A solution of the unsolved mathematical mystery Divide by zero and zero to the power zero

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Abstract: *In this paper, we find a reasonable solution on the unsolved mathematical mystery divide by zero and zero to the power zero. From the time of discovery of zero, it is taught that we cannot divide by zero and we find another mathematical mystery zero to the power zero is undefined. In this article, we apply our best opinions to find a popular solution of these operations in the light of basic mathematical (Arithematic) operations and with the application of newly introduced algebraic formulas.*

Keywords: *Divide by Zero, zero to the power zero, two step method, multiplicative identity.*

Introduction

Let us study first the mathematical operations of our basic digits 0, 1, 2, ……8, 9, we find all the digits except "0" take part in all four major operations namely, addition, substraction, multiplication and division. We cannot divide any number or even "0" by "0". Another operation "0" to the power "0" is kept undefined.

In mathematical operations, division is the inverse operation of multiplication. If we find result after multiplication, it is necessary to find the result after multiplication. Keeping this inverse view in mind, we proceed our study to find the result.

Question Arise- Whether our present multiplication results is perfect?
To find answer, we compare the multiplicational result of various systems (methods)-
In our general method (Long multiplication)

1x2 = 2
1x2x3 = 6
2x0x5 = 0
0x0x0 = 0

In Lattice method –

1x2 = 02
2x0 = 00

In decimal system (without putting "0" before decimal point) –

˙1x ˙2= ˙02
˙1x˙2x˙3 = ˙006
˙5x˙2x˙0 = ˙000

Comparing the above three methods of multiplicational result, we observe disimilarities on creation of digits on products after multiplication.

Question Arise- Which method is perfect?

1. Natural creation of digits on products after multiplication-

To find answer, which method is perfect? We think it is the Indian decimal system of multiplication. In general multiplication too, the creation of digits on product maintain the rule of decimal system. Our present system neglects zero or zeroes before natural numbers.

In favour of our opinion, we apply a newly introduced method namely, **two step method**. The two step method is a perfect method to show the natural creation of digits on products after multiplication.

The two step method is a method based on digit wise multiplication. This two step method contains two steps- 1. Step1 and 2. Step2. Step 1 of this method contains the product of vertical digit or digits and step2 contains the result of the sum of crosswise products. After adding step1 and step2, we find the final result. A symbolic "o" is putted on block on one's place on step2 which is vary in number depending on digit or digits.

We find the two step method on a series of algebraic formulas. Here we apply only the formulas of 2 digits (ab) numbers-

Numbers		Step1 Step2
$(ab)^2$	=	$a^2b^2 + 2ab/0$
$(ab)^3$	=	$a^3b^3 + 3a^2b/3ab^2/0$
$(ab)^4$	=	$a^4b^4 + 4a^3b/6a^2b^2/4ab^3/0$

And so on.

The number as power on "a" and "b" of Step1 indicate the creation of digits on products after multiplication i.e. a^2 = 2 digits number, b^2 = 2 digit number, a^3 = 3 digits number and so on in their respective places.

Step1 of these formulas are very important to reach our goal. Here we apply the formulas on a 2 digits number 12 and observe -

1. $(12)^2 = 1^2 2^2 + 2.1.2/0$
   = 0104 + 004/0 (1x1 = 01, 2x2 = 04)
   = 0104 + 0040 (withdrawing block symbol)
   = 0144

2. $(12)^3 = 1^3 2^3 + 3.\ 1^2.2/\ 3.1.2^2/0$
   = 001008+0006/0012/0 (1x1x1 = 001, 2x2x2 = 008)
   = 001008+0007/2/0 (1 digit in each block)
   = 001008+000720 (withdrawing block symbols)
   = 001728

3. $(12)^4 = 1^4\ 2^4 + 4.1^3 2/\ 6.1^2.\ 2^2/\ 4.1.2^3/0$
   = 00010016 + 00008/00024/00032/0 (1x1x1x1 = 0001, 2x2x2x2 = 0016)
   = 00010016+ 00010/7/2/0 (1 digit in each block)
   = 00010016 + 00010720 (Withdrawing block symbols)
   = 00020736

[We may apply our present rule of multiplicational result on step 2]
Step1 of the above examples clearly show the natural creation of digits on product after multiplication.

Question Arise-Does "O" behave like other basic digits?

