Common Fixed Point Theorem in Intuitionistic Fuzzy Metric Space

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Abstract: *The purpose of this paper is to obtain common fixed point theorem in* □□*chainable intuitionistic fuzzy metric space using the concept of occasionally weak compatibility for six self-maps and generalizes the result of Manroet. al[12].*

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I. INTRODUCTION

In fuzzy set theory of Zadeh [1] the membership of an element to a fuzzy set is a single value between zero and one. However in reality, it may not always be true that the degree of non-membership of an element in a fuzzy set is equal to (1- membership degree)because there may be some hesitation degree (hesitation margin). Therefore in 1986 Atanassav [2] introduced the concept of intuitionistic fuzzy sets as powerful tool to deal with vagueness. A special future of intuitionistic fuzzy set is that assign to Each element a membership degree and non-membership degree and thus intuitionistic fuzzy set constitutes an extension of Zadeh's fuzzy set by treating membership as a fuzzy logical value rather than a single truth value. For an intuitionistic set the logical value has to be consistent (in the sense $\gamma_A(\mathrm{x})+\mu_A(\mathrm{x})\geq 1$), where $\gamma_A(\mathrm{x})$ and $\mu_A(\mathrm{x})$ denotes degree of membership and degree of non-membership respectively.All results which hold of fuzzy sets can be transformed intuitionistic fuzzy sets but converse need not be true. The knowledge and semantic representation of intuitionistic fuzzy sets become more meaningful and applicable since it includes the degree of belongingness (membership), degree of non-belongingness (non-membership) and the hesitation margin.Intuitionisticfuzzy set in an important concept in fuzzy mathematics because of its wide applications in real world problems such as pattern, recognition, marketing prediction, sale analysis, negotiation process, machine learning, decision making, medical diagnosis and psychological interventions etc.In 2004, Park defined the notion of Intuitionistic fuzzy metric spaces with the help of continuous t-norm and continuous t-conorm as a generalization of G.V. fuzzy metric space [3].Alaca et. al. [4] proved the well-known fixed point theorem of Banach in the setting of Intuitionistic fuzzy metric spaces. Later on Turkogluet. al. [5]proved Jungck"s [6] common fixed point theorem for Intuitionistic fuzzy metric space. Torkoglu et. al. [5] further formulated the notion of weakly commuting and R-weakly commuting mappings in Intuitionistic fuzzy metric spaces and proved the intuitionistic fuzzy version of Pant's theorem [7].In 2008, Alaca et. al. [8, 9] proved common fixed point theorems for compatible maps of type (I), type (II) and compatible mappings in intuitionistic fuzzy metric space. In2010, Park [10] and Kumar [11] proved common fixed point theorems using the notions of semi-compatibility and weak compatibility in intuitionistic fuzzy metric space respectively. In the same year, Manro [12] introduced$\in -$chainable intuitionistic fuzzy metric space and proved a common fixed point theorem for four weakly compatible maps of $\in -$chainable intuitionistic fuzzy metric space. In this paper we prove common fixed point theorem in $\in -$ chainable intuitionistic fuzzy metric space using the concept of occasionally weak compatibility for six self-maps and generalizes the result of Manroet. al [12] and some previous result.

II. PRELIMINARIES

Definition 2.1[1] Let X be any set. A fuzzy set A in X is a function with domain in X and values in$[0,1]$.
Definition 2.2. Intuitionistic fuzzy set:Let X be a nonempty set. An intuitionistic fuzzy set A in X is an object having the form $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle : x\in X\}$ where the functions $\mu_A(x),\nu_A(x): X\to[0,1]$,define respectively, the degree of membership and degree of non-membership of the element $x\in X$ to the set A, which is a subset of X, and for every element $x\in X, 0\leq \mu_A(x)+\nu_A(x)\leq 1$. Furthermore, we have

$\pi_A(x)=1-\mu_A(x)-\nu_A(x)$ called the intuitionistic fuzzy set index or hesitation margin of x in A. $\pi_A(x)$ is the degree of indeterminacy of $x\in X$ to the IFS A and $\pi_A(x)\in[0,1]$ i.e., $\pi_A(x):X\to[0,1]$ and $0\le\pi_A(x)\le1$ for every $x\in X$. $\pi_A(x)$ expressesthe lack of knowledge of whether x belongs to IFS A or not.

