# Effect of Rotation and Suspended Particles on Micropolar Fluid Heated and Soluted from Below Saturating Porous Medium

Pushap Lata Sharma<sup>1</sup>, Sumit Gupta<sup>2</sup>

<sup>1,2</sup>Department of Mathematics, Rajiv Gandhi Govt. Degree College Kotshera, Shimla -17 1004, India

Abstract - This paper deals with the convection of micropolar fluids heated and soluted from below in the presence of suspended particles (fine dust) and uniform vertical rotation  $\overline{\Omega}(0,0,\Omega)$  in a porous medium and using the Boussinesq approximation, the linearized stability theory and normal mode analysis, the exact solutions are obtained for the case of two free boundaries. It is found that the presence of the suspended particles number density, the rotation parameter, stable solute parameter and medium permeability bring oscillatory modes which were non-existent in their absence. It is found that the presence of coupling between thermal and micropolar effects, rotation parameter, solute parameter and suspended particles may introduce overstability in the system. Graphs have been plotted by giving numerical values to the parameters accounting for rotation  $\overline{\Omega}(0,0,\Omega)$ , solute parameter, the dynamic microrotation viscosity  $\kappa$  and coefficient of angular viscosity  $\gamma'$  to depict the stability characteristics, for both the cases of stationary convection and overstability. It is found that Rayleigh number for the case of overstability and stationary convection increases with increase in rotation parameter, solute parameter, solute parameter and medium permeability, for a fixed wave number, implying thereby the stabilizing effect of rotation parameter, solute parameter and destabilizing effect of micropolar coefficients and medium permeability.

*Keywords* - *Micropolar fluid, rotation, suspended particles (fine dust), solute parameter, medium permeability, micro rotation, coefficient of angular viscosity.* 

# I. INTRODUCTION

Micropolar theory was introduced by Eringen<sup>1</sup> in order to describe some physical systems which do not sastisfy the Navier Stokes equations. These fluids are able to describe the behaviour of colloidal solutions, liquid crystals, animal blood etc. The equations governing the flow of micropolar fluid theory involve a spin vector and a microinertia tensor in addition to velocity vector. A generalization of the theory including thermal effects has been developed by Kazakia and Ariman<sup>2</sup> and Eringen<sup>3</sup>. Micropolar fluid stabilities have become an important field of research these days. A particular stability problem is the Rayleigh-Bénard instability in a horizontal thin layer of fluid heated from below.

A detailed account of thermal convection in a horizontal thin layer of Newtonian fluid heated from below has been given by Chandrasekhar<sup>4</sup>. Ahmadi <sup>5</sup> and Pérez-Garcia *et al*<sup>6</sup> have studied the effects of the microstructures on the thermal convection and have found that in the absence of coupling between thermal and micropolar effects, the principle of exchange of stabilities may not be fulfilled and consequently micropolar fluids introduce oscillatory motions. The existence of oscillatory motions in micropolar fluids has been depicted by Lekkerkerker in liquid crystals<sup>7, 8</sup>, Bradley in dielectric fluids<sup>9</sup> and Laidlaw in binary mixture<sup>10</sup>. In the study of problems of thermal convection, it is frequent practice to simplify the basic equations by introducing an approximation which is attributed to Boussinesq<sup>11</sup>. In geophysical situations, the fluid is often not pure but contains suspended particles. Saffman<sup>12</sup> has considered the stability of laminar flow of a dusty gas. Scanlon and Segel<sup>13</sup> have considered the effects of suspended particles, rotation and solute gradient on thermal instability of fluids saturating a porous medium have been discussed by Sharma and Sharma<sup>14</sup>. The suspended particles were thus found to destabilize the layer. Palaniswami and Purushotham<sup>15</sup> have studied the stability of shear flow of stratified fluids with the fine dust and found that the presence of dust particles

increases the region of instability. On the other hand, multiphase fluid systems are concerned with the motion of liquid or gas containing immiscible inert identical particles. The theoretical and experimental results of the onset of thermal instability (Bénard convection) in a fluid layer under varying assumptions of hydromagnetics, has been depicted in a treatise by Chandrasekhar<sup>4</sup>. Lapwood <sup>16</sup> has studied the convective flow in porous medium using linearized stability theory. The Rayleigh instability in flow through a porous medium has been considered by Wooding <sup>17</sup>. The problem of thermal convection in a fluid in porous medium is of importance in geophysics, soil–science, ground–water, hydrology and astrophysics. The physical property of comets, meteororites and inter–planetary dust strongly suggests the importance of porosity in the astrophysical context McDonnel <sup>18</sup>.

Moreover, Saffman and Taylor <sup>19</sup> have shown that the motion in a Hele–Shaw cell is mathematically analogous to two dimensional flow in porous medium. In recent years, there has been a considerable interest in the study of breakdown of the stability of a layer of a fluid subjected to a vertical temperature gradient in a porous medium and also in the possibility of convective flow. When a fluid permeates a porous material, the gross effect is represented by Darcy's law. As a result of this macroscopic law, the usual viscous term in the equations of motion of microscopic fluid is replaced by the resistance term  $\left[-\frac{1}{k_1}(\mu+\kappa) q_1\right]$ , where  $\mu$  and  $\kappa$  are viscosity and dynamic

microrotation viscosity respectively,  $k_1$  is the medium permeability and  $q_1$  is the Darcian (filter) velocity of the fluid. The heat and solute being two diffusing components, thermosolutal convection is the general term dealing with such phenomena. The buoyancy forces can arise not only from density differences due to variations in temperature, but also from those due to variations in solute concentration. Brakke <sup>20</sup> explained a double–diffusive instability that occurs when solution of a slowly diffusing protein is layered over a dense solution of more diffusing sucrose. Convection that is dominated by the presence of two components is very common in geophysical systems.

The problem of thermosolutal convection (double–diffusive convection) in a layer of fluid heated from below and subjected to a stable solute gradient has been studied by Veronis <sup>21</sup>. Thermosolutal convection problems arise in oceanography, limnology and engineering. The case of fluids with uniform salinity gradients when the fluxes are driven by other mechanisms has been looked at by McDougall <sup>22</sup> who assumed that the fluxes were proportional to the salinity difference between the convective layers and independent of the layer thickness and by Holyer <sup>23</sup>, who assumed that the fluxes were driven by molecular diffusivities. In all these cases where an unbounded fluid has uniform horizontal and vertical compositional gradients, the fluid is always unstable and so considerations of marginal stability are inappropriate. From the physical point of view the effect of rotation on the micropolar fluids in the presence of suspended particles is interesting because there is a competition between the large enough stabilizing effect (to a smaller extent) of suspended particles. Moreover, rotation introduces Coriolis acceleration which plays an important role on the stability on the system and a centrifugal force which is neglected due to its small magnitude. The rotating fluid also finds its application in meteorphysics and oceanography.