The present system teaches us that zero does not follow the same rules as all the other numbers and the multiplication property states that the product of any number and zero is zero. It does not matter what the number is, when we multiply it to zero, we get the result as zero.

We think our present system is unable to find our result. To find the answer of the above question, again we apply the formulas of two step method on the number 10 and 1002.

4. $(10)^2 = 1^2\ 0^2 + 2.1.0/0$
   = 0100 + 000/0 (1x1= 01, 0x0 = 00)
   = 0100 (As Step 2 = 0000)

5. $(10)^3 = 1^3\ 0^3 + 3.1^2.0/3.1.0^2/0$
   = 001000 + 0000/0000/0 (1x1x1= 001, 0x0x0 = 000)
   = 001000 (Step 2 = 0000)

6. $(10)^4 = 1^4\ 0^4 + \ldots..$ (Step2 results zeroes)
   = 00010000 (1x1x1x1 = 0001, 0x0x0x0 = 0000)

7. $(1002)^2 = (10)^2\ (02)^2 = 2.10.02/00$ (2 digit as b)
   = 0100 0004 + 40/00 $(10)^2 = 0100$, $(02)^2 = 0004$)
   = 01000004 + 4000 (withdrawing block)
   = 01004004

Comparing the examples 4,5,6,7 with the examples 1, 2, 3, we find that "0" creates equal number of digits like other basic digits and find-

1^2 = 1x1 = 01 -> 2 digits number
1^3 = 1x1x1 = 001 -> 3 digits number
2^2 = 2x2 = 04 -> 2 digits number

2^3 = 2x2x2 = 008 -> 3 digits number

Or 0^2 = 0x0 = 00 -> 2 digits number
0^3 = 0x0x0 = 000 -> 3 digits number
0^4 = 0x0x0x0 = 0000 -> 4 digits number

2. Zero Participate in zero divide by zero-

In our present system, we cannot divide by zero even zero by zero. We can apply our two step method to proceed our goal.

In multiplication, unit's digit always results the unit's digit of the product. If division is inverse operation of multiplication then unit's digit will also take part in division and result the quotient.

To find whether "O" results quotient like the other basic digits or not, we compare a study with natural numbers. Firstly we multiply and thereafter divide the products by multiplier.

Multiply		Divide the products	unit'sdigit / unit digits
2x14	=28	28÷14 = 2	8÷4 = 2
2x13	= 26	26÷13 = 2	6÷3 = 2
2x11	= 22	22÷11 = 2	2÷1 = 2
6x5	=30	30÷5 = 6	(3)0÷5 = 6
10x10	= 100	100÷10 = 10	0÷0 = ?
100x100	=10000	1000÷100 = 100	00÷00 = ?

(00 as unit's digit using multi-digit as single digit)

We observe that when unit's digit of numerator and denominator are natural number it result the quotient as natural number and when we divide "0" by "0" or "00" by "00", we find equal number of zero or zeroes vanish (disappear) from the result.

We forward studying by digit-wise of the step1 of the examples 1.4, 1.5 and 1.6.

0^2 = 00 or, 1^2= 01
0^3 = 000 1^3= 001
0^4 = 0000 1^4= 0001

We again proceed backward from 0^4 to 0^0 and 1^4 to 1^1

0^4 = 0000 0r, 1^4 = 0001
0^3 = 0000÷0 = 000 1^3 = 0001 ÷ 1 = 001
0^2 = 000÷0 = 00 1^2 = 001÷1 = 01
0^1 = 00÷0=0 1^1 = 01÷1 = 1
0^0 = 0÷0 = ?

It is proved that zero creates equal number of digits like other digits after multiplication and here we observe it maintain equal rule after division .

From the above patterns, we observe that zero takes part in division and results the quotient as 1 (one) zero lesser than the dividend.

3. Result of "O" divide by "0" and "0" to the power "0"-

To find the answer how they vanish from the result, we forward the following patterns-

$(10)^4$ = 10000
$(10)^3$ = 10000÷10 = 1000.0 = 1000 (withdrawing ˙0)
$(10)^2$ = 1000÷10 = 100.0 = 100
$(10)^1$ = 100÷10 = 10.0 = 10
$(10)^0$ = 10÷10 = 1.0 = 1

We again forward another patterns to reach our goal-

0^4 = 0000
0^3 = 0000÷0 = 000.0 = 000 (withdrawing ˙0)
0^2 = 000÷0 = 00.0 = 00
0^1 = 00÷0 = 0.0 = 0
0^0 = 0÷0 = ˙0 ....= Nothing (Vanishing result)

We find 0^0 and 0/0 result the same answer ˙0 (point zero).

