

Fuzzy Soft Hyperideals In Join Hyperlattices

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Abstract:

In this paper, we introduce the notion of join hyperlattice and some properties related to it.

Keywords:

Fuzzy soft hyperideal [1], Fuzzy soft homomorphism, join hyperlattice, hyperideals.

I. INTRODUCTION

We develop the theory of fuzzy soft hyperideals in join hyperlattices by introducing the novel concept of fuzzy soft hyperideals. The theory introduced here is one of the initial ideas to be introduced in the development of the theory of fuzzy soft hyperideals. The properties and structural characteristics of these concepts are also investigated and discussed here.

II. PRELIMINARIES

Definition 1.1. Let (L, \wedge, \vee) be a join hyperlattice and (f, X) be a fuzzy soft set over L.

- (f, X) is called a fuzzy soft \vee -hyperideal over L if for all $x \in X$ and $a, b \in L$
 - (i) $\bigcap_{c \in a \wedge b} f_x(c) \geq f_x(a) \cap f_x(b)$,
 - (ii) $\bigcap_{c \in a \vee b} f_x(c) \geq f_x(a) \cup f_x(b)$.

That is, for each $x \in X$, f_x is a fuzzy \vee -hyperideals of L.

- (f, X) is called a fuzzy soft \wedge -hyperideals over L if for all $x \in X$ and $a, b \in L$,
 - (i) $\bigcap_{c \in a \vee b} f_x(c) \geq f_x(a) \cap f_x(b)$,
 - (ii) $\bigcap_{c \in a \wedge b} f_x(c) \geq f_x(a) \cup f_x(b)$.

That is, for each $x \in X$, f_x is a fuzzy \wedge -hyperideals of L.

Next, let us illustrate this definition by the following examples.

Example 1.1. A fuzzy soft \vee -hyperideals (f, X) , for which X is a singleton, is a fuzzy \vee -hyperideal. Hence a fuzzy \vee -hyperideal is a particular type of fuzzy soft \vee -hyperideal. In a similar way, a fuzzy \wedge -hyperideal is a particular type of fuzzy soft \wedge -hyperideals.

Example 1.2. Let (L, \wedge, \vee) be the join hyperlattice. Set $X = \{a, b\}$.

- (1) Let (f, X) be a fuzzy soft set on L, where fuzzy sets f_α and f_β are as follows.

$$f_\alpha(a) = \begin{cases} 0.8, & a \in \{x, y\} \\ 0.4, & a \in \{z, s\} \end{cases}, f_\beta(a) = \begin{cases} 0.6, & a \in \{x, y\} \\ 0.3, & a \in \{z, s\} \end{cases}$$

Then (f, X) is a fuzzy soft \wedge -hyperideal over L.

- (2) Let (f, X) be a fuzzy soft set on L, where fuzzy sets f_α and f_β are as follows.

$$f_\alpha(a) = \begin{cases} 0.5, & a \in \{x, y\} \\ 0.7, & a \in \{z, s\} \end{cases}, f_\beta(a) = \begin{cases} 0.2, & a \in \{x, y\} \\ 0.4, & a \in \{z, s\} \end{cases}$$

Then (f, X) is a fuzzy soft \vee -hyperideal over L.

In what follows, we shall investigate some properties of fuzzy soft hyperideals.

Proposition 1.1. Let (f, X) and (g, Y) be two fuzzy soft \mathbf{V} -hyperideals (\wedge -hyperideals) over the join hyperlattice (L, \wedge, \mathbf{V}) . Then $(f, X) \sqcap (g, Y)$ is a fuzzy soft \mathbf{V} -hyperideal (\wedge -hyperideal) over L .