For example, let A be an intuitionistic fuzzy set with $\mu_A(x)=0.5$ and $\nu_A(x)=0.3$,

$\Rightarrow\pi_A(x)=1-(0.5+0.3)=0.2$. It can be interpreted as "the degree that the object x belongs to IFS A is 0.5, the degree that the object x does not belong to IFS A is 0.3, the degree of hesitancy or indeterminacy of$x$ belonging to IFS A is 0.2.

Definition 2.3.[3] A binary operation $*:\ [0,1]\times[0,1]\to[0,1]$ is called a continuous t-norm if it satisfies the following conditions:

(i) $*$ is associative and commutative;

(ii) $*$ is continuous;

(iii) $a*1=a$, for all $a\in[0,1]$;

(iv) $a*b\le c*d$, whenever $a\le c$ and $b\le d$ for all $a,b,c,d\in[0,1]$.

Example: $a*b=min\{a,b\}$ (minimum t-norm), $a*b=ab$ (product t-norm).

Definition 2.4[3]A binary operation $\Diamond:\ [0,1]\times[0,1]\to[0,1]$is continuous t-conorm if $\Diamond$ is satisfying the following conditions:

(i) $\Diamond$ is commutative andassociative;

(ii) $\Diamond$ iscontinuous;

(iii) $a\Diamond0=a$ for all $a\in[0,1]$;

(iv) $a\Diamond b\le c\Diamond d$ whenever $a\le c$ and $b\le d$ for all $a,b,c,d\in[0,1]$.

Definition 2.5[4]A 5-tuple $(X,M,N,*,\Diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\Diamond$ is a continuous t-conorm and M,N are fuzzy sets on $X^2\times[0,\infty)$ satisfying the following conditions:

1) $M(x,y,t)+N(x,y,t)\le1$ for all $x,y\in X$ and $t>0$;

2) $M(x,y,0)=0$ for all $x,y\in X$;

3) $M(x,y,t)=1$ for all $x,y\in X$ and $t>0$ if and only if $x=y$;

4) $M(x,y,t)=M(y,x,t)$ for all $x,y\in X$ and $t>0$;

5) $M(x,y,t)*M(y,z,s)\le M(x,z,t+s)$ for all $x,y,z\in X$ and $t>0$;

6) For all $x,y\in X$, $M(x,y,.):[0,\infty)\to[0,1]$ is left continuous;

7) $\lim_{n\to\infty}M(x,y,t)=1$ for all $x,y\in X$ and $t>0$;

8) $N(x,y,0)=1$ for all $x,y\in X$;

9) $N(x,y,t)=0$ for all $x,y\in X$ and $t>0$ if and only if $x=y$;

10) $N(x,y,t)=N(y,x,t)$ for all $x,y\in X$ and $t>0$;

11) $N(x,y,t)\Diamond N(y,z,s)\ge N(x,z,t+s)$ forall $x,y,z\in X$ and $s,t>0$;

12) For all $x,y\in X$, $N(x,y,.):[0,\infty)\to[0,1]$ is right continuous;

13) $\lim_{n\to\infty}N(x,y,t)=0$ for all x,y in X.

(M,N) iscalled an intuitionistic fuzzy metric on X. The functions $M(x,y,t)$ and $N(x,y,t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t,respectively.

Remark 2.1 [8]An intuitionistic fuzzy metric spaces with continuous t-norm $*$ and continuous t-conorm $\Diamond$

defined by $a * a \geq a$ and $(1-a)\Diamond(1-a) \leq (1-a)$ for all $a \in [0,1]$. Then for all $x, y \in X$, $M(x, y, *)$ is non- decreasing and $N(x, y, \Diamond)$ is non-increasing.
Alaca et. al. [4] introduced the following notions:

Definition 2.6 [4]Let $(X, M, N, *, \Diamond)$ be an intuitionistic fuzzy metric space. Then

(1) A sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all $t > 0$ and $p > 0$,

$$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1, \ \lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0.$$