A broad view of the subject of double–diffusive convection is given by Brandt and Fernando<sup>24</sup>. Sharma and Gupta<sup>25</sup> have studied the thermal convection in micropolar fluids in porous medium and have found that medium permeability has stabilizing effect on stationary convection and destabilizing effect on the overstable case. Sharma and Gupta<sup>26</sup> have studied the effect of rotation on thermal convection in micropolar fluids in the presence of suspended particles.

Sharma and Gupta <sup>27</sup> have studied the thermosolutal convection of micropolar fluids in the presence of suspended particles. Keeping in mind the importance and relevance of porosity, solute parameter and rotation in chemical engineering, geophysics and biomechanics, thermal instability of micropolar fluids in the presence of rotation to include the effect of solute parameter, suspended particles (dust particles) in porous medium has been considered in the present paper.

## II. MATHEMATICAL FORMULATION AND ANALYSIS

Consider an infinite, horizontal layer of an micropolar fluid of thickness d permeated with suspended particles (or fine dust) in an isotropic and homogeneous medium of porosity  $\mathcal{E}$  and medium permeability  $k_1$ . This fluid-particles layer is heated and soluted from below but convection sets in when the temperature gradient  $\left(\beta = \left| \frac{dT}{dz} \right| \right)$  between

the lower and upper boundaries exceeds a certain critical value. A uniform vertical rotation  $\vec{\Omega}(0,0,\Omega)$  pervades the system .This is the Rayleigh-Bénard instability problem in presence of salinity gradient and fine dust in micropolar

fluids. Both the boundaries are taken to be free and perfect conductor of heat. The mass, momentum, internal angular momentum, internal energy balance equations and analogous solute equation using the Boussinesq approximation are

$$\nabla \cdot q_1 = 0 \tag{1}$$

$$\frac{1}{\varepsilon} \left( \frac{\partial}{\partial t} + \frac{q_1}{\varepsilon} \cdot \nabla \right) q_1 = -\frac{1}{\rho_0} \nabla p - \frac{1}{\rho_0 k_1} \left( \mu + \kappa \right) q_1 + \frac{\kappa}{\rho_0} \nabla \times \mathcal{G} - \left( 1 + \frac{\delta \rho}{\rho_0} \right) g \hat{e}_z + \frac{1}{\rho_0} \frac{KN}{\varepsilon} \left( u - q_1 \right) + 2\rho_0 \left( \vec{v} \times \vec{\Omega} \right)$$
(2)

$$\rho_0 j_1 \left( \frac{\partial}{\partial t} + \frac{q_1}{\varepsilon} \cdot \nabla \right) \mathcal{G} = \left( \varepsilon' + \beta' \right) \nabla \left( \nabla \cdot \mathcal{G} \right) + \gamma' \nabla^2 \mathcal{G} + \frac{\kappa}{\varepsilon} \nabla \times q_1 - 2 \kappa \mathcal{G}$$
(3)

$$\left[\rho_{0}c_{v}\varepsilon + \rho_{s}c_{s}\left(1-\varepsilon\right)\right]\frac{\partial T}{\partial t} + \rho_{0}c_{v}q_{1}\cdot\nabla T = k_{T}\nabla^{2}T + \delta\left(\nabla\times\vartheta\right)\cdot\nabla T$$

$$\tag{4}$$

$$\left[\rho_{0}c_{v}\varepsilon + \rho_{s}c_{s}\left(1-\varepsilon\right)\right]\frac{\partial C}{\partial t} + \rho_{0}c_{v}q_{1}\cdot\nabla C = k_{T}^{\prime}\nabla^{2}C + \delta^{\prime}\left(\nabla\times\vartheta\right)\cdot\nabla C$$

$$\tag{5}$$

Where  $q_1$ ,  $\mathcal{G}$ , p,  $\rho$ ,  $\mathcal{G}$ ,  $\rho_0$ ,  $\mu_e$  and u, denote the filter (seepage) velocity, the spin, the pressure, the fluid density, the acceleration due to gravity, the reference density, magnetic permeability and velocity of the suspended particles, respectively. N(x,t) denotes the number density of dust particles and  $\kappa$  is the dynamic microrotation viscosity, x = (x, y, z).  $K = 6 \pi \mu r$ , r being the particle radius, is the Stokes drag coefficient and  $k_T$ ,  $k'_T$ ,  $c_v$ ,  $c_s c_{pt}$ ,  $\delta$ ,  $\delta'$ ,  $\dot{J}_1$  denote, respectively, the thermal conductivity, the solute conductivity, the specific heat at constant volume, the heat capacity of solid matrix, the heat capacity of particles, the coefficient giving account of coupling between spin flux with heat flux, spin flux with solute flux and microinertial constant.  $\varepsilon', \beta', \gamma'$  are the coefficients of angular viscosity.

Assuming dust particles of uniform size, spherical shape and small relative velocities between the two phases (fluid and particles), the net effect of the particles on the fluid is equivalent to an extra body force term per unit volume  $KN(u-q_1)$ , as has been taken in equation [2]. We also use the Boussinesq approximation by allowing the density to change only in the gravitational body force term. The density equation of the state is

$$\rho = \rho_0 [1 - \alpha (T - T_0) + \alpha' (C - C_0)],$$

Where  $\rho_0$ ,  $T_0$  are reference density, reference temperature at the lower boundary and  $\alpha$ ,  $\alpha'$  is the coefficient of thermal expansion and analogous solvent coefficient, respectively. Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid. The distance between the particles is assumed to be so large compared with their diameter that interparticle reactions are ignored. The buoyancy force on the particles is also neglected. If mN is the mass of suspended particles per unit volume, then the equations of motion and continuity for the particles, under the above assumptions, are

$$mN\left(\frac{\partial}{\partial t} + \frac{u}{\varepsilon} \cdot \nabla\right) u = KN\left(q_1 - u\right) \tag{6}$$

$$\varepsilon \frac{\partial N}{\partial t} + \nabla \cdot (N u) = 0 \tag{7}$$

In the quiescent state, the solution of equations [1] - [7] is

$$q_{1} = 0, \ u = 0, \ \mathcal{G} = 0, \ N = N_{0} \text{ (constant)}, T = T_{0} - \beta z, \ C = \beta' z - C_{0}, \ p = p_{0} - g\rho_{0} \left( z + \frac{\alpha \beta z^{2}}{2} - \frac{\alpha' \beta' z^{2}}{2} \right)$$
(8)