To verify our opinion (result), we again apply the multiplication of 10 by 10 and 300 by 200 using two step methods. Step1 of the formula is responsible for our study.

We know, 10x10 = (1x1) (0x0) = 0100 -> Step1
300x200 = (3x2) (00x00) = 060000 -> Step1

(00 = multi-digit as single digit)

To find multiplicand, we divide the product by the multiplier –

In 10x10, we find 1x1= 01 and 0x0= 00. We divide 01 by multiplier 1 and 00 by 0, we get 01÷1= 1 and 00÷0= 0˙0= 0. Setting in order, we find the multiplicand 10.

In 300x200, we find 3x2 = 06 and 00x00 = 0000. We divide by the multiplier 06 by 2 and 0000 by 00, we get 06÷2= 3 and 0000÷00 = 00˙00 = 00. Setting in order, we find the multiplicand 300.

From the above study, we find that zero or zeroes as unit's digit of denominator is responsible for decimal point only for unit's digit zero or zeroes on numerator.

In number system, "0" is the least digit. So ˙0 (point zero) is not a digit, it vanishes from the result and it cannot take part in any mathematical operations lonely. We may call ˙0 (point zero) as vanishing digit (or number) or cancellation form.

To prove Multiplicative Identity-
Multiplication inverse or reciprocal for a number x, denoted by 1/x or x^{-1}, is a number which when multiplied by x yields the multiplicative Identity 1. The multiplicative inverse of 0 is1/0 (where x = 0)
To find multiplicative identity-
0x 1/0 = 1x 0/0
= 1x ˙0 (0/0 = ˙0)
= 1 (˙0 does take part in multiplication)

Or, 0 x 1/0 = 00÷0 (1x0 = 00)
= 1 0÷0 (1x0 = 1 0), [hidden 1 in a (bigger) zero]
= 1˙0 (00 = 1x0), (As per question only)
To verify 1x0 = 00, we apply step1 of two step method on the following illustration-

21x10 = (2x1) (1x0) = 0200

In 02= 2x1 and 00= 1x0 (As per rule)
We divide the product by the multiplier to find back our multiplicand and observe –
02 ÷ 1 = 2 (multiplier is 1)
00÷0 = 0˙0 = 0 (multiplier is 0)
In 00÷0, we find the multiplicand 0 in place of 1. Where is our fault? To find correct answer we should follow the rule that multiplicand and multiplier always hidden on the product. Applying the same logic i.e. after multiplication of 1x0, 1 is hidden in a zero in this form
1x0 = 1 0 but not in the form 0 1 as 1x1 = 01.
Therefore, we may write 1 0÷0 = 1˙0 =1
Or, 00÷0 = (1x0) ÷0 = 1x 0/0 = 1x˙0 = 1.

Question may arise, we may write 00 equal to 2x0, 3x0 or 7x0 and so on. It is quite wrong because a product is the multiplication result of the multiplicand and multiplier as per question only. If we do not follow this rule it yields us mathematical puzzle (fun).

3. Any number except zero divide by zero-
In our numbers system, 0 is the least digit and it is denoted for nothing. We know, zero takes part in multiplication and result zero (zeroes). In mathematics, division is the inverse operation of multiplication.
Our query is whether zero takes part in division of all numbers except zero. We proceed considering the two important properties of division-
1. Dividend is the product of divisor and quotient where remainder is zero.
2. Dividend is equal to the product of divisor and quotient plus remainder.

In our present system, 1 digit number divide by 1 digit number, we may get 1 digit number quotient. For instance 8÷2 = 4. If we apply the above 3.1 property of division, we observe that the dividend must be larger in digits than the divisor. For instance 4÷2 = 2. When we multiply divisor 2 by quotient 2, we get the product 04. What does it mean? In present system, we are always neglecting “0” before natural number.