(2) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if, for all $t > 0$,

$$\lim_{n\to\infty} M(x_n, x, t) = 1, \ \lim_{n\to\infty} N(x_n, x, t) = 0.$$

Since $*$ and $\Diamond$ are continuous, the limit is uniquely determined form (5) and (11) of definition 2.3, respectively. An intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to be complete if and only if every Cauchy sequence in X is convergent.
Turkoglu et. al. [5] introduced the notions of compatible mappings in intuitionistic fuzzy metric space, akin to the concept of compatible mappings introduced by Jungck [6] in metric spaces as follows:

Definition 2.7[5] A pair of self-mappings (f, g) of an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to be compatibleif

$\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1 \textit{ and } \lim_{n\to\infty} N(fgx_n, gfx_n, t) = 0$ for every $t > 0$,

Whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ for some $z \in X$.

Definition 2.8[5]A pair of self-mappings (f, g) of an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is said to be non-compatible if $\lim_{n\to\infty} M(fgx_n, gfx_n, t) \neq 1$ or non-existent and $\lim_{n\to\infty} N(fgx_n, gfx_n, t) \neq 0$ or non-existent for every $t > 0$, whenever $\{x_n\}$ is a sequence in X Xsuchthat $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$ forsome $z \in X$.
In 1998, Jungck and Rhoades [13] introduced the concept of weakly compatible maps as follows:

Definition 2.9[13]Two self-maps f and g are said to be weakly compatible if they commute at coincidence points.

Lemma 2.1 Let $(X, M, N, *, \Diamond)$ be intuitionistic fuzzy metric space and for all x, y in X,

$t > 0$ and if for a number k in $(0,1)$,

$M(x, y, kt) \geq M(x, y, t) \textit{ and } N(x, y, kt) \leq N(x, y, t)$ Then $x = y$.

Definition 2.10 [14]: Two self-mappings A and B of an intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ are said to be occasionally weakly compatible(owc) iff there is a point x in X which is coincidence point of A and B at which A and B commute.
Now, we define □– chainable intuitionistic fuzzy metricspace as follows:

Definition 2.11[12]Let $(X, M, N, *, \Diamond)$ be intuitionistic fuzzy metric space.

A finite sequence $x = x_0, x_1, x_2, \ldots, x_n = y$ is called $\in -$chain from x to y if there exists a positive number $\in > 0$ such that $M(x_i, x_{i-1}, t) > 1 - \in$ and $N(x_i, x_{i-1}, t) < 1 - \in$

for all $t > 0$ and $i = 1, 2, \ldots, n$.

An intuitionistic fuzzy metric space $(X, M, N, *, \Diamond)$ is called $\in -$chainable if for any $x, y \in X$, there exists an $\in -$chain from x to y.

S. Manroet. al. [12] **proved the following result:**

Theorem 2.11 Let A, B, S and T be self-maps of a complete $\in -$chainable intuitionistic fuzzy metric spaces $(X, M, N, *, \Diamond)$ with continuous t-norm $*$ and continuous t-conorm $\Diamond$ definedby $a * a \geq a$ and $(1-a) \Diamond (1-a) \leq (1-a)$ for all $a \in [0,1]$ satisfying the following conditions:

(2.11.1) $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$,

(2.11.2) A and S arecontinuous,

(2.11.3) the pairs (A, S) and (B, T) are weaklycompatible,

(2.11.4) There exist $q \in (0,1)$ such that

$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t)$,

$N(Ax, By, qt) \leq N(Sx, Ty, t) \Diamond N(Ax, Sx, t) \Diamond N(By, Ty, t) \Diamond N(Ax, Ty, t)$, for every x, y in X and $t > 0$.

Then A, B, S and T have a unique common fixed point in X.

