Computation of *ABC*, *AG* and Augmented Status Indices of Graphs

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Abstract: *The status of a vertex u is the sum of distances between u and all other vertices in a graph. In this paper, we introduce the atom bond connectivity (ABC) status index, arithmetic-geometric (AG) status index and augmented status index of a graph. Also these indices of some standard graphs and friendship graphs are computed.*

Keywords: *Status, ABC status index, AG status index, augmented status index, graphs.*

Mathematics Subject Classification: 05*C*05, 05*C*12, 05*C*35, 05*C*90.

I. Introduction

Let G be a finite, simple, connected graph with vertex set $V(G)$ and edge set $E(G)$. The degree $d_G(u)$ of a vertex u is the number of vertices adjacent to u. The distance $d(u, v)$ between any two vertices u and v is the length of the shortest u-v path in a graph G. The status $\sigma(u)$ of a vertex u in G is the sum of the distances of all other vertices from u in G. For undefined term and notation, we refer [1]. Several status indices of a graph such as first and second status connectivity indices [2], first and second status coindices [3], harmonic status index [4], first and second hyper status indices [5], geometric-arithmetic status index [6], F-status index [7], (a, b)-status index [8] were introduced and studied in the literature.

We introduce the atom bond connectivity status index, arithmetic-geometric status index, augmented status index of a graph as follows:

The atom bond connectivity status index of a graph G is defined as

$$ABCS(G) = \sum_{uv \in E(G)} \sqrt{\frac{\sigma(u) + \sigma(v) - 2}{\sigma(u)\sigma(v)}}.$$

The arithmetic-geometric status index of a graph G is defined as

$$AGS(G) = \sum_{uv \in E(G)} \frac{\sigma(u) + \sigma(v)}{2\sqrt{\sigma(u)\sigma(v)}}.$$

The augmented status index of a graph G is defined as

$$ASI(G) = \sum_{uv \in E(G)} \left(\frac{\sigma(u)\sigma(v)}{\sigma(u) + \sigma(v) - 2}\right)^3.$$

Recently many different topological indices were studied, for example, in [9, 10, 11, 12, 13, 14, 15, 16, 17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37]. In this paper, we compute the atom bond connectivity status index, arithmetic-geometric status index, augmented status index of some standard graphs and friendship graphs.

II. Results for complete graphs

Theorem 1. Let K_n be a complete graph with n vertices. Then

(1) $ABCS(K_n) = \frac{1}{\sqrt{2}} n\sqrt{n-2}.$

(2) $AGS(K_n) = \frac{n(n-1)}{2}.$

(3) $ASI(K_n) = \dfrac{n(n-1)^7}{16(n-2)^3}$.

Proof: Let K_n be a complete graph with n vertices. Then it has $\dfrac{n(n-1)}{2}$ edges and for any vertex u in K_n, $\sigma(u) = n-1$. Thus

(1) $$ABCS(K_n) = \sum_{uv\in E(K_n)} \sqrt{\frac{\sigma(u)+\sigma(v)-2}{\sigma(u)\sigma(v)}} = \left(\sqrt{\frac{(n-1)+(n-1)-2}{(n-1)(n-1)}}\right)\frac{n(n-1)}{2} = \frac{1}{\sqrt{2}}n\sqrt{n-2}.$$

(2) $$AGS(K_n) = \sum_{uv\in E(K_n)} \frac{\sigma(u)+\sigma(v)}{2\sqrt{\sigma(u)\sigma(v)}} = \frac{n-1+n-1}{2\sqrt{(n-1)(n-1)}} \times \frac{n(n-1)}{2} = \frac{n(n-1)}{2}.$$

(3) $$ASI(K_n) = \sum_{uv\in E(K_n)} \left(\frac{\sigma(u)\sigma(v)}{\sigma(u)+\sigma(v)-2}\right)^3 = \left(\frac{(n-1)(n-1)}{n-1+n-1-2}\right)^3 \frac{n(n-1)}{2} = \frac{n(n-1)^7}{16(n-2)^3}.$$

III. Results for cycles

Theorem 2. Let C_n be a cycle with n vertices and n edges. Then

(1) $ABCS(C_n) = \dfrac{2\sqrt{2(n^2-4)}}{n}$, if n is even,

$\quad = \dfrac{2n\sqrt{2(n^2-5)}}{n^2-1}$, if n is odd.

(2) $AGS(C_n) = n$, if n is even,

$\quad = n$, if n is odd.