Considering the lattice method, decimal system and two step method, we may say,“ Sum of the digits of divisor and quotient is equal to total digits of the dividend (accounting neglecting zero or zeroes).” If we divide 2 digits number by 1 digit number, we get 1 digit number as quotient and 3 digits number by 1 digit number , we get 2 digits quotient and so on.

Remembering the above rule, we apply the properties of division by taking dividend 25 and divisor 6. We get quotient 4 and remainder 1.

Application of properties of division-
1. Dividend = DivisorXQuotient+Remainder
25 = 6X4+1 = 25
2. Quotient = (Dividend-Remainder) ÷ Divisor
4 = (25-1) ÷ 6 = 4
3. Divisor = (Dividend-Remainder) ÷ Quotient
6 = (25-1) ÷ 4 = 6
4. Remainder = Dividend – (Divisor X Quotient)
1 = 25 – (6X4) = 1
The above illustrations satisfy all the properties of division.

To find answer, any number divide by zero, we forward remembering “0” is a digit denotes for nothing. If we divide by “0” the quotient must be 1 digit lesser than the dividend.

Considering the rule, quotient must be 1 digit lesser then the dividend, we cannot consider a natural number divide by zero result the same number as quotient or if we consider „0‟ was neglected before the dividend then also we will get 2nd property “Dividend equal to product of divisor and quotient plus remainder” results zeroes. This opinion will negate our all properties of division.

We proceed considering the view in mind, no divide, no quotient, all are remainder and observe the properties of division. We take dividend 25 and divisor “0”. 25 is a two digit number and “0” is a one digit number. We consider 25÷0 equal to quotient 0 and remainder 25.

Application of properties of division-
1. Dividend = DivisorXQuotient+Remainder
25 = 0x0+25 =00+25 = 25
2. Quotient = (Dividend-Remainder) ÷ Divisor
0 = (25-25) ÷ 0 = 00†0 = 0˙0 = 0
3. Divisor = (Dividend-Remainder) ÷ Quotient
0 = (25-25) ÷ 0 = 00÷0 = 0
4. Remainder = Dividend – (Divisor X Quotient)
25 = 25 – (0X0) = 25-00 = 25
The above illustrations satisfy all the properties of division.

We again apply 1 digit number 4 and divide it by “0”. We get quotient 0 and remainder 4. We observe the properties of division-

Application of properties of division-
1. Dividend = DivisorXQuotient+Remainder
4 = 0x0 +4 = 00+4 = 04
2. Quotient = (Dividend-Remainder)÷Divisor
0 = (4-4) ÷ 0 = 0÷0 = ˙0 (point zero)
3. Divisor = (Dividend-Remainder)÷Quotient
0 = (4-4) ÷ 0 = 0÷0 = ˙0 (point zero)
4. Remainder = Dividend – (Divisor X Quotient)
4 = 4 – (0X0) = 4 - 00 = 04

The above illustration 4÷0, does not satisfy the properties of division. In properties 1 and 4, we get unequal digits in both sides and in 2 and 3, we got “0” equal to ˙0 (point zero).

Where is our fault? We apply the dividend only as a number but not written in product form.

To verify, we write the dividend 4 in product form 04 and divide it by 0. We apply the properties of division and observe-

Application of properties of division-

1. Dividend = DivisorXQuotient+Remainder
   04 = 0x0+04 = 00+04 = 04
2. Quotient = (Dividend-Remainder)÷Divisor
   0 = (04-04) ÷ 0 = 00÷0 = 0˙0 = 0
3. Divisor = (Dividend-Remainder)÷Quotient
   0 = (04-04) ÷ 0 = 00÷0 = 0˙0 =0
4. Remainder = Dividend – (Divisor X Quotient)
   04 = 04 – (0X0) =04 - 00 = 04

We find the above illustration satisfy all the properties of division. When we divide any natural number (Dividend in product form) by zero it satisfies all the properties of division.

Observations-

1. 0/0 and 0^0 results ˙0 (point zero)
2. 1/0 or any number except zero divide by zero results quotient "0" or zeroes and the number (Dividend) becomes remainder.

Conclusion-

In this article, we focus the natural creation of digits on products of all the basic digits, including zero, after multiplication. We know it is somewhere against the rule of present system but what is true, we cannot deny. We hope our new way of thinking and our reasonable solution to the hidden mathematical mystery will help the mathematical world in future.

References –

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