III. MAIN RESULTS

Theorem 3.1 Let $(X, M, N, *, \Diamond)$ be a complete $\in -$chainable intuitionistic fuzzy metric spacewith continuous t-norm $*$ and continuous t-conorm $\Diamond$ definedby $a * a \geq a$ and $(1-a) \Diamond (1-a) \leq (1-a)$ for all $a \in [0, 1]$. Let A, B, P, Q, S and T be six self-mappings on X, satisfying the following conditions:

(3.1.1) $A(X) \subseteq TP(X)$ and $B(X) \subseteq SQ(X)$,

(3.1.2) SQ iscontinuous,

(3.1.3) $TP = PT$, $SQ = QS$, $AQ = QA$, $BP = PB$,

(3.1.4) (A, SQ) and (B, TP) are occasionally weakly compatible,

(3.1.5) There exist $q \in (0, 1)$ such that $\forall x, y \in X$ and $t > 0$

$M(Ax, By, qt) \geq M(SQx, TPy, t) * M(Ax, SQx, t) * M(By, TPy, t) * M(Ax, TPy, t)$

$N(Ax, By, qt) \leq N(SQx, TPy, t) \Diamond N(Ax, SQx, t) \Diamond N(By, TPy, t) \Diamond N(Ax, TPy, t)$,

Then A, B, P, Q, S and T have a unique common fixed point in X.

Proof: Let $x_0 \in X$ from (3.1.1) there exist $x_1, x_2 \in X$,

Such that $y_0 = A(x_1) = TP(x_1)$ and $y_1 = B(x_1) = SQ(x_2)$ inductively are construct sequence $\{x_n\}$ and $\{y_n\}$ in X such that

$$y_{2n} = A(x_{2n}) = TP(x_{2n+1}) \text{ and } y_{2n+1} = B(x_{2n+1}) = SQ(x_{2n+1}), n = 1, 2, 3, \ldots\ldots$$

Taking $x = x_{2n}$ and $y = x_{2n+1}$ in equation (3.1.4) we get

$$M(Ax_{2n}, Bx_{2n+1}, qt) \geq \left\{ \begin{array}{l} M(SQx_{2n}, TPx_{2n+1}, t) * M(Ax_{2n}, SQx_{2n}, t) * M(Bx_{2n+1}, TP_{2n+1}, t) \\ * M(Ax_{2n}, TPx_{2n+1}, t) \end{array} \right\}$$

$$N(Ax_{2n}, Bx_{2n+1}, qt) \leq \left\{ \begin{array}{l} N(SQx_{2n}, TPx_{2n+1}, t) \Diamond N(Ax_{2n}, SQx_{2n}, t) \Diamond N(Bx_{2n+1}, TP_{2n+1}, t) \\ \Diamond N(Ax_{2n}, TPx_{2n+1}, t) \end{array} \right\}$$

$M(y_{2n}, y_{2n+1}, qt) \geq M(y_{2n-1}, y_{2n}, t) * M(y_{2n}, y_{2n-1}, t) * M(y_{2n+1}, y_{2n}, t) * M(y_{2n}, y_{2n}, t)$

$N(y_{2n}, y_{2n+1}, qt) \leq N(y_{2n-1}, y_{2n}, t) \Diamond N(y_{2n}, y_{2n-1}, t) \Diamond N(y_{2n+1}, y_{2n}, t) \Diamond N(y_{2n}, y_{2n}, t)$

$M(y_{2n}, y_{2n+1}, qt) \geq M(y_{2n-1}, y_{2n}, t)$

$N(y_{2n}, y_{2n+1}, qt) \leq N(y_{2n-1}, y_{2n}, t)$

Similarly,

$M(y_{2n+1}, y_{2n+2}, qt) \geq M(y_{2n}, y_{2n+1}, t)$

$N(y_{2n+1}, y_{2n+2}, qt) \leq N(y_{2n}, y_{2n+1}, t)$

Thus we have $M(y_n, y_{n+1}, qt) \geq M(y_{n-1}, y_n, t)$

$N(y_n, y_{n+1}, qt) \leq N(y_{n-1}, y_n, t)$ for $n = 1, 2, 3\ldots..$

Therefore we have

$M(y_n, y_{n+1}, t) \geq M(y_{n-1}, y_n, t/q)$

$N(y_n, y_{n+1}, t) \geq N(y_{n-1}, y_n, t/q)$

$M((y_n, y_{n+1}, t) \geq M(y_{n-1}, y_n, t/q) \geq M(y_{n-2}, y_{n-1}, t/q^2) \ldots. \geq M(y_0, y_1, t/q^n)$

$N(y_n, y_{n+1}, t) \leq N(y_{n-1}, y_n, t/q) \leq N(y_{n-2}, y_{n-1}, t/q^2) \ldots.. \leq N(y_0, y_1, t/q^n)$

