

# Expressions of Combinations

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## Abstract

The operation Combination is a powerful operation in mathematics. Combination operation as an infinite series with  $2n$  and  $n$  result in many mathematical constants. Some of them have been demonstrated in this paper.

## Keywords

Combinations, Wolfram, Expressions, Infinite series, constants,  $\pi$ ,  $e$ , golden ratio,  $\sqrt{2}$ ,  $\sqrt{3}$ ,  $\sqrt{5}$ ,  $\sqrt{17}$ ;

## Introduction

Humans have wondered about presence and absence. I have written papers on “Expressions for 1” for presence and “Expressions for 0” for absence. These are very important expressions. In fact, Wikipedia treats 0 and 1 as general mathematical constants.

Advancement from 1 is essentially a combination exercise that a human undergoes. He sees similarity between two things to count as 2 from 1. This is an important thought process of an individual. Another important thing a man realizes is that some things are desirable and some things are not desirable. And the human begins to wonder the world of his as a mixture of these two aspects.

The above thought process of a human is the combination. The combination  $\text{comb}(2n,n)$  which is also written as  $\binom{2n}{n}$ .

In this paper I have considered the combination of  $n$  and  $2n$  over an infinite series to express the mathematical constants that they give. Indeed, the expression  $\text{comb}(2n,n)$  or  $\binom{2n}{n}$  is very beautiful which gives many mathematical constants.

The expressions for mathematical constants  $\pi$ ,  $e$ , golden ratio,  $\sqrt{2}$ ,  $\sqrt{3}$ ,  $\sqrt{5}$ ,  $\sqrt{17}$  are obtained.

My special thanks to Wolfram for their brilliant mathematical widget without which these expressions could not have been tested and confirmed for correctness. <sup>[1]</sup>

## The expressions on $\pi$

$$\sum_2^{inf} \left( \frac{2^n}{\binom{2n}{n}} \right) = \pi/2$$

The above expression has been checked in Wolfram alpha. <sup>[1]</sup>



If we take  $1/\binom{2n}{n}$ ,  $(n \cdot n)/\binom{2n}{n}$ ,  $(n \cdot n \cdot n)/\binom{2n}{n}$ ,  $(n \cdot n \cdot n \cdot n)/\binom{2n}{n}$ ,  $\dots \dots (n \cdot n \cdot n \cdot n \cdot n \cdot n)/\binom{2n}{n} \dots \dots$  and so on the result of summation from 0 to infinity is some rational number + some number  $\cdot \pi$  in all the cases. Some snapshots from Wolfram alpha is attached below. [1]

WolframAlpha computational intelligence

sum of  $\{1/\text{comb}(2n,n)\}$ , for n = 0 to infinity

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Infinite sum:

$$\sum_{n=0}^{\infty} \frac{1}{\binom{2n}{n}} = \frac{2}{27} (18 + \sqrt{3} \pi)$$

$\binom{2n}{n}$  is the binomial coefficient

Decimal approximation: 1.736399858718715077909795168364923490063125832909497905682...

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sum of  $\{(n \cdot n)/\text{comb}(2n,n)\}$ , for n = 0 to infinity

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Infinite sum:

$$\sum_{n=0}^{\infty} \frac{n \cdot n}{\binom{2n}{n}} = \frac{2}{81} (54 + 5\sqrt{3} \pi)$$

$\binom{2n}{n}$  is the binomial coefficient

Decimal approximation: 2.005110875642302907627436391719316937882987499293607620581...

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sum of  $\{(n \cdot n \cdot n)/\text{comb}(2n,n)\}$ , for n = 0 to infinity

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Infinite sum:

$$\sum_{n=0}^{\infty} \frac{n \cdot n \cdot n}{\binom{2n}{n}} = \frac{18}{3} + \frac{74 \pi}{81 \sqrt{3}}$$

$\binom{2n}{n}$  is the binomial coefficient

Decimal approximation: 4.990384604362124949925454210685435224555813609368676575211...

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sum of  $\left(\frac{n!n^n}{\text{comb}(2n,n)}\right)$ , for  $n = 0$  to infinity

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infinite sum:

$$\sum_{n=0}^{\infty} \frac{n!n^n}{\binom{2n}{n}} = \frac{32}{3} + \frac{238\pi}{81\sqrt{3}}$$

$\binom{n}{k}$  is the binomial coefficient

Decimal approximation: More digits

15.99610183565115862273321759652880326276058971661817501216...

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sum of  $\left(\frac{n!n^n n^n}{\text{comb}(2n,n)}\right)$ , for  $n = 0$  to infinity

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infinite sum:

$$\sum_{n=0}^{\infty} \frac{n!n^n n^n}{\binom{2n}{n}} = \frac{3170}{3} + \frac{23594\pi}{81\sqrt{3}}$$

$\binom{n}{k}$  is the binomial coefficient

Decimal approximation: More digits

1584.99731110792760452983558531862990652481801305557051111...

**The expression on e**

$$\sum_0^{inf} \left( \frac{\binom{2n}{n}}{\text{permutation}(2n,n)} \right) = e$$

The above expression has been checked in Wolfram alpha.<sup>[1]</sup>

WolframAlpha computational intelligence.

sum of  $\left(\frac{\text{comb}(2n,n)}{\text{PI}(2n)}\right)$ , for  $n = 0$  to infinity

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infinite sum:

$$\sum_{n=0}^{\infty} \frac{\binom{2n}{n}}{\text{PI}(2n)} = e$$

$\binom{n}{k}$  is the binomial coefficient  
 $\text{PI}(n)$  is the factorial function

Decimal approximation: More digits

2.718281828459045235360287471352662497757247093699818574466...

Convergence tests: More digits

By the ratio test, the series converges.

**The expression on 1.61803. . . . , the Golden Ratio**

$$\frac{1}{2} + 1/2 \sum_0^{inf} (((5^n) \binom{2n}{n})) = 1.61803 \dots\dots$$

The above expression has been checked in Wolfram alpha.<sup>[1]</sup>



**The expressions on irrational numbers of the type  $\sqrt{n}$**

$$\sum_0^{inf} ((2^{-3n}) \binom{2n}{n}) = \sqrt{2}$$

$$3/2 \sum_0^{inf} ((4^{-2n}) \binom{2n}{n}) = \sqrt{3}$$

$$\sum_0^{inf} ((5^{-2n}) \binom{2n}{n}) = \sqrt{5}$$

$$\sum_0^{inf} (((17^{-3n}) / (34^{-2n})) \binom{2n}{n}) = \sqrt{17}$$

The above expression has been checked in Wolfram alpha.<sup>[1]</sup>





**References**

[1] <https://www.wolframalpha.com/>