Where  $p_0, C_0, T_0, \rho_0, N_0$  are their respective reference values at z=0 and  $\beta = \frac{T_0 - T_1}{d} (T_0 > T_1)$  is the magnitude of uniform temperature gradient. Assume small perturbations around the basic state, and let  $q_1 = (u, v, w), \ u = (\ell, r, s), \ \omega, \ p', \ \rho', \ \theta$  denote, respectively, the perturbations on fluid velocity  $q_1$ , particles velocity u, spin  $\mathcal{G}$ , pressure p, density  $\rho$ , temperature T so that the change in density  $\rho'$  caused mainly by the perturbations  $\theta$  and  $\gamma$  in temperature and solute concentration, is given by

$$\rho' = -\rho_0 \left( \alpha \,\theta - \alpha' \gamma \right) \tag{9}$$

Then the linearized perturbation equations of the microplar fluid become

$$\nabla \cdot q_1 = 0 \tag{10}$$

$$\frac{\rho_0}{\varepsilon} \left( \frac{\partial}{\partial t} + \frac{q_1}{\varepsilon} \cdot \nabla \right) q_1 = -\nabla p' - \frac{1}{k_1} \left( \mu + \kappa \right) q_1 + \kappa \left( \nabla \times \omega \right) + g \rho_0 \alpha \, \theta \, \hat{e}_z + \frac{K N_0}{\varepsilon} \left( u - q_1 \right) + 2 \left( \vec{v} \times \vec{\Omega} \right) \tag{11}$$

$$\rho_0 j_1 \left( \frac{\partial}{\partial t} + \frac{q_1}{\varepsilon} \cdot \nabla \right) \omega = \left( \varepsilon' + \beta' \right) \nabla \left( \nabla \cdot \omega \right) + \gamma' \nabla^2 \omega + \frac{\kappa}{\varepsilon} \nabla \times q_1 - 2 \kappa \omega$$
<sup>(12)</sup>

$$H_{1}\left[\rho_{0} c_{v} \varepsilon + \rho_{s} c_{s} (1 - \varepsilon)\right] \left(\frac{\partial}{\partial t} + \frac{q_{1}}{\varepsilon} \cdot \nabla\right) \theta = \beta (w + h_{1}s) + k_{T} \nabla^{2} \theta + \delta \left[\nabla \theta \cdot (\nabla \times \omega) - (\nabla \times \omega)_{z} \cdot \beta\right]$$
(13)

$$H_{1}\left[\rho_{0} c_{v}\varepsilon + \rho_{s}c_{s}\left(1-\varepsilon\right)\right]\left(\frac{\partial}{\partial t} + \frac{q_{1}}{\varepsilon}\cdot\nabla\right)\gamma = \beta'\left(w+h_{1}s\right) + k_{T}'\nabla^{2}\theta + \delta'\left[\nabla\gamma\cdot\left(\nabla\times\omega\right) - \left(\nabla\times\omega\right)_{z}\cdot\beta'\right]$$
(14)

$$mN_0 \left(\frac{\partial}{\partial t} + \frac{u}{\varepsilon} \cdot \nabla\right) u = KN_0 \left(q_1 - u\right) \tag{15}$$

$$\varepsilon \frac{\partial M}{\partial t} + \nabla \cdot u = 0 \tag{16}$$

Where  $H_1 = 1 + h_1, h_1 = \frac{fc_{pt}}{c_v}, \quad f = \frac{mN_0}{\rho_0} \text{ and } M = \frac{N}{N_0}.$ 

Using the non-dimensional numbers

$$z = z^* d, \quad \theta = \beta \, d\theta^*, \quad \gamma = \beta' d\gamma^*, \quad t = \frac{\rho_0 d^2}{\mu} t^*, \quad q_1 = \frac{\kappa_T}{d} q_1^*, \quad \nabla = \frac{\nabla}{d}, \quad u = \frac{\kappa_T}{d} u^*, \quad p = \frac{\mu \kappa_T}{d^2} p^*, \quad \omega = \frac{\kappa_T}{d^2} \omega$$
(17)

Equations [11] - [16] in the non-dimensional form are

$$\nabla \cdot q_1 = 0 \tag{18}$$

$$\frac{1}{\varepsilon} \left( \frac{\partial}{\partial t} + \frac{q_1}{\varepsilon} \cdot \nabla \right) q_1 = -\nabla p' - \frac{1}{\bar{k}_1} (1 + K_1) \nabla^2 q_1 + K_1 \nabla \times \omega + \hat{e}_z \left( R\theta - \frac{p_1}{q} S\gamma \right) + \frac{N_2}{\varepsilon} (u - q_1) + 2 \left( \vec{v} \times \vec{\Omega} \right)$$
(19)

$$\bar{j}_{2}\left(\frac{\partial}{\partial t} + \frac{q_{1}}{\varepsilon} \cdot \nabla\right)\omega = C_{1}' \nabla \left(\nabla \cdot \omega\right) - C_{0}' \nabla \times \left(\nabla \times \omega\right) + K_{1}\left(\frac{1}{\varepsilon} \nabla \times q_{1} - 2\omega\right)$$
(20)

$$EH_{1}p_{1}\left(\frac{\partial}{\partial t} + \frac{q_{1}}{\varepsilon} \cdot \nabla\right)\theta = \beta\left(w + h_{1}s\right) + \kappa_{T}\nabla^{2}\theta + \overline{\delta}\left[\nabla\theta \cdot \left(\nabla \times \omega\right) - \left(\nabla \times \omega\right)_{z}\right]$$
(21)

$$EH_{1}q\left(\frac{\partial}{\partial t} + \frac{q_{1}}{\varepsilon} \cdot \nabla\right)\gamma = \beta'\left(w + h_{1}s\right) + \kappa'_{T}\nabla^{2}\theta + \overline{\delta}'\left[\nabla\gamma \cdot \left(\nabla \times \omega\right) - \left(\nabla \times \omega\right)_{z}\right]$$
(22)

$$\left[a\left(\frac{\partial}{\partial t} + q_1 \cdot \nabla\right) + 1\right]u = q_1 \tag{23}$$

where the following non-dimensional parameters are introduced

$$K_{1} = \frac{\kappa}{\mu}, \quad \bar{j}_{2} = \frac{j}{d^{2}}, \quad \bar{\delta} = \frac{\delta}{\rho_{0}c_{v}d^{2}}, \quad \bar{\delta}' = \frac{\delta'}{\rho_{0}c_{v}d^{2}}, \quad C_{0}' = \frac{\gamma'}{\mu d^{2}}, \quad C_{1}' = \frac{\varepsilon' + \beta' + \gamma'}{\mu d^{2}}, \quad E = \varepsilon + (1 - \varepsilon)\frac{\rho_{s}c_{s}}{\rho_{0}c_{v}}, \quad \bar{\kappa}_{1} = \frac{k_{1}}{\mu d^{2}}, \quad E = \varepsilon + (1 - \varepsilon)\frac{\rho_{s}c_{s}}{\rho_{0}c_{v}}, \quad \bar{\kappa}_{1} = \frac{k_{1}}{\mu d^{2}}, \quad \bar{\kappa}_{2} = \kappa N_{0}\frac{d^{2}}{\mu}, \quad a = \frac{m}{Kd^{2}}, \quad \bar{\kappa}_{1} = \frac{g\alpha\beta\rho_{0}d^{4}}{\mu\kappa_{T}}, \quad S = \frac{g\alpha'\beta'\rho_{0}d^{4}}{\mu\kappa_{T}'}, \quad p_{1} = \frac{\upsilon}{\kappa_{T}}, \quad \kappa_{T} = \frac{k_{T}}{\rho_{0}c_{v}}, \quad q = \frac{\mu}{\rho_{0}\kappa_{T}'} \quad (24)$$

where R is known as dimensionless Rayleigh number, S is analogous solute number,  $p_1$  is thermal Prandtl number and q is analogous Schmidt number. Eliminating s between equations [21] and [22] with the help of [23] and applying the curl operator twice to resulting equation, we obtain

$$L_{2}\left[EH_{1}p_{1}\frac{\partial}{\partial t}-\nabla^{2}\right]\theta = \left(a\frac{\partial}{\partial t}+H_{1}\right)\beta w - L_{2}\overline{\delta}\Omega_{z}.$$
(25)

$$L_{2}\left[EH_{1}q\frac{\partial}{\partial t}-\nabla^{2}\right]\gamma = \left(a\frac{\partial}{\partial t}+H_{1}\right)\beta' w - L_{2}\overline{\delta}'\Omega_{z}.$$
(26)