Hence,

$lim_{n\to\infty} M(y_n, y_{n+1}, t) = 1, \ \forall \ t > 0$

$lim_{n\to\infty} N(y_n, y_{n+1}, t) = 0, \forall \ t > 0$

Now for any positive integer we have

$M(y_n, y_{n+p}, t) \geq M(y_n, y_{n+1}, t/p) * M(y_{n+1}, y_{n+2}, t/p) * \ldots. * M(y_{n+p-1}, y_{n+p}, t/p)$

$N(y_n, y_{n+p}, t) \leq N(y_n, y_{n+1}, t/p) \Diamond N(y_{n+1}, y_{n+2}, t/p) \Diamond \ldots.. \Diamond N(y_{n+p-1}, y_{n+p}, t/p)$

$lim_{n\to\infty} M(y_n, y_{n+p}, t) = 1 * 1 \ldots\ldots.. * 1 = 1.$

$lim_{n\to\infty} N(y_n, y_{n+p}, t) = 0 \Diamond 0 \ldots\ldots.. \Diamond 0 = 0.$

Thus $\{y_n\}$ is a Cauchy Sequence in X .

By the completeness of X , $\{y_n\}$ is converges to z .

Hence then all subsequences $\{A(x_{2n})\}, \{TP(x_{2n+1})\}, \{B(x_{2n+1})\}$ and $\{SQ(x_{2n+2})\}$ are also converges to $z \in X$.

$\lim_{n\to\infty} Ax_{2n} = \lim_{n\to\infty} TP(x_{2n+1}) = \lim_{n\to\infty} B(x_{2n+1}) = \lim_{n\to\infty} SQ(x_{2n+2}) = \lim_{n\to\infty} SQ(x_{2n}) = z$. …. (i)

Since X is $\in -$chainable there exist $\in -$chain from x_n to x_{n+1} that is there exist a finite sequence.

$x_n = y_1, y_2, \ldots\ldots., y_l = x_{n+1}$ for $\in > 0$ such that

$M(y_i, y_{i-1}, t) > 1 - \in$ and $N(y_i, y_{i-1}, t) < 1 - \in$, for all $t > 0$ and $i = 1, 2, \ldots., l$.

Thus we have

$M(x_n, x_{n+1}, t) \geq M(y_1, y_2, t/l) * M(y_2, y_3, t/l) * \ldots\ldots.. * M(y_{l-1}, y_l, t/l).$

$N(x_n, x_{n+1}, t) \leq N(y_1, y_2, t/l) \Diamond N(y_2, y_3, t/l) \Diamond \ldots\ldots\ldots. \Diamond N(y_{l-1}, y_l, t/l).$

$M(x_n, x_{n+1}, t) \geq (1-\in) * (1-\in) * (1-\in) * \ldots\ldots\ldots.. * (1-\in) \geq (1-\in).$

$N(x_n, x_{n+1}, t) \leq (1-\in) \Diamond (1-\in) \Diamond (1-\in) \Diamond \ldots\ldots\ldots.. \Diamond (1-\in) \leq (1-\in).$

For $m > n$

$$M(x_n, x_m, t) \geq M(x_n, x_{n+1}, t/m-n) * M(x_{n+1}, x_{n+2}, t/m-n) * .... * M(x_{m-1}, x_m, t/m-n)$$
$$\geq (1-\in) * (1-\in) * (1-\in) * .... * (1-\in) \geq (1-\in).$$

$$N(x_n, x_m, t) \leq N(x_n, x_{n+1}, t/m-n) \lozenge N(x_{n+1}, x_{n+2}, t/m-n) \lozenge .... \lozenge N(x_{m-1}, x_m, t/m-n)$$
$$\leq (1-\in) \lozenge (1-\in) \lozenge (1-\in) \lozenge .... \lozenge (1-\in) \leq (1-\in).$$

Therefore $\{x_n\}$ is a Cauchy sequence in X and hence there exist x in X such that $x_n \to x$.

