Geo Chromatic Number for certain Cartesian product of Graphs

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ABSTRACT

In this paper we found minimum cardinality among all geodetic sets which is known as geodetic number and is denoted by $g(G)$ and the minimum cardinality among all the chromatic sets which is known as chromatic number and is denoted by $\chi(G)$. The combination of these concepts we have the geo chromatic number $\chi_{gc}(G)$ for certain cartesian product of graphs.

KEYWORDS

Geodetic number; Chromatic number; Geo-Chromatic number.

INTRODUCTION

We found the geo chromatic number $\chi_{gc}(G)$ for cartesian product of Path, Cycle, Star graph, ladder graph, Comb graph.

PRELIMINARIES

- The **Cartesian product of graphs** and is the graph G □ H with vertices V (G □ H) = V (G) × V (H) , and for which (x , u) (y , v) is an edge if and u v ∈ E (H) , or x y ∈ E (G) and u= v.[citation]
- The **distance** $d(u,v)$ between two vertices u and v in a connected cartesian product of graph G is the length of a shortest $u-v$ path in G. An $u-v$ path of length $d(u,v)$ is called an geodesic. A vertex is said to lie on an $u-v$ geodesic if x is an internal vertex of P. The closed interval $I(u,v)$ consists of u,v and all vertices lying on some $u-v$ geodesic of G, and for a non-empty set, $I(S)=\bigcup_{u,v\in S} I(u,v)$. If G is connected graph, then a set S of vertices is a geodetic set if $I(S)=V(G)$.The geodetic number $g(G)$ is the minimum cardinality of a geodetic set of G [citation]
- A p **- vertex coloring** of G is an assignment of p colors, $1,2,\ldots\ldots p$ to the vertices of G the coloring is proper if no two distinct adjacent vertices have the same color. [citation]If $\chi(G)=p$ is said to be - p chromatic, where $p\le k$. A set $c\subseteq V(G)$ is called chromatic set if C contains all p vertices of distinct colors in G. Chromatic number of G is the minimum cardinality among all the chromatic sets of G.
- Let P_n be a path graph with n vertices. The **comb graph** is defined as Pn⊙K1. It has 2n vertices and 2n - 1 edges
- **Ladder graph** L_n is a planar undirected graph with 2n vertices and 3n-2 edges. The ladder graph can be obtained as the Cartesian product of two path graphs, one of which has only one edge: $L_{n,1} = P_n \times P_2$

THEOREM 1

For any cartesian product between P_n and P_m then

$\chi_{gc}(P_n \square P_m)$ = $\begin{cases} 2 & when\ m=2 \\ 3 & when\ m=3 \\ 4 & when\ m>3 \end{cases}$ for all n = even &

$\chi_{gc}(P_n \square P_m)$= $\begin{cases} 3 & when\ m=2 \\ 4 & when\ m>3 \end{cases}$ for all n= odd

PROOF:

Case 1:

When n = 2k $\forall$ k = 1,2,..which is even then we assume that,

Let P_n be the path with vertex set $\{v_1, v_2, ...v_n\}$ and P_m be the path with vertex set $\{u_1, u_2, ...u_m\}$.

Subcase 1.1:

For $m=2$, then ($P_n \square P_2$), we get the geodetic set $\{v_1, v_m\}$ which is minimum. Also the set S is a minimum chromatic set $\chi = 2$ of ($P_n \square P_2$) (i.e.) $g(G) = \chi(G)$ then χ_{gc}($P_n \square P_2$) = $|S| = 2$.

Subcase 1.2:

For $m=3$, then ($P_n \square P_3$), we get the geodetic set $\{v_1, ..., v_m\}$ which is minimum. Also the set S is a minimum chromatic set $\chi = 3$ of ($P_n \square P_3$) (i.e.) $g(G) = \chi(G)$ then χ_{gc}($P_n \square P_3$) = $|S_c|$. If the set is not chromatic set, S has vertices with distinct colour and also satisfies the condition of the geodetic set then we have, χ_{gc}($P_n \square P_3$) = 3.

Subcase 1.3:

For $m>3$, then ($P_n \square P_m$), we get the geodetic set $\{v_1, ..., v_m\}$ which is minimum and also satisfies the minimum chromatic set $g(G) = \chi(G)$ then we have , χ_{gc}($P_n \square P_m$) = 4 ,where $m>3$.