Eliminating u between equations [19] and [23] and on linearizing, we obtain

$$\varepsilon^{-1}L_{1} q_{1} = L_{2} \left[ -\nabla p' - \frac{1}{\bar{k}_{1}} (1 + K_{1}) q_{1} + K_{1} \nabla \times \omega + \left( R\theta - \frac{p_{1}}{q} S\gamma \right) \hat{e}_{z} + 2(\bar{v} \times \vec{\Omega}) \right]$$
(27)  
Where  $L_{1} = a \frac{\partial^{2}}{\partial t^{2}} + F \frac{\partial}{\partial t}, \quad L_{2} = a \frac{\partial}{\partial t} + 1 \text{ and } F = f + 1.$ 

Applying the curl operator to equations [19], [20] and taking z –component, we get

$$\varepsilon^{-1}L_2 \frac{\partial}{\partial t} \zeta_z + \varepsilon^{-1} N_2 \zeta_z \left( L_2 - 1 \right) = -\frac{1}{\bar{k}_1} \left( 1 + K_1 \right) \nabla^2 \zeta_z L_2 - 2\Omega \frac{\partial \zeta_z}{\partial z}$$
<sup>(28)</sup>

$$\bar{j}_2 \frac{\partial \Omega_z}{\partial t} = C_0' \nabla^2 \Omega_z - K_1 \left( \frac{1}{\varepsilon} \nabla^2 w + 2\Omega_z \right)$$
<sup>(29)</sup>

where,  $\zeta_z = (\nabla \times q_1)_z$  are the *z*-components of vorticity, respectively.  $K_1$  and  $C'_0$  account for coupling between vorticity and spin effects and spin diffusion, respectively.

Applying the curl operator twice to equations [19] and taking z –component, we get

$$\varepsilon^{-1}L_{1}\nabla^{2}w = L_{2}\left[R\nabla_{1}^{2}\theta - \frac{p_{1}}{q}S\nabla_{1}^{2}\gamma - \frac{1}{\bar{k}_{1}}\left(1 + K_{1}\right)\nabla^{2}w + K_{1}\nabla^{2}\Omega_{z} - 2\Omega\frac{\partial\zeta_{z}}{\partial z}\right]$$
(30)

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}, \ \Omega_z = (\nabla \times \omega)_z.$$
(31)

The boundaries are considered to be free. The case of two free boundaries is little artificial except in astrophysical situations but it enables us to find analytical solutions. Thus the boundary conditions appropriate to problem are

$$w = \frac{\partial^2 w}{\partial z^2} = \frac{\partial}{\partial z} (\nabla \times q_1)_z = 0, = (\nabla \times \omega)_z = 0, \ \theta = \gamma = 0 \text{ at } z = 0 \text{ and } z = d.$$
(32)

Now we analyze the perturbations into a complete set of normal modes and then examine the stability of each of these modes individually. We ascribe to all quantities describing the perturbation dependence on x, y and t of the form  $\exp[i(k_x x + k_y y) + nt]$ , where  $k_x, k_y$  are the wave numbers along the x- and y- directions, respectively,  $k = (k_x^2 + k_x^2)^{\frac{y}{2}}$  is the resultant wave number, n is the stability parameter which can be, complex, in general. The solution of the stability problem requires the specifications of the state for each k. The above considerations allow us to suppose that the perturbation quantities have the form

$$[w, \Omega_z, \zeta_z, \xi_z, \theta, h, \gamma] = [W(z), \Omega_2(z), Z(z), G(z), \Theta(z), \Gamma(z)] \exp\left(ik_x x + ik_y y + nt\right)$$
(33)

Then the equations [28] - [30], using equation [33] become

$$(an+1)\left\{EH_1p_1n - \left(D^2 - k^2\right)\right\}\Theta = (an+H_1)W - (an+1)\overline{\delta}\Omega_2$$
(34)

$$(an+1)\left\{EH_1qn - \left(D^2 - k^2\right)\right\}\Gamma = (an+H_1)W - (an+1)\overline{\delta}'\Omega_2$$
(35)

$$\left(D^{2}-k^{2}\right)\left\{\left(an^{2}+Fn\right)+\frac{1}{\bar{k}_{1}}\left(an+1\right)\left(1+K_{1}\right)\left(D^{2}-k^{2}\right)\right\}W=\left(an+1\right)\left\{-Rk^{2}\Theta+\frac{p_{1}}{q}Sk^{2}\Gamma+K_{1}\left(D^{2}-k^{2}\right)\Omega-2\Omega DZ\right\}$$
(36)

$$\left\{\varepsilon^{-1}\left(an^{2}+Fn\right)+\left(an+1\right)\frac{1}{\bar{k}_{1}}\left(1+K_{1}\right)\right\}Z=2\Omega DW$$
(37)

$$\left\{\ell_1 n + 2A - \left(D^2 - k^2\right)\right\} \Omega_2 = -A\varepsilon^{-1} \left(D^2 - k^2\right) W$$
(38)

$$\left\{n - \frac{1}{p_2} \left(D^2 - k^2\right)\right\} G = \varepsilon^{-1} H DZ$$
(39)

Where 
$$A = \frac{K_1}{C'_0}$$
,  $\ell_1 = \overline{j}_2 \frac{A}{K_1}$ ,  $D = \frac{d}{dz}$ ,  $\frac{\partial}{\partial t} = n$ ,  $L_2 = a \frac{\partial}{\partial t} + 1 = an + 1$ ,  $L_1 = a \frac{\partial^2}{\partial t^2} + F \frac{\partial}{\partial t} = an^2 + Fn$ .

The boundary conditions [33] transform to

$$W = 0, D^2 W = 0, DZ = 0, G = 0, \Omega_2 = 0, \Theta = 0, \Gamma = 0 \text{ at } z = 0 \text{ and } = 1.$$
 (40)

Using boundary conditions [40], equations [34]-[39] transform to

$$D^{2}\Theta = 0, \ D^{2}\Omega_{2} = 0, \ D^{3}Z = 0, \ D^{3}G = 0, \ D^{3}\Gamma = 0.$$
(41)

Differentiating [36] twice with respect to Z and using boundary conditions [41], it can be shown that  $D^4W = 0$ . It can be shown from equations [34]–[39] and boundary conditions [40], [41] that all even order derivatives of W vanish on the boundaries. The proper solution of W belonging to the lowest mode is

$$W = W_0 \sin \pi z \tag{42}$$

Where  $W_0$  is a constant.