Proof:

Let (f, X) and (g, Y) be two fuzzy softsets over A such that $X \cup Y \neq \emptyset$. The restricted intersection of (f, X) and (g, Y) is the fuzzy soft set (h, Z) , where $Z = X \cap Y$ and $h_z = f_z \cap g_z$, for all $z \in Z$. This is denoted by $(h, Z) = (f, X) \sqcap (g, Y)$.

$$(f, X) \sqcap (g, Y) = (h, Z),$$

$$\text{Where } Z = X \cap Y \text{ and } h_z = f_z \cap g_z,$$

$$\text{That is, } h_z(a) = f_z(a) \cap g_z(a) \text{ for all } z \in Z \text{ and } a \in L.$$

Suppose that (f, X) and (g, Y) are two fuzzy soft \mathbf{V} -hyperideals over the join hyperlattice (L, \wedge, \mathbf{V}) .

If any $a, b \in L$ and $c \in a \wedge b$, for all $z \in Z$, we have $h_z(c) = f_z(c) \cap g_z(c) \geq (f_z(a) \cap g_z(b)) \cap (g_z(a) \cap g_z(b)) \geq (f_z(a)g_z(a)) \cap (f_z(b) \cap g_z(b)) = h_z(a) \cap h_z(b)$.

Then we obtain $\bigcap_{c \in a \wedge b} h_z(c) \geq h_z(a) \cap h_z(b)$ for all $z \in Z$.

On the other hand, for all $c \in a \vee b$ and $z \in Z$, we have $h_z(c) = f_z(c) \cap g_z(c) \geq (f_z(a) \cup g_z(b)) \cap (g_z(a) \cup g_z(b)) = (f_z(a) \cap g_z(a)) \cup (f_z(b) \cap g_z(b)) = h_z(a) \cup h_z(b)$, which implies $\bigcap_{c \in a \vee b} h_z(c) \geq h_z(a) \cup h_z(b)$ for all $z \in Z$. Therefore, $(f, X) \sqcap (g, Y)$ is a fuzzy soft \wedge -hyperideals over L .

The case for \wedge -hyperideals can be similarly proved.

Proposition 1.2. Let (f, X) and (g, Y) be two fuzzy soft \mathbf{V} -hyperideals (\wedge -hyperideals) over the join hyperlattice (L, \wedge, \mathbf{V}) . Then $(f, X) (g, Y)$ is a fuzzy soft \mathbf{V} -hyperideals (\wedge -hyperideals) over L .

Proof:

Suppose that (f, X) and (g, Y) are two fuzzy soft \mathbf{V} -hyperideals over the join hyperlattice (L, \wedge, \mathbf{V})

The extended intersection of two fuzzy soft sets (f, X) and (g, Y) over A is the fuzzy soft set (h, Z) , where $Z = X \cup Y$

$$h_z = \begin{cases} f_z, & \text{if } Z \in X - Y \\ g_z, & \text{if } Z \in Y - X \\ f_z \cap g_z, & \text{if } Z \in X \cap Y \end{cases}$$

For all $z \in Z$. This is denoted by $(f, X) (g, Y) = (h, Z)$

$$(f, X) (g, Y) = (h, Z), \text{ where } Z = X \cup Y \text{ and}$$

$$h_z = \begin{cases} f_z, & \text{if } Z \in X - Y \\ g_z, & \text{if } Z \in Y - X \\ f_z \cap g_z, & \text{if } Z \in X \cap Y \end{cases} \text{ for all } z \in Z$$

Now, for all $z \in Z$ and $a, b \in L$, we consider the following cases.

Case 1: $Z \in X - Y$, then $h_z = f_z$. Since (f, X) is a fuzzy soft \mathbf{V} -hyperideal over the join hyperlattice (L, \wedge, \mathbf{V}) , h_z is a fuzzy soft \mathbf{V} -hyperideal over (L, \wedge, \mathbf{V}) .

Case 2: $Z \in Y - X$, then $h_z = g_z$. Analogous to the proof of case 1, we have h_z is a fuzzy soft \mathbf{V} -hyperideal over (L, \wedge, \mathbf{V}) .