SQ is continuous, since (3.1.2)

$$\Rightarrow SQx_n \to SQx$$
$$\Rightarrow SQx_{2n} \to SQx$$

By uniqueness of limits we have $SQx = z.$ .... (ii)

Now we have to show that $Ax = z$ put $x = x$ (same)and $y = x_{2n+1}$ in (3.1.5)

$$M(Ax, Bx_{2n+1}, qt) \geq M(SQx, TPx_{2n+1}, t) * M(Ax, SQx, t) * M(Bx_{2n+1}, TP_{2n+1}, t)$$
$$* M(Ax, TPx_{2n+1}, t).$$

$$N(Ax, Bx_{2n+1}, qt) \leq N(SQx, TPx_{2n+1}, t) \lozenge N(Ax, SQx, t) \lozenge N(Bx_{2n+1}, TP_{2n+1}, t)$$
$$\lozenge N(Ax, TPx_{2n+1}, t).$$

Letting $n \to \infty$ and using (ii)

By Lemma (2.1) we have $Ax = z$ and therefore $Ax = SQx = z.$ i.e. x is coincidence point of A and SQ. (A, SQ) is occasionally weak compatible, therefore we have

$$ASQx = SQAx \Rightarrow Az = SQz.$$ .... (iii)

Put $x = z$ and $y = x_{2n+1}$ in (3.1.5)

$$M(Az, Bx_{2n+1}, qt) \geq M(SQz, TPx_{2n+1}, t) * M(Az, SQz, t) * M(Bx_{2n+1}, TPx_{2n+1}, t) * M(Az, TPx_{2n+1}, t).$$
$$N(Az, Bx_{2n+1}, qt) \leq N(SQz, TPx_{2n+1}, t) \lozenge N(Az, SQz, t) \lozenge N(Bx_{2n+1}, TPx_{2n+1}, t) \lozenge N(Az, TPx_{2n+1}, t).$$

Letting $n \to \infty$ and using (iii)

$$M(Az, z, qt) \geq M(Az, z, t) * M(Az, Az, t) * M(z, z, t) * M(Az, z, t).$$
$$N(Az, z, qt) \leq N(Az, z, t) \lozenge N(Az, Az, t) \lozenge N(z, z, t) \lozenge N(Az, z, t).$$

$$M(Az, z, qt) \geq M(Az, z, t).$$

$$N(Az, z, qt) \leq N(Az, z, t).$$

By using Lemma (2.1) $Az = z$

$Az = SQz = z$. .... (iv)

Again since $A(X) \subseteq TP(X)$ then there exist $u \in X$ such that $TP(u) = z$. .... (v)

Now, we show that: $Bu = z$.

Put $x = x_{2n}$ and $y = u$ in (3.1.5)

$$M(Ax_{2n}, Bu, qt) \geq M(SQx_{2n}, TPu, t) * M(Ax_{2n}, SQx_{2n}, t) * M(Bu, TPu, t) * M(Ax_{2n}, TPu, t)$$
$$N(Ax_{2n}, Bu, qt) \leq N(SQx_{2n}, TPu, t) \lozenge N(Ax_{2n}, SQx_{2n}, t) \lozenge N(Bu, TPu, t) \lozenge N(Ax_{2n}, TPu, t),$$

Taking $n \to \infty$ and using (v)

$$M(z, Bu, qt) \geq M(z, z, t) * M(z, z, t) * M(Bu, z, t) * M(z, z, t).$$
$$N(z, Bu, qt) \leq N(z, z, t) \lozenge N(z, z, t) \lozenge N(Bu, z, t) \lozenge N(z, z, t).$$
$$M(z, Bu, qt) \geq M(Bu, z, t).$$
$$N(z, Bu, qt) \leq N(Bz, z, t).$$

Therefore $Bu = z$.

$$\Rightarrow Bu = TPu = z.$$

Thus $'u'$ is a coincident point of B and TP in X.