Eliminating  $\Theta$ ,  $\Gamma$ ,  $\Omega_2$  from equations [34]–[39] and substituting the solution given by equation [42], we obtain the dispersion relation  $Rk^2 \left\{ n + \frac{b}{p_2} \right\} \left\{ (H_1qn + b)(an + H_1)(\ell_1n + 2A + b) - (an + 1)\varepsilon^{-1}\overline{\delta}Ab \right\} - Rk^2(an + 1)Ab\overline{\delta} = b \left\{ \varepsilon^{-1}(an^2 + Fn) + \frac{1}{\overline{k_1}}(an + 1)(1 + K_1) \right\} (EH_1p_1n + b)(\ell_1n + 2A + b) \left\{ n + \frac{b}{p_2} \right\} - \varepsilon^{-1}K_1Ab^2 \frac{p_1}{q}Sk^2(an + 1) (EH_1p_1n + b) - 4\Omega^2\pi^2(EH_1p_1nb + b^2)(H_1qn + b)K_1Ab^2\overline{\delta}(an + 1)\varepsilon^{-1}(\ell_1n + 2A + b) \left\{ n + \frac{b}{p_2} \right\} - \frac{p_1}{q}Sk^2(an + 1)Ab\overline{\delta}'$  (43) Where  $\boldsymbol{b} = \pi^2 + \boldsymbol{k}^2$ .

$$\begin{aligned} \text{The case of oscillatory modes-} & \text{Here we examine the possibility of oscillatory modes, if any, in the stability problem due to the presence of salinity gradient, rotation and suspended particles number density. Equating the imaginary parts of equation [43], we have  $n_i \bigg[ n_i^4 EabH_1 p_1 q \ell_1 \varepsilon^{-1} + n_i^2 \bigg( -2Aab^2 \varepsilon^{-1} \frac{p_1}{q} Sk^2 - ab^3 \varepsilon^{-1} - 2EH_1 p_1 A \frac{ab^2}{p_2} \varepsilon^{-1} - 2EH_1 p_1 A \frac{ab^3}{p_2} \varepsilon^{-1} - EFH_1 p_1 b^2 \varepsilon^{-1} - Fb^2 \ell_1 \varepsilon^{-1} - EH_1 p_1 \ell_1 \varepsilon^{-1} \frac{b^2}{p_2} + \frac{1}{\bar{k}_1} \bigg\{ -2qAEH_1 - \frac{p_1}{q} Sk^2 p_1 ab - EH_1 p_1 ab^2 - EH_1 p_1 \ell_1 q \bar{\delta}' \frac{ab^2}{p_2} - 2qAEH_1 p_1 abK_1 - \frac{p_1}{q} Sk^2 EH_1 p_1 K_1 b^2 - ab^2 \ell_1 K_1 q \bar{\delta}' - EH_1 p_1 \ell_1 bK_1 + \frac{b^3 a}{p_2} K_1 \bigg\} + 2Aq \frac{b^5}{p_2} F\varepsilon^{-1} + \frac{b^4}{p_2} F\varepsilon^{-1} \bigg\} + \frac{ab^4}{p_2} \bigg\{ Aa + \frac{1}{\bar{k}_1} \bigg\{ AqaK_1 \bar{\delta} + AEH_1 p_1 K_1 \bigg\} \bigg\} + \frac{b^2}{p_2} \bigg( \frac{1}{\bar{k}_1} \frac{p_1}{q} Sk^2 2b^3 EH_1 p_1 K_1 + -4\pi^2 \Omega^2 H_1 p_1 \ell_1 + Rk^2 a \bigg) + b^2 \frac{1}{\bar{k}_1} 2AK_1 + b \bigg( Rk^2 \bigg\{ -H_1 \frac{p_1}{q} Sk^2 + \bar{\delta}A \varepsilon^{-1} - \frac{2A}{p_2} a - \frac{1}{p_2} H_1 \ell_1 4\pi^2 \Omega^2 b^2 + 8\pi^2 \Omega^2 A \bigg\} \bigg\} - 2Rq\bar{\delta}' k^2 A H_1 \bigg\} = 0$ (44)$$

It is evident from equation [44] that  $n_i$  may be either zero or non-zero, meaning thereby that the modes may be non-oscillatory or oscillatory. In the absence of suspended particles number density, rotation parameter, magnetic permeability and solute parameter, equation [44] reduces to  $n_i \left(2Ab^2K_1 + Rk^2\overline{\delta}Ab\right) = 0$ 

(45) and term within the brackets is definitely positive, which implies that  $n_i = 0$ . Therefore, the oscillatory modes are not allowed and principal of exchange of stabilities is satisfied for porous medium in the absence of suspended particles, solute parameter and rotation. The presence of the suspended particles number density, rotation parameter, medium permeability and solute parameter bring oscillatory modes (as  $n_i$  may not be zero) which were non–existent in their absence.

The case of over-stability-The present section is devoted to the possibility that instability may occur as overstability. Since we wish to determine the Rayleigh number for onset of instability via a state of pure oscillations, it suffices to find the conditions for which equation [43] will admit of solutions with  $n_i$  real. Substituting  $n = in_i$  in equation [43], and then equating the real and imaginary parts of equation [43] we obtain

$$\begin{split} & n_{i}^{\ 4} \Bigg[ EH_{1}p_{1}\ell_{1}b^{2}\varepsilon^{-1}a\frac{(1+K_{1})}{\bar{k}_{1}} + \bar{\delta}'\varepsilon^{-1}ab\Bigg\{ \frac{p_{1}}{q}qSk^{2}b\ell_{1}\Bigg(1 + \frac{EH_{1}p_{1}}{p_{2}}\Bigg) \Bigg\} + EH_{1}p_{1}\varepsilon^{-1}(2aA + F\ell_{1})\Bigg] \\ & - n_{i}^{\ 2} \Bigg[ \Bigg\{ (2A+b)\varepsilon^{-1}\Bigg(1 + \frac{EH_{1}p_{1}}{p_{2}}\Bigg) + \frac{EH_{1}p_{1}\ell_{1}}{p_{2}}\bar{\delta}' \Bigg\} \Bigg\{ 2aAEH_{1}\frac{p_{1}}{q}Sk^{2}\frac{(1+K_{1})}{\bar{k}_{1}}b^{3} + b^{3}\varepsilon^{-1}(2aA + F\ell_{1}) \Bigg\} + \\ & \Bigg\{ EH_{1}p_{1}q\frac{(1+K_{1})}{\bar{k}_{1}}\varepsilon^{-1} + \frac{\varepsilon^{-1}aE}{p_{2}} \Bigg\} + b^{4}a\ell_{1}E\frac{(1+K_{1})}{\bar{k}_{1}}\Bigg(1 + \frac{EH_{1}p_{1}}{p_{2}}\Bigg) - K_{1}A\varepsilon^{-1}b^{3} + \end{split}$$