Case 3: $Z \in X \cap Y$, then $h_z = f_z \cap g_z$

Proposition 1.3: Let (f, X) and (g, Y) be two fuzzy soft \mathbf{V} -hyperideals (\wedge -hyperideals) over the join hyperlattice (L, \wedge, \mathbf{V}) . Then $(f, X) (g, Y)$ is a fuzzy soft \mathbf{V} -hyperideals (\wedge -hyperideals) over L .

Proof:

Definition of (f, X) and (g, Y) are two fuzzy soft sets. Then $(f, X) (g, Y)$ is defined as $(h, X \times Y)$, where $h(x, y) = f_x \cap g_y$, for all $(x, y) \in X \times Y$. we denote $(f, X) (g, Y) = (h, X \times Y)$.

We know that for all $x \in X, y \in Y, f_x$ and g_y are fuzzy \mathbf{V} -hyperideals of L and so is $h(x, y) = f_x \cap g_y$, for $(x, y) \in X \times Y$, because intersection of two fuzzy \mathbf{V} -hyperideals is also a fuzzy \mathbf{V} -hyperideal.

Therefore, $(h, X \times Y) = (f, X) (g, Y)$ is a fuzzy soft \mathbf{V} -hyperideals over L .

Similarly, we can prove that $(h, X \times Y) = (f, X) (g, Y)$ is a fuzzy soft \wedge -hyperideal over L .

Proposition 1.4:

Let (f, X) and (g, Y) be two fuzzy soft \mathbf{V} -hyperideals (\wedge -hyperideals) over the join hyperlattice (L, \wedge, \mathbf{V}) . If for all $x \in X$ and $y \in Y, f_x \subseteq g_y$ (or) $g_y \subseteq f_x$, then $(f, X) (g, Y)$ is a fuzzy soft \mathbf{V} -hyperideals (\wedge -hyperideals) over L .

Proof:

If (f, X) and (g, Y) are two fuzzy soft sets. Then $(f, X) \tilde{\cup} (g, Y)$ is defined as $(\tilde{O}, X \times Y)$, where $\tilde{O}(x, y) = f_x \cup g_y$, for all $(x, y) \in X \times Y$. we can write $(\tilde{O}, Z) = (f, X) \tilde{\cup} (g, Y)$, where $Z = X \times Y$, for all $(x, y) \in Z$, we have $\tilde{O}(x, y) = f_x \cup g_y$.

By hypothesis, for all $(x, y) \in Z, f_x \subseteq g_y$ (or) $g_y \subseteq f_x$.

Now, we assume that $f_x \subseteq g_y$ for any $a, b \in L$ and $c \in a \wedge b$, we have $\tilde{O}(x, y)(c) = f_x(c) \cup g_y(c) = g_y(c) \geq g_y(a) \cap g_y(b)$

$= (f_x(a) \cup g_y(a)) \cap (f_x(b) \cup g_y(b)) = \tilde{O}(x, y)(a) \cap \tilde{O}(x, y)(b)$. Then we obtain $\bigcap_{c \in a \wedge b} \tilde{O}(x, y)(c) \geq \tilde{O}(x, y)(a) \cap \tilde{O}(x, y)(b)$.

On the other hand, for all $c \in a \mathbf{V} b$, we have $\tilde{O}(x, y)(c) = f_x(c) \cup g_y(c) = g_y(c) \geq g_y(a) \cup g_y(b) = (f_x(a) \cup g_y(a)) \cup (f_x(b) \cup g_y(b)) = \tilde{O}(x, y)(a) \cup \tilde{O}(x, y)(b)$. Hence, $\bigcap_{c \in a \mathbf{V} b} \tilde{O}(x, y)(c) \geq \tilde{O}(x, y)(a) \cup \tilde{O}(x, y)(b)$. Therefore, $(f, X) \tilde{\cup} (g, Y)$ is a fuzzy soft \mathbf{V} -hyperideal over L .

The case for \wedge -hyperideals can be similarly proved.