(3) $ASI(C_n) = \dfrac{n^{13}}{512(n^2-4)^3}$, if n is even,

$\quad = \dfrac{n(n^2-1)^6}{512(n^2-5)^3}$, if n is odd.

Proof: Let C_n be a cycle with n vertices and n edges.

Case 1. Suppose n is even. Then $\sigma(u) = \dfrac{n^2}{4}$ for any vertex u in C_n. Thus

(1) $$ABCS(C_n) = \sum_{uv\in E(C_n)} \sqrt{\frac{\sigma(u)+\sigma(v)-2}{\sigma(u)\sigma(v)}} = \left(\frac{\frac{n^2}{4}+\frac{n^2}{4}-2}{\frac{n^2}{4}\times\frac{n^2}{4}}\right)^{1/2} n = \frac{2\sqrt{2(n^2-4)}}{n}.$$

(2) $$AGS(C_n) = \sum_{uv\in E(C_n)} \frac{\sigma(u)+\sigma(v)}{2\sqrt{\sigma(u)\sigma(v)}} = \left(\frac{\frac{n^2}{4}+\frac{n^2}{4}}{2\sqrt{\frac{n^2}{4}\times\frac{n^2}{4}}}\right) n = n.$$

(3) $$ASI(C_n) = \sum_{uv\in E(C_n)} \left(\frac{\sigma(u)\sigma(v)}{\sigma(u)+\sigma(v)-2}\right)^3 = \left[\frac{\frac{n^2}{4}\times\frac{n^2}{4}}{\frac{n^2}{4}+\frac{n^2}{4}-2}\right]^3 n = \frac{1}{512}\times\frac{n^{13}}{(n^2-4)^3}.$$

Case 2: Suppose n is odd. Then $\sigma(u) = \dfrac{n^2-1}{4}$ for any vertex u in C_n.

(1) $ABCS(C_n)$ $= \sum_{uv\in E(C_n)} \sqrt{\frac{\sigma(u)+\sigma(v)-2}{\sigma(u)\sigma(v)}} = \left(\sqrt{\frac{\frac{n^2-1}{4}+\frac{n^2-1}{4}-2}{\frac{n^2-1}{4}\times\frac{n^2-1}{4}}}\right) n = \frac{2n\sqrt{2(n^2-5)}}{n^2-1}.$

(2) $AGS(C_n)$ $= \sum_{uv\in E(C_n)} \frac{\sigma(u)+\sigma(v)}{2\sqrt{\sigma(u)\sigma(v)}} = \left(\frac{\frac{n^2-1}{4}+\frac{n^2-1}{4}}{2\sqrt{\frac{n^2-1}{4}\times\frac{n^2-1}{4}}}\right) n = n.$

(3) $ASI(C_n)$ $= \sum_{uv\in E(C_n)} \left(\frac{\sigma(u)\sigma(v)}{\sigma(u)+\sigma(v)-2}\right)^3 = \left(\frac{\frac{n^2-1}{4}\times\frac{n^2-1}{4}}{\frac{n^2-1}{4}+\frac{n^2-1}{4}-2}\right)^3 n = \frac{1}{512}\times\frac{n(n^2-1)^6}{(n^2-5)^3}.$

IV. Results for complete bipartite graphs

Theorem 3. Let $K_{p,q}$ be a complete bipartite graph with $p+q$ vertices and pq edges. Then

(1) $ABCS(K_{p,q}) = pq\left[\frac{3(p+q)-6}{2(p^2+q^2)-6(p+q)+5pq+4}\right]^{\frac{1}{2}}.$

(2) $AGS(K_{p,q}) = \frac{pq[3(p+q)-4]}{2\sqrt{2(p^2+q^2)-6(p+q)+5pq+4}}.$

(3) $ASI(K_{p,q}) = \frac{pq[2(p^2+q^2)-6(p+q)+5pq+4]^3}{[3(p+q)-6]^3}.$

Proof: The vertex set of $K_{p,q}$ can be partitioned into two independent sets V_1 and V_2 such that $u \in V_1$ and $v \in V_2$ for every edge uv in $K_{p,q}$. Thus $d_G(u) = q$, $d_G(v) = p$. Then $\sigma(u) = q + 2(p - 1)$ and $\sigma(v) = p+2(q - 1)$. Therefore

(1) $ABCS(K_{p,q})$ $= \sum_{uv\in E(K_{p,q})} \sqrt{\frac{\sigma(u)+\sigma(v)-2}{\sigma(u)\sigma(v)}}$