$$\left\{ Ep_{1}H_{1}q\bar{\delta}' + a\varepsilon^{-1}\left(1 + \frac{Ep_{1}H_{1}}{p_{2}}\right) \right\} + \frac{H^{2}\pi}{4}\varepsilon^{-1}\left[ -n_{i}^{2}\left\{\bar{\delta}'bq\ell_{1}\left(EH_{1}p_{1}+a\right) + \frac{p_{1}}{q}Sk^{2}EH_{1}p_{1}\varepsilon^{-1}\left(2A+b\right) \right\} \right] + \left[ -4\pi^{2}\Omega^{2}H_{1}p_{1}\ell_{1}\frac{1}{p_{2}}\left\{ \frac{\left(1+K_{1}\right)}{\bar{k}_{1}} - \varepsilon^{-1}K_{1}A\right\} b^{5} + \frac{b^{4}}{p_{2}}\left\{ 2\bar{\delta}'Aq\frac{\left(1+K_{1}\right)}{\bar{k}_{1}} + 4\pi^{2}\Omega^{2}b^{2} + 8\pi^{2}\Omega^{2}Ab \right\} \right]$$
(46)

and

$$Rk^{2} \bigg[ -a\ell_{1}qn_{i}^{3} + 2AH_{1}n_{i} + \frac{p_{1}}{q}Sk^{2}H_{1}bn_{i} - n_{i}\overline{\delta}Ab\varepsilon^{-1}\overline{\delta'} + \frac{2Ab}{p_{2}}an_{i}q + \frac{p_{1}}{q}Sk^{2}\frac{b}{p_{2}}H_{1}\ell_{1}n_{i} - \frac{b^{2}}{p_{2}}\varepsilon^{-1}an_{i}\overline{\delta}A \bigg] = -8\pi^{2}\Omega^{2}AEH_{1}p_{1} - 4EH_{1}p_{1}b\pi^{2}\Omega^{2} - 4\pi^{2}\Omega^{2} - ab^{3}n_{i}^{3}\varepsilon^{-1}\overline{\delta'} - 2EH_{1}p_{1}n_{i}^{3}A\frac{ab^{2}}{p_{2}}\varepsilon^{-1} - 2EH_{1}p_{1}n_{i}^{3}A\frac{ab^{3}}{p_{2}}\varepsilon^{-1} - \frac{a^{2}b^{3}}{p_{2}} + \frac{p_{1}}{q}Sk^{2}\ell_{1}n_{i}^{3}\varepsilon^{-1} - EFH_{1}p_{1}n_{i}^{3}b\varepsilon^{-1} - qEFn_{i}^{3}H_{1}p_{1}b^{2}\varepsilon^{-1} - Fb^{2}\ell_{1}n_{i}^{3}\varepsilon^{-1} - EH_{1}p_{1}\ell_{1}n_{i}^{3}\varepsilon^{-1}\frac{b^{2}}{p_{2}} + 2A\frac{b^{5}}{p_{2}} \\ Fn_{i}\varepsilon^{-1}\overline{\delta'} + \frac{b^{4}}{p_{2}}Fn_{i}\varepsilon^{-1} + \frac{1}{k_{1}}\bigg[ -2qAEH_{1}p_{1}abn_{i}^{3} - \frac{p_{1}}{q}Sk^{2}EH_{1}p_{1}n_{i}^{3}ab^{2} - \frac{an_{i}^{3}b^{2}\ell_{1}}{p_{2}} - EH_{1}p_{1}\ell_{1}n_{i}^{3}\frac{ab^{2}}{p_{2}} \\ + 2qAan_{i}\frac{b^{3}}{p_{2}} + \frac{ab^{4}}{p_{2}}\overline{\delta'}n_{i}K_{1} - 2AEH_{1}p_{1}n_{i}^{3}abK_{1} - EH_{1}p_{1}n_{i}^{3}K_{1}b^{2} - ab^{2}\ell_{1}n_{i}^{3}K_{1}q + 2Aan_{i}\frac{b^{3}}{p_{2}}K_{1} \\ + \frac{ab^{4}}{p_{2}}n_{i}K_{1} - EH_{1}p_{1}n_{i}^{3}\ell_{1}b + 2\frac{p_{1}}{q}Sk^{2}Ab^{2}n_{i} + b^{3}n_{i} + 2\overline{\delta'}AEH_{1}p_{1}n_{i}\frac{b^{2}}{p_{2}} + EH_{1}p_{1}n_{i}\frac{b^{3}}{p_{2}} + \frac{b^{3}}{p_{2}}\ell_{1}n_{i}K_{1} \\ - \frac{p_{1}}{q}Sk^{2}\overline{\delta'}EH_{1}p_{1}n_{i}^{3}\ell_{1}bK_{1} + 2Ab^{2}n_{i}K_{1} + 2qAEH_{1}p_{1}n_{i}\frac{b^{2}}{p_{2}}K_{1} + 2AEH_{1}p_{1}n_{i}\frac{b^{3}}{p_{2}}K_{1} \\ + \frac{ab^{4}}{p_{2}}n_{i}K_{1} - EH_{1}p_{1}n_{i}^{3}\ell_{1}bK_{1} + 2Ab^{2}n_{i}K_{1} + 2qAEH_{1}p_{1}n_{i}\frac{b^{2}}{p_{2}}K_{1} + 2AEH_{1}p_{1}n_{i}\frac{b^{3}}{p_{2}}K_{1} + \frac{b^{3}}{p_{2}}}n_{i}K_{1} \\ - \frac{P_{1}}{q}Sk^{2}\overline{\delta'}EH_{1}p_{1}n_{i}^{3}\ell_{1}bK_{1} + 2Ab^{2}n_{i}K_{1} + 2qAEH_{1}p_{1}n_{i}\frac{b^{2}}{p_{2}}K_{1} + 2AEH_{1}p_{1}n_{i}\frac{b^{3}}{p_{2}}K_{1} \\ + \frac{Ab^{4}}{p_{2}}}n_{i}K_{1} + \frac{Ab^{4}}{p_{2}}n_{i}K_{1} + 2Ab^{2}n_{i}K_{1} + 2Ab^{2}n_{i}K_{1} + 2Ab^{2}n_{i}K_{1} + 2AEH_{1}p_{1}n_{i}\frac{b^{3}}{p_{2}}K_{1} + \frac{b^{3}}{p_{2}}}n_{i}K_{1} \\ \end{bmatrix}$$