Definition 1.2. Let (f, X) be a fuzzy soft set over L . The soft $(f, X)_t = \{(f_x)_{(t)} : x \in X\}$ for all $t \in [0, 1)$, are called the t -level soft set and strong t -level soft set of the fuzzy soft set (f, X) respectively, where $(f_x)_t$ and $(f_x)_{(t)}$ are the t -level set and strong t -level set of the fuzzy set f_x , respectively.

Theorem 1.1. Let (f, X) be a fuzzy soft set over the join hyperlattice (L, \wedge, \mathbf{V}) . Then (f, X) is a fuzzy soft \mathbf{V} -hyperideals (\wedge -hyperideals) over L if and only if for all $x \in X$ and $t \in [0, 1]$ with $(f_x)_t \neq \emptyset$, the t -level soft set $(f, X)_t$ is a soft \mathbf{V} -hyperideals (\wedge -hyperideals) over L .

Proof:

Let (f, X) be a fuzzy soft \mathbf{V} -hyperideals over the join hyperlattice (L, \wedge, \vee) . Then for all $x \in X$, f_x is a fuzzy \mathbf{V} -hyperideal of L . For $t \in [0,1]$ with $(f_x)_t \neq \phi$,

Let $a, b \in (f_x)_t$, then $f_x(a) \geq t$ and $f_x(b) \geq t$. Hence, for all $c \in a \wedge b$, we have $f_x(c) \geq f_x(a) \wedge f_x(b) \geq t \wedge t = t$, that is, $c \in (f_x)_t$, which implies $a \wedge b \in (f_x)_t$.

On the other hand, let $y \in L$, for all $c \in y \vee a$, we have $f_x(c) \geq f_x(y) \wedge f_x(a) \geq t$, that is $c \in (f_x)_t$, which implies $y \vee a \subseteq (f_x)_t$, then we obtain that $(f_x)_t$ is a \mathbf{V} -hyperideal of L , for all $x \in X$. Therefore, $(f, X)_t$ is a soft \mathbf{V} -hyperideal over L . For all $x \in X$. Let $\alpha = f_x(a) \wedge f_x(b)$, then we have $f_x(a) \geq \alpha$, $f_x(b) \geq \alpha$, which implies $a, b \in (f_x)_\alpha$. Since $(f_x)_\alpha$ is a \mathbf{V} -hyperideal of L , then $a \wedge b \in (f_x)_\alpha$. Hence, for all $c \in a \wedge b$, we have $c \in (f_x)_\alpha$. Thus, we can obtain $f_x(c) \geq \alpha = f_x(a) \wedge f_x(b)$, which implies $\bigcap_{c \in a \wedge b} f_x(c) \geq f_x(a) \wedge f_x(b)$.

On the other hand, let $\beta = f_x(a)$, then we have $f_x(a) \geq \beta$, that is, $a \in (f_x)_\beta$, and $\beta \in [0,1]$. Then for all $y \in L$, $y \vee a \subseteq (f_x)_\beta$. Hence for all $c \in y \vee a$, we have $c \in (f_x)_\beta$. Thus, we have $f_x(c) \geq \beta = f_x(a)$. Similarly, $f_x(c) \geq f_x(y)$, which implies $\bigcap_{c \in y \vee a} f_x(c) \geq f_x(y) \wedge f_x(a)$. Therefore, (f, X) is a fuzzy soft \mathbf{V} -hyperideals over L .

The case for \wedge -hyperideals can be similarly proved.

Theorem 1.2.

Let (f, X) be a fuzzy soft set over the join hyperlattice (L, \wedge, \vee) . Then (f, X) is a fuzzy soft \mathbf{V} -hyperideal (\wedge -hyperideal) over L if and only if for all $x \in X$ and $t \in [0,1]$ with $(f_x)_{(t)}$ is a soft \mathbf{V} -hyperideal (\wedge -hyperideal) over L .