(B, TP) is occasionally weak compatible therefore we have

$BTPu = TPBu \Rightarrow Bz = TPz.$ ………..(vi)

Now, we show that $Bz = z$

Put $x = z$ and $y = z$ in (3.1.5)

$$M(Az, Bz, qt) \geq M(SQz, TPz, t) * M(Az, SQz, t) * M(Bz, TPz, t) * M(Az, TPz, t)$$
$$N(Az, Bz, qt) \leq N(SQz, TPz, t) \lozenge N(Az, SQz, t) \lozenge N(Bz, TPz, t) \lozenge N(Az, TPz, t)$$

Using (iv) and (vi)

$$M(z, Bz, qt) \geq M(z, Bz, t) * M(z, z, t) * M(Bz, Bz, t) * M(z, Bz, t)$$
$$N(z, Bz, qt) \leq N(z, Bz, t) \lozenge N(z, z, t) \lozenge N(Bz, Bz, t) \lozenge N(z, Bz, t)$$

$M(z, Bz, qt) \geq M(z, Bz, t)$ and $N(z, Bz, qt) \leq N(z, Bz, t)$.

By using Lemma (2.1) $Bz = z$.

Thus, $Bz = TPz = z$.

Therefore $Az = Bz = SQz = TPz = z$. …….(vii)

Now we claim that $Qz = z$.

For this, putting $x = Qz$ and $y = z$ in (3.1.5)

$$M(AQz, Bz, qt) \geq M(SQ(Qz), TPz, t) * M(AQz, SQ(Qz), t) * M(Bz, TPz, t) * M(AQz, TPz, t)$$
$$N(AQz, Bz, qt) \leq N(SQ(Qz), TPz, t) \lozenge N(AQz, SQ(Qz), t) \lozenge N(Bz, TPz, t) \lozenge N(AQz, TPz, t)$$

Using (3.1.3) and (vii)

$$M(QAz, z, qt) \geq M(QSQz, z, t) * M(QAz, QSQz, t) * M(z, z, t) * M(QAz, TPz, t).$$
$$N(QAz, z, qt) \leq N(QSQz, z, t) \lozenge N(QAz, QSQz, t) \lozenge N(z, z, t) \lozenge N(QAz, z, t).$$
$$M(Qz, z, qt) \geq M(Qz, z, t) * M(Qz, Qz, t) * M(z, z, t) * M(Qz, z, t)$$
$$N(Qz, z, qt) \leq N(Qz, z, t) \lozenge N(Qz, Qz, t) \lozenge N(z, z, t) \lozenge N(Qz, z, t)$$
$$M(Qz, z, qt) \geq M(Qz, z, t) \text{ and } N(Qz, z, qt) \leq N(Qz, z, t).$$

Using Lemma (2.1), $Qz = z$.

$SQz = z \Rightarrow Sz = z.$

$Qz = Sz = z$. ……. (viii)

Again, we show that $Pz = z.$

Putting $x = z$ and $y = Pz$ in (3.1.5)

$$M(Az, BPz, qt) \geq M(SQz, TPPz, t) * M(Az, SQz, t) * M(BPz, TPPz, t) * M(Az, TPPz, t),$$
$$N(Az, BPz, qt) \leq N(SQz, TPPz, t) \lozenge N(Az, SQz, t) \lozenge N(BPz, TPPz, t) \lozenge N(Az, TPPz, t).$$

Using (3.1.3) and (vii)

$$M(z, PBz, qt) \geq M(z, PTPz, t) * M(z, z, t) * M(PBz, PTPz, t) * M(z, PTPz, t).$$
$$N(z, PBz, qt) \leq N(z, PTPz, t) \lozenge N(z, z, t) \lozenge N(PBz, PTPz, t) \lozenge N(z, PTPz, t).$$
$$M(z, Pz, qt) \geq M(z, Pz, t) * M(z, z, t) * M(Pz, Pz, t) * M(z, Pz, t)$$
$$N(z, Pz, qt) \leq N(z, Pz, t) \lozenge N(z, z, t) \lozenge N(Pz, Pz, t) \lozenge N(z, Pz, t)$$
$$M(z, Pz, qt) \geq M(z, Pz, t) \text{ and } N(z, Pz, qt) \leq N(z, Pz, t)$$

Using Lemma (2.1), $Pz = z$.

As, $TPz = z \Rightarrow Tz = z$.

Thus, $Pz = Tz = z.$ ……(ix)

Hence, by using (viii), (viii) and (ix)

$$Az = Bz = Sz = Qz = Pz = Tz = z.$$

Uniqueness: Let ‘ w ’ be another common fixed point of A, B, S, Q, P and T .