$= pq\left(\frac{q+2p-2+p+2q-2-2}{(q+2p-2)(p+2q-2)}\right)^{\frac{1}{2}} = pq\left(\frac{3(p+q)-6}{2(p^2+q^2)-6(p+q)+5pq+4}\right)^{\frac{1}{2}}.$

(2) $AGS(K_{p,q})$ $= \sum_{uv\in E(K_{p,q})} \frac{\sigma(u)+\sigma(v)}{2\sqrt{\sigma(u)\sigma(v)}}$

$= \frac{pq(q+2p-2+p+2q-2)}{2\sqrt{(q+2p-2)(p+2q-2)}} = \frac{pq[3(p+q)-4]}{2\sqrt{2(p^2+q^2)-6(p+q)+5pq+4}}.$

(3) $ASI(K_{p,q})$ $= \sum_{uv\in E(K_{p,q})} \left(\frac{\sigma(u)\sigma(v)}{\sigma(u)+\sigma(v)-2}\right)^3$

$= \frac{pq[(q+2p-2)(q+2p-2)]^3}{(q+2p-2+p+2q-2-2)^3} = \frac{pq[2(p^2+q^2)-6(p+q)+5pq+4]^3}{[3(p+q)-6]^3}$

V. Results for wheel graphs

A wheel graph W_n is the join of K_1 and C_n. Clearly W_n has n+1 vertices and 2n edges. A graph W_n is shown in Figure 1.

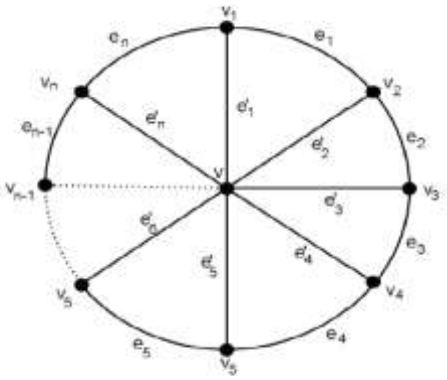

Figure 1. Wheel graph W_n

In W_n, there are two types of edges as follows:

$E_1 = \{uv \in E(W_n) \mid d_{W_n}(u) = d_{W_n}(v) = 3\}$, $\qquad |E_1| = n$.

$E_2 = \{uv \in E(W_n) \mid d_{W_n}(u) = 3, d_{W_n}(v) = n\}$, $\qquad |E_2| = n$.

Therefore there are two types of status edges in W_n as given in Table 1.

$\sigma(u), \sigma(v) \setminus uv \in E(W_n)$	$(2n-3, 2n-3)$	$(n, 2n-3)$
Number of edges	n	n

Table 1. Status edge partition of W_n

Theorem 4. Let W_n be a wheel graph with n+1 vertices and 2n edges. Then

(1) $\quad ABCS(W_n) = \dfrac{2n\sqrt{n-2}}{2n-3} + \sqrt{\dfrac{n(3n-5)}{2n-3}}.$

(2) $\quad AGS(W_n) = n + \dfrac{n(3n-3)}{2\sqrt{n(2n-3)}}.$

(3) $\quad ASI(W_n) = n\left[\dfrac{(2n-3)^2}{4(n-2)}\right]^3 + n\left[\dfrac{n(2n-3)}{3n-5}\right]^3.$

Proof: By definitions and by using Table 1, we deduce

(1) $\quad ABCS(W_n)$

$$= \sum_{uv \in E(W_n)} \sqrt{\frac{\sigma(u)+\sigma(v)-2}{\sigma(u)\sigma(v)}}$$

$$= \left(\frac{2n-3+2n-3-2}{(2n-3)(2n-3)}\right)^{\frac{1}{2}} n + \left(\frac{n+2n-3-2}{n(2n-3)}\right)^{\frac{1}{2}} n$$

$$= \frac{2n\sqrt{n-2}}{2n-3} + \sqrt{\frac{n(3n-5)}{2n-3}}.$$

(2) $\quad AGS(W_n)$

$$= \sum_{uv \in E(W_n)} \frac{\sigma(u)+\sigma(v)}{2\sqrt{\sigma(u)\sigma(v)}}$$

$$= \left(\frac{2n-3+2n-3}{2\sqrt{(2n-3)(2n-3)}}\right) n + \left(\frac{n+2n-3}{2\sqrt{n(2n-3)}}\right) n$$

$$= n + \frac{n(3n-3)}{2\sqrt{n(2n-3)}}.$$

(3) $ASI(W_n)$

$$= \sum_{uv \in E(W_n)} \left(\frac{\sigma(u)\sigma(v)}{\sigma(u)+\sigma(v)-2} \right)^3$$

$$= \left(\frac{(2n-3)(2n-3)}{2n-3+2n-3-2} \right)^3 n + \left(\frac{n(2n-3)}{n+2n-3-2} \right)^3 n$$

$$= n\left[\frac{(2n-3)^2}{4(n-2)} \right]^3 + n\left[\frac{n(2n-3)}{3n-5} \right]^3.$$