Eliminating R between equations [46] and [47], we get

$$\begin{split} &n_{i}^{-6} \left[ b^{2} \left\{ \varepsilon^{-1} a^{2} H_{1}^{-2} \ell_{1}^{2} \frac{(1+K_{1})}{\bar{k}_{1}} \right\} + b \left\{ \frac{\varepsilon^{-1} a E H_{1} g p_{1}}{\bar{k}_{1}} \frac{p_{1}}{q} S k^{2} (1-\bar{\delta} A) - \varepsilon^{-1} b^{3} a \ell_{1} H_{1} E \left( \frac{Eq}{p_{2}} + \frac{(1+K_{1})}{\bar{k}_{1}} \right) \right\} \right] \\ &+ n_{i}^{-4} \left[ b^{5} \left\{ E H_{1} q p_{1} a^{2} (1-\varepsilon^{-1} \bar{\delta} A) + \frac{Ep_{1}}{p_{2}} \bar{\delta} A (H_{1}-1) \frac{p_{1}}{q} S k^{2} (1-\bar{\delta} A) \right\} + b^{4} \left\{ 2 E H_{1} q p_{1} a^{2} A \frac{(1+K_{1})}{\bar{k}_{1}} \right] \\ &- \frac{Ep_{1}}{p_{2}} a^{2} p_{1} H_{1}^{2} q (H_{1}-1) \right\} + b^{3} \left\{ \frac{1}{p_{2}^{2}} E H_{1} p_{1} q a F (1-\varepsilon^{-1} \bar{\delta} A) + E H_{1} p_{1} \ell_{1}^{2} a^{2} \frac{1}{p_{2}^{2}} (H_{1}-1) - E H_{1} q p_{1} \ell_{1} K_{1} a \\ &\left( 2-\varepsilon^{-1} \bar{\delta} A \right) + \frac{E}{p_{2}^{2}} H_{1} p_{1} q a \ell_{1}^{2} (1-\varepsilon^{-1} \bar{\delta} A) \frac{p_{1}}{q} S k^{2} \right\} + b^{2} \left\{ \frac{-a E^{2}}{p_{2}} 2 A F \ell_{1} (H_{1}-1) - \varepsilon^{-1} \ell_{1} a \left( a \ell_{1} - \frac{Ep_{1}}{p_{2}} a \ell_{1} + p_{1} \varepsilon^{-1} \bar{\delta} A \right) \\ &- 2A a \ell_{1} H_{1} E^{2} \frac{(1+K_{1})}{\bar{k}_{1}} + 2A E^{2} \varepsilon^{-1} (2-K_{1} E p_{1}) \right\} + b^{4} \left\{ \frac{1}{p_{2}^{2}} - 2A^{2} E^{2} a H_{1} q p_{1} (H_{1}-F) - \frac{2H^{2}\pi}{4} \varepsilon^{-1} a \ell_{1} A \\ &\left( a \ell_{1} - \frac{Ep_{1}}{p_{2}} \frac{p_{1}}{q} S k^{2} (1-\bar{\delta} A) + p_{1} \varepsilon^{-1} \bar{\delta} A \right) \right\} \right] + n_{i}^{2} \left[ b^{7} \left\{ \left( 1-\varepsilon^{-1} \bar{\delta} A \right) H_{1} q - \varepsilon^{-1} \bar{\delta} A \frac{a E}{p_{2}^{2}} \frac{(1+K_{1})}{\bar{k}_{1}} \right\} + b^{6} \left\{ \frac{E^{2} a^{2}}{p_{2}^{2}} \varepsilon^{-1} \bar{\delta} A \\ &\left( \frac{1+K_{1}}{\bar{k}_{1}} \right) - EH_{1} q p_{1} \ell_{1} \frac{1}{p_{2}^{2}} \left( 2-\varepsilon^{-1} \bar{\delta} A \right) - \frac{2Ep_{1}}{p_{2}} H_{1}^{2} a A \frac{(1+K_{1})}{\bar{k}_{1}} \right\} + b^{4} \left\{ - \varepsilon^{-1} \bar{\delta} A \right\} - EH_{1} p p_{1} \bar{\delta} A \\ &\frac{1}{p_{2}^{2}} \varepsilon^{-1} (F - a K_{1}) \frac{p_{1}}{p_{2}} S k^{2} - \bar{\delta} A \varepsilon^{-1} H_{1} q \frac{1}{p_{2}} \ell_{1} \frac{(1+K_{1})}{\bar{k}_{1}} \right\} + b^{5} \left\{ -\frac{E^{2} H_{1} p_{1}}{p_{2}^{2}} a \ell_{1} (H_{1}-1) + \varepsilon^{-1} H_{1} p_{1} \frac{a^{2}}{p_{2}^{2}} E^{2} \frac{(1+K_{1})}{\bar{k}_{1}} \right\} \\ &+ \bar{\delta} A \frac{\varepsilon^{-1} a}{\bar{k}_{1}} \frac{(1+K_{1})}{\bar{p}_{2}^{2}} - \frac{E^{2} \ell_{1}^{2}}{2} (2aA + F \ell_{1}) - 2A^{2} \varepsilon^{-1} \frac{\bar{\delta} E}{p_{2}} (H_{1} - F) + E^{2} H_{1} a p_{1} \frac{1}{\bar{k}_{1} F_{2}^{2}} \left\{ (1-K_{1}-1) + \frac{E^{2} H_{$$