Then, $Aw = Bw = Sw = Qw = Pw = Tw = w$ and $Az = Bz = Sz = Qz = Pz = Tz = z.$ …… (x)

Put $x = z$ and $y = w$ in (3.1.5), we get

$$M(Az, Bw, qt) \geq M(SQz, TPw, t) * M(Az, SQz, t) * M(Bw, TPw, t) * M(Az, TPw, t).$$
$$N(Az, Bw, qt) \leq N(SQz, TPw, t) \lozenge N(Az, SQz, t) \lozenge N(Bw, TPw, t) \lozenge N(Az, TPw, t).$$
$$M(z, w, qt) \geq M(z, w, t) * M(z, z, t) * M(w, w, t) * M(z, w, t),$$
$$N(z, w, qt) \leq N(z, w, t) \lozenge N(z, z, t) \lozenge N(w, w, t) \lozenge N(z, w, t), \text{since(x)}$$
$$M(z, w, qt) \geq M(z, w, t)$$

$N(z,w, qt) \le N(z,w,t)$.

Using Lemma (2.1) we conclude that $z = w$.

Hence,' z ' is a unique common fixed point of self-maps A, B, S, Q, P and T.

Take $P = Q = I$ (Identity mappings).

Corollary (3.2) Let $(X, M, N, *, \lozenge)$ be a complete $\in -$chainable intuitionistic fuzzy metric space with continuous t-norm $*$ and continuoust-conorm $\lozenge$ defined by $a * a \ge a$ and $(1-a)\lozenge(1-a) \le (1-a)$ for all $a \in [0,1]$. Let A, B, S and T be four mappings on X. Satisfying following conditions.

(3.2.1) $A(X) \subseteq T(X)$ and $B(x) \subseteq S(x)$,
(3.2.2) S is continuous,
(3.2.3) (A, S) and (B, T) are occasionally weakly compatible.
(3.2.4) There exist $q \in (0,1)$ such that $\forall x, y \in X$ and $t > 0$

$$M(Ax, By, qt) \ge M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(Ax, Ty, t),$$
$$N(Ax, By, qt) \le N(Sx, Ty, t) \lozenge N(Ax, Sx, t) \lozenge N(By, Ty, t) \lozenge N(Ax, Ty, t),$$

Then, A, B, S and T have a unique common fixed points in X.

On putting $A = B$ in corollary (3.2).

We get the following corollary for three self-maps.

Corollary 3.3 Let A, S and T be self-maps of a complete $\in -$chainable intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ and satisfying following conditions.

(3.3.1) $A(X) \subseteq S(X) \cap T(X)$

(3.3.2) S is continuous,

(3.3.3) (A, S) and (A, T) are occasionally weakly compatible,

(3.3.4) There exist $q \in (0,1)$ such that for every $x, y \in X$ and $t > 0$.

$$M(Ax, Ay, qt) \ge M(Sx, Ty, t) * M(Ax, Sx, t) * M(Ay, Ty, t) * M(Ax, Ty, t),$$
$$N(Ax, Ay, qt) \le N(Sx, Ty, t) \lozenge N(Ax, Sx, t) \lozenge N(Ay, Ty, t) \lozenge N(Ax, Ty, t),$$

Then A, S and T have a unique common fixed points in X.

Putting $S = T$ in corollary (3.3)

Corollary 3.4 Let A and S be self-maps of a complete $\in -$chainable intuitionistic fuzzy metric space $(X, M, N, *, \lozenge)$ and satisfying following conditions.

(3.4.1) $A(X) \subseteq S(X)$,

(3.4.2) S is continuous,

(3.4.3) (A, S) are occasionally weakly compatible, (3.4.4) There exist $q \in (0,1)$ such that for every $x, y \in X$ and $t > 0$.

$$M(Ax, Ay, qt) \geq M(Sx, Sy, t) \text{ and } N(Ax, Ay, qt) \leq N(Sx, Sy, t),$$

Then A and S have a unique common fixed points in X .

CONCLUSION

This paper is generalization of the result of Manroet. al. [12]in the sense of replace weakly compatible to occasionally weak compatible (owc) to prove atheorem on common fixed point theorems for sixself-mappings in complete chainable intuitionistic fuzzymetric space.

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