VI. Results for friendship graphs

A friendship graph F_n is the graph obtained by taking $n \geq 2$ copies of C_3 with vertex in common. This graph has $2n + 1$ vertices and $3n$ edges. A graph F_4 is presented in Figure 2.

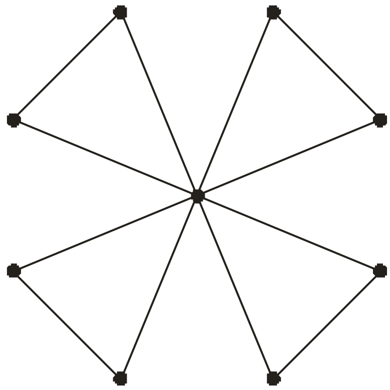

Figure 2. Friendship graph F_4

In F_n, there are two types of edges as follows:

$E_1 = \{uv \in (F_n) \mid d_{F_n}(u) = d_{F_n}(v) = 2\}$, $|E_1| = n$.

$E_2 = \{uv \in (F_n) \mid d_{F_n}(u) = 2, d_{F_n}(v) = 2n\}$, $|E_1| = 2n$.

Therefore in F_n, we obtain two types of status edges as given in Table 2.

$\sigma(u), \sigma(v) \setminus uv \in E(F_n)$	$(4n-2, 4n-2)$	$(2n, 4n-2)$
Number of edges	n	$2n$

Table 2. Status edge partition of F_n

Theorem 5. Let F_n be a friendship graph with $2n+1$ vertices and $3n$ edges. Then

(1) $ABCS(F_n)$ $= \dfrac{n\sqrt{8n-6}}{4n-2} + \sqrt{\dfrac{2n(3n-2)}{2n-1}}.$

(2) $AGS(F_n)$ $= n + (3n-1)\left(\dfrac{n}{2n-1} \right)^{\frac{1}{2}}.$

(3) $ASI(F_n)$ $= n\left[\dfrac{(4n-2)^2}{4n-6} \right]^3 + 2n\left[\dfrac{n(4n-2)}{3n-2} \right]^3.$

Proof: By using definitions and Table 2, we deduce

(1) $ABCS(F_n)$ $= \displaystyle\sum_{uv \in E(F_n)} \sqrt{\frac{\sigma(u)+\sigma(v)-2}{\sigma(u)\sigma(v)}}$

$$= \left(\frac{4n-2+4n-2-2}{(4n-2)(4n-2)}\right)^{\frac{1}{2}} n + \left(\frac{2n+4n-2-2}{2n(4n-2)}\right)^{\frac{1}{2}} 2n$$

$$= \frac{n\sqrt{8n-6}}{4n-2} + \sqrt{\frac{2n(3n-2)}{2n-1}}.$$

(2) $AGS(F_n)$

$$= \sum_{uv\in E(F_n)} \frac{\sigma(u)+\sigma(v)}{2\sqrt{\sigma(u)\sigma(v)}}$$

$$= \left(\frac{4n-2+4n-2}{2\sqrt{(4n-2)(4n-2)}}\right) n + \left(\frac{2n+4n-2}{2\sqrt{2n(4n-2)}}\right) 2n$$

$$= n + (3n-1)\left(\frac{n}{2n-1}\right)^{\frac{1}{2}}.$$

(3) $ASI\,(F_n)$

$$= \sum_{uv\in E(F_n)} \left(\frac{\sigma(u)\sigma(v)}{\sigma(u)+\sigma(v)-2}\right)^3$$

$$= \left(\frac{(4n-2)(4n-2)}{4n-2+4n-2-2}\right)^3 n + \left(\frac{2n(4n-2)}{2n+4n-2-2}\right)^3 2n$$

$$= n\left[\frac{(4n-2)^2}{8n-6}\right]^3 + 2n\left[\frac{n(4n-2)}{3n-2}\right]^3.$$

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