Case 2:

When n = 2k-1 $\forall$ k = 1,2,..which is odd then we assume that,

Let P_n be the path with vertex set $\{v_1, v_2, ...v_n\}$ and P_m be the path with vertex set $\{u_1, u_2, ...u_m\}$.

Subcase 2.1:

For $m=2$, then ($P_n \square P_2$), we get the geodetic set $\{v_1, ...v_m\}$ in which is minimum. Also the set S is a minimum chromatic set $\chi = 3$ of ($P_n \square P_2$) (i.e.) $g(G) = \chi(G)$ then χ_{gc}($P_n \square P_2$) = $|S| = 3$.

Subcase 2.2:

For $m > 3$, then ($P_n \square P_m$), we get the geodetic set $\{v_1, \ldots\ldots, v_m\}$ which is minimum and also satisfies the minimum chromatic set $g(G) = \chi(G)$ then we have , $\chi_{gc}(P_n \square P_m) = 4$.

THEOREM 2:

For any cartesian product between C_n and C_m then $\chi_{gc}(C_n \square C_m) = m$ for n = 3,4,..n.

PROOF:

Let C_n be the cycle with vertex set $\{v_1, v_2, \ldots\ldots v_n\}$ and C_m be the cycle with vertex set $\{u_1, u_2, \ldots\ldots u_m\}$.

For m = 3,4,. then ($C_n \square C_m$), we get the geodetic set {$\{v_1, v_2, \ldots\ldots v_n\}$, $\{u_1, u_2, \ldots\ldots u_m\}$} which is minimum set . The set S has the chromatic set $\chi = m$ of ($C_n \square C_m$) (i.e.) $g(G) = \chi(G)$ then we have, $\chi_{gc}(C_n \square C_m)$ = m

Remarks 2.1

The above results are true for both odd number of cycle and as well as even number of cycles.

THEOREM 3:

For any cartesian product between $(P_n \odot k_1)$ and $(P_m \odot k_1)$ then $\chi_{gc}(P_n \odot k_1 \square P_m \odot k_1) = m + n$ for n,m = 1,2,3,4,..n.

PROOF:

Let $P_n \odot k_1$ be the comb graph with vertex set $\{v_1, v_2, \ldots\ldots v_n\}$ and $P_m \odot k_1$ be the comb graph with vertex set $\{u_1, u_2, \ldots\ldots u_m\}$.

For m = 1,2,3, 4,…. then ($P_n \odot k_1 \square P_m \odot k_1$), we get the geodetic set {$\{v_1, v_2, \ldots\ldots v_n\}$, $\{u_1, u_2, \ldots\ldots u_m\}$} which is minimum set . The set S has the chromatic set $\chi = m + n$ of ($P_n \odot k_1 \square P_m \odot k_1$) (i.e.) $g(G) = \chi(G)$ then we have, $\chi_{gc}(P_n \odot k_1 \square P_m \odot k_1) = m + n$

THEOREM 4:

For cartesian product between L_n and L_m then,

$$\chi_{gc}(L_n \square L_m) = \begin{cases} 2 & when\ \ n, m = 1 \\ 4 & when\ \ n = 1, m > 2 \\ 4 & when\ \ n, m > 2 \end{cases}$$

PROOF:

Let L_n be the ladder graph with vertex set $\{v_1, v_2, ...v_n\}$ and L_m be the ladder graph with vertex set $\{u_1, u_2, ...u_m\}$.

Case 1:

For n=1 and m=1 we have the minimum geodetic set $\{v_1, v_2\}$ or $\{u_1, u_2\}$ in which is minimum and we also have the minimum chromatic set which satisfies the condition that $g(G) = \chi(G)$ such that we have, $\chi_{gc}(L_n \,\square\, L_m) = 2$

SUBCASE 1.1

Ifn = 1 and m=2,3,4…then we have the minimum geodetic set among the vertices of L_n and L_m which is minimum and also the minimum chromatic set which satisfies the condition that $g(G) = \chi(G)$ in such cases we have the results as, $\chi_{gc}(L_n \,\square\, L_m) = 4$.

CASE 2:

For n=2,3…. and m=2,3…… we have to satisfy the above condition exists and we have the following results to be $\chi_{gc}(L_n \,\square\, L_m) = 4$.

CONCLUSION:

In this paper, we extend the concept of geo chromatic number based on vertex colouring for some cartesian product of graphs such as Path, Cycle, Comb graph, Ladder Graph, Star graph. This concept can be extended to several other operations on graphs such a, total colouring graphs, Johan colouring, Hardy colouring etc.,

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