$$\left(1 - \frac{EH_{1}p_{1}}{p_{2}}\right) - \left(\left\{F\ell_{1}\left(1 - \frac{EH_{1}qp_{1}}{p_{2}}\right)\frac{p_{1}}{q}Sk^{2}\left(1 - \bar{\delta}'A\right) - \frac{EH_{1}p_{1}\ell_{1}}{p_{2}}\right\}\left(2 - \varepsilon^{-1}\bar{\delta}A\right)\right\} + \frac{H_{1}q\ell_{1}E^{2}}{p_{2}^{2}} \\ \left(\ell_{1} + \varepsilon^{-1}H_{1}p_{1}\bar{\delta}A\right) + H_{1}^{2}p_{1}F\frac{1}{p_{2}}2A\left(1 - \varepsilon^{-1}\bar{\delta}A\right)\right\} + b^{2}\left\{\frac{EH_{1}qp_{1}\ell_{1}}{p_{2}}\left(2 - \varepsilon^{-1}\bar{\delta}A\right) + \left\{\ell_{1} + Ep_{1}\left(\frac{\ell_{1}}{p_{2}} - \bar{\delta}A\varepsilon^{-1}\right)\right\} \\ + b^{8}\left\{\frac{E^{2}}{p_{2}}\left(1 + H_{1}qp_{1}\frac{\left(1 + K_{1}\right)}{\bar{k}_{1}}\left(2 - \varepsilon^{-1}\bar{\delta}A\right)\frac{p_{1}}{q}Sk^{2} - F\ell_{1}\varepsilon^{-1}\frac{\left(1 + K_{1}\right)}{\bar{k}_{1}}\bar{\delta}Aa\right)\right\} \\ + b^{7}\left[\frac{2E^{2}}{p_{2}^{2}}\left\{\frac{\left(1 + K_{1}\right)}{\bar{k}_{1}} - EK_{1}p_{1}\right\}\left(1 - \bar{\delta}'A\right)\frac{p_{1}}{q}Sk^{2}A\left(1 - \varepsilon^{-1}\bar{\delta}A\right) + 2Aa\varepsilon^{-1}\left\{1 + EH_{1}qp_{1}\frac{\left(1 + K_{1}\right)}{\bar{k}_{1}}\right\}\right] \\ + b^{6}\left[\frac{2A^{2}FE^{2}}{p_{2}^{2}}\left\{\left(1 - \varepsilon^{-1}\bar{\delta}A\right) + \frac{p_{1}}{q}Sk^{2}\left(1 - \bar{\delta}'A\right)\frac{E}{p_{2}^{2}}H_{1}p_{1}q\left(H_{1} - 1\right)\right\} + \frac{H^{2}\pi}{4}\varepsilon^{-1}\left\{\left(\frac{EH_{1}qp_{1}}{p_{2}} - 1\right)\right) \\ \left(2 - \varepsilon^{-1}\bar{\delta}A\right) - \frac{EH_{1}qp_{1}F}{p_{2}}\varepsilon^{-1}\bar{\delta}A\ell_{1}\right\}\right] + b^{5}\left[\frac{2A^{2}\varepsilon^{-1}H_{1}}{p_{2}^{2}}\left(2 - K_{1}Ep_{1}\right) + \frac{2E^{2}}{p_{2}^{2}}\frac{p_{1}}{q}Sk^{2}\left(1 - \bar{\delta}'A\right)AH_{1}\varepsilon^{-1} + \frac{\varepsilon^{-1}H^{2}\pi}{4}\left\{2AF\left(\frac{EH_{1}qp_{1}}{p_{2}} - 1\right)\left(2 - \varepsilon^{-1}\bar{\delta}A\right) - \frac{EH_{1}p_{1}\ell_{1}}{p_{2}}\varepsilon^{-1}\bar{\delta}AF\right\}\right] + b^{4}\left[-\frac{2EH_{1}}{p_{2}}\frac{p_{1}}{q}Sk^{2}\left(1 - \bar{\delta}'A\right)\frac{\left(1 + K_{1}\right)}{\bar{k}_{1}} + \left\{\left(1 - \varepsilon^{-1}\bar{\delta}A\right)\left(\frac{EH_{1}qp_{1}}{p_{2}} - 1\right)\left(2 - \varepsilon^{-1}\bar{\delta}A\right) + 4\pi^{2}\Omega^{2}F\ell_{1}\right\}\right] + b^{3}\left[-\varepsilon^{-1}\frac{H^{2}\pi}{2}A\frac{a}{p_{2}}\left(H_{1} - 1\right) + \frac{EH_{1}a}{p_{2}}\frac{\left(1 + K_{1}\right)}{\bar{k}_{1}}\right] \\ + b^{2}\left[\frac{P_{1}}{q}Sk^{2}\left(1 - \bar{\delta}'A\right)\frac{H^{2}\pi}{4}\varepsilon^{-1}\left(2 - \varepsilon^{-1}\bar{\delta}A\right) + 4\pi^{2}\Omega^{2}\left\{2A(1 + K_{1})a - \varepsilon^{-1}A^{2}H^{2}\pi\frac{a}{2}EH_{1}\right\}\right] = 0$$

$$(48)$$

*The case of stationary convection-* When the instability sets in as stationary convection, the marginal state is characterized by  $n_i = 0$ . Putting  $n_i = 0$  in equation [46], we obtain

$$R = \frac{b^{4} \left[\frac{p_{1}}{q} S k^{2} A \left\{-\overline{\delta'} b + H_{1} (2A+b)\right\}\right] + b^{3} \left[\frac{(1+K_{1})}{\bar{k}_{1}} - \varepsilon^{-1} K_{1} A\right] + 2b^{2} A \frac{(1+K_{1})}{\bar{k}_{1}} + 4\pi^{2} \Omega^{2} b + 8\pi^{2} \Omega^{2} A}{k^{2} \left\{2H_{1} A + b \left(1 - \varepsilon^{-1} \overline{\delta} A\right)\right\}}$$
(49)

In the absence of stable solute parameters  $(S = 0, \overline{\delta}' = 0)$  and rotation  $(\vec{\Omega} = 0)$  equation [49] reduces to

$$R = \frac{b^{3} \left[ \frac{(1+K_{1})}{\bar{k}_{1}} - \varepsilon^{-1} K_{1} A \right] + 2A \frac{(1+K_{1})}{\bar{k}_{1}} b^{2}}{k^{2} \left\{ 2A + b \left( 1 - \varepsilon^{-1} \bar{\delta} A \right) \right\}},$$
(50)

a result in good agreement with Sharma and Gupta [25].

#### **III. RESULTS AND DISCUSSION**

Equation [48] has been examined numerically using the Newton–Raphson method through the software Fortran 90. We have plotted the variation of Rayleigh number with respect to wave-number using equation [46] satisfying [48] for overstable case and equation [49] for stationary case, for the fixed permissible values of the dimensionless parameters  $p_2 = 1, a = 10, F = 1.005, \overline{\delta}' = 1.5, q = 0.035$   $H_1 = 1.01, \varepsilon = 0.5, E = 0.9, \overline{k_1} = 2.$ 

**Figs. 1–3** correspond to three values of rotation parameter i.e.  $\bar{\Omega} = 20$ , 16, 10 rev/minute, respectively, which shows that Rayleigh number increases with increase in rotation parameter depicting thereby the stabilizing effect of rotation parameter. **Figs. 4–6** correspond to three values of medium permeability  $\bar{k}_1 = 5$ , 10 and 30. The graphs show that the Rayleigh number for the stationary convection and for the case of over-stability decreases with the increase in medium permeability depicting thereby destabilizing effect of medium permeability. **Figs. 7–9** correspond to three values of micropolar coefficient  $\kappa = 0.5$ , 0.7 and 1.0, respectively, accounting for dynamic microrotation viscosity. The graphs show that the Rayleigh number for the case of

overstability decreases with the increase in micropolar coefficient  $\kappa$  implying thereby the destabilizing effect of dynamic microrotation viscosity.



Fig- 1.



Fig- 3.





















Fig- 8.





**Figures 10–12** correspond to three values of micropolar coefficient  $\gamma' = 1.0$ , 1.2 and 1.4, respectively. The graphs show that the Rayleigh number for the stationary convection and for the case of overstability decreases with the increase in micropolar coefficient  $\gamma'$  implying thereby the destabilizing effect of coefficient of angular viscosity, therefore micropolar coefficients have destabilizing effects on the system.

Figures 13–14 correspond to two different values of the solute parameter i.e. S = 30 and 10, respectively. It is evident from the graphs that Rayleigh number increases with the increase in stable solute parameter even in the presence of suspended particles (fine dust) number density depicting the stabilizing effect of solute parameter.



## IV CONCLUSION

There is a competition between the large enough stabilizing effect of rotation parameter, stable solute parameter and the destabilizing effect of the micropolar coefficients and medium porosity. The presence of coupling between thermal and micropolar effects, rotation parameter, solute parameter, medium permeability and suspended particles number density may bring over stability in the system. It is also noted from the **Figures 3**, **4**, **7** and **10** that the Rayleigh number for overstability is always less than the Rayleigh number for stationary convection, for a fixed wave-number. However, the reverse may also occur for large wave-numbers, as has been depicted in **Figures1**, **2**, **5**, **6**, **8**, **9**, **11** and **12**.

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