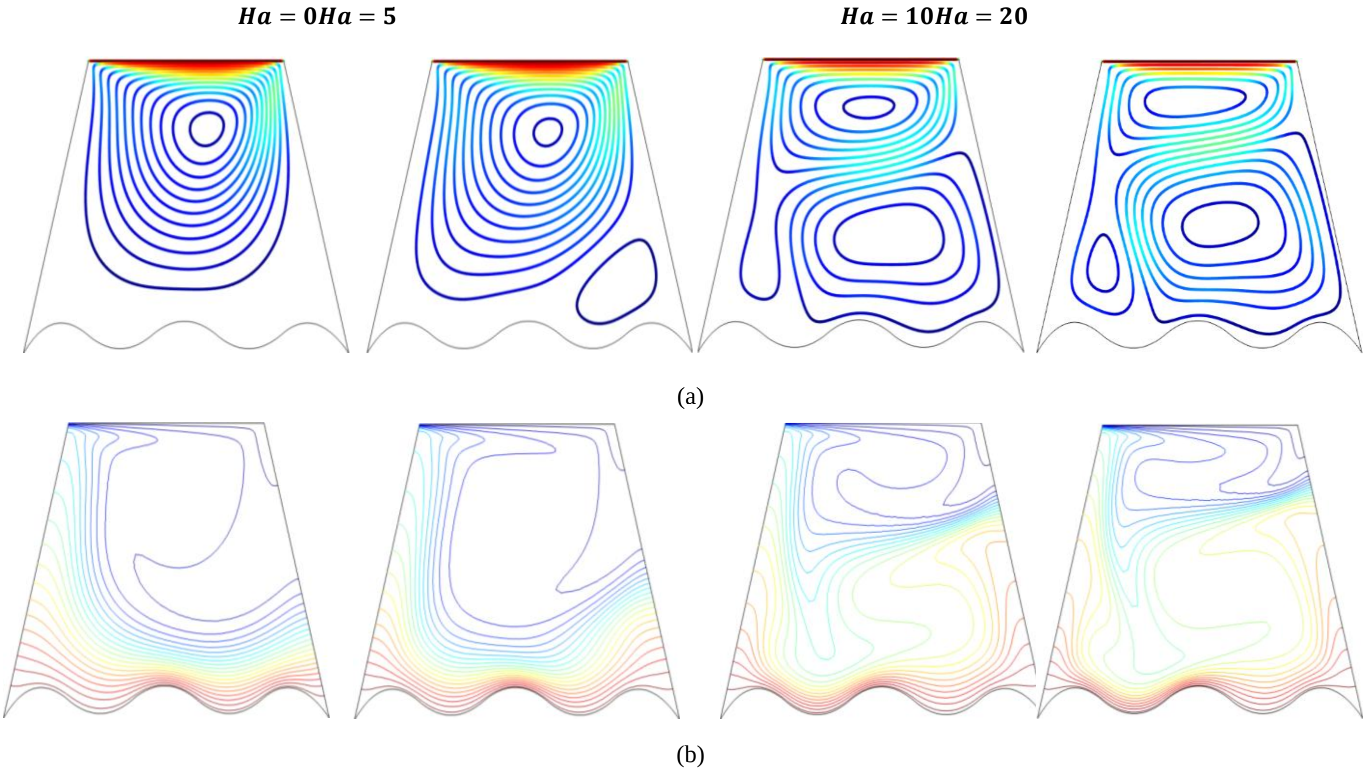


Fig. 3 (a) Stream lines and (b) Isotherms for various values of magnetic parameter Ha, the Hartmann number at *Ri*=1.0, *Re =100, Pr=6.2, K = 7*

VI. CONCLUSION

MHD effect on hybrid nanofluid flow in convective lid-driven sinusoidal trapezoidal enclosure is investigated numerically. Varying the parameters $Ri$and φ have leads to the conclusion that the corrugated lid-driven cavity could be considered as an significantly effective heat transfer apparatus for the bottom wavy surface amplitude.The mixed convection parameter Riaffects extensively on the flow patterns and heat transfer mechanism inside the hybrid nanofluid filled cavity.The increment of Richardson number promoted heat transfer by convection.Compared with regular nanofluid, a hybrid nanofluid dispersed in water base fluid has very modest enhancement on the Average Nusselt Number values.The effect of volume fraction of the hybrid nanofluid becomes significant in the situation of the case of small buoyancy force. The induction of the applied magnetic field reduces the flow rate and decreases further when the intensity of magnetic field increases. Also the flow rate of the nanofluid reduces for the increasing values of Hartmann number Ha.

VII. ACKNOWLEDGMENTS

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