Proof:

\Leftarrow Now, assume that (f, X) is not a fuzzy soft \mathbf{V} -hyperideal over L . Then there exists $x \in X$ such f_x is not a fuzzy \mathbf{V} -hyperideal of L . That is, there exists $a_0, b_0 \in L$, such that $\bigcap_{c \in a_0 \wedge b_0} f_x(c) < f_x(a_0) \wedge f_x(b_0)$ or $\bigcap_{c \in a_0 \vee b_0} f_x(c) < f_x(a_0) \vee f_x(b_0)$

Now, we consider the following cases

- (i) If $\bigcap_{c \in a_0 \wedge b_0} f_x(c) < f_x(a_0) \wedge f_x(b_0)$, Let $t = \bigcap_{c \in a_0 \wedge b_0} f_x(c)$. Then there exists $c_0 \in a_0 \wedge b_0$ such that $f_x(c_0) = t$. Hence $t = f_x(c_0) < f_x(a_0) \wedge f_x(b_0)$. Then we get $a_0, b_0 \in (f_x)_{(t)}$, but $c_0 \notin (f_x)_{(t)}$. Thus, we obtain $a_0 \wedge b_0 \notin (f_x)_{(t)}$.
- (ii) If $\bigcap_{c \in a_0 \vee b_0} f_x(c) < f_x(a_0) \vee f_x(b_0)$, let $t = \bigcap_{c \in a_0 \vee b_0} f_x(c)$. Then there exists $c_0 \in a_0 \vee b_0$ such that $f_x(c_0) = t$. Hence $t = f_x(c_0) < f_x(a_0) \vee f_x(b_0)$. Then we have $t \in [0,1]$ and $f_x(a_0) > t$ or $f_x(b_0) > t$, that is, $a_0 \in (f_x)_{(t)}$ or $b_0 \in (f_x)_{(t)}$, but $c_0 \notin (f_x)_{(t)}$. Thus, we also obtain $a_0 \vee b_0 \notin (f_x)_{(t)}$.

Thus, results in case (i) and (ii) contradict the fact that $(f, X)_{(t)}$ is a soft \mathbf{V} -hyperideal over L . Therefore, (f, X) is a fuzzy soft \mathbf{V} -hyperideal over L .

The case for \wedge -hyperideals can be similarly proved.

Definition 1.3.

Let (f, X) and (g, Y) be two fuzzy soft sets over L_1 and L_2 respectively and Let (ϕ, ψ) be a fuzzy soft function from L_1 to L_2 .

- (1) The image of (f, X) under (ϕ, ψ) , denoted by $(\phi, \psi)(f, X)$, is a fuzzy soft over L_2 defined by $(\phi, \psi)(f, X) = (\phi(f), \psi(X))$, where $\phi(f)_k(b) = \begin{cases} \bigcup_{\phi(a)=b} \bigcup_{\psi(x)=k} f_x(a), & \text{if } a \in \phi^{-1}(b) \text{ for all } k \in \psi(x) \text{ } b \in L_2 \\ 0 & \text{otherwise} \end{cases}$
- (2) The pre-image of (g, Y) under (ϕ, ψ) , denoted by $(\phi, \psi)^{-1}(g, Y)$, is a fuzzy soft set over L_1 , defined by $(\phi, \psi)^{-1}(g, Y) = (\phi^{-1}(g), \psi^{-1}(Y))$, where $\phi^{-1}(g)_x(a) = g_{\psi(x)}(\psi(a))$, for all $x \in \psi^{-1}(Y), a \in L_1$.

Definition 1.4

Let (f, X) and (g, Y) be two fuzzy soft sets over the join hyperlattice L_1 and the join hyperlattice L_2 , respectively. Let (φ, ψ) be a fuzzy soft function from L_1 to L_2 . If φ is a homomorphism from L_1 to L_2 , then (φ, ψ) is said to be a fuzzy soft homomorphism from L_1 to L_2 .

Theorem 1.3.

Let (φ, ψ) be a fuzzy soft homomorphism from the join hyperlattice (L_1, Λ_1, V_1) to the join hyperlattice (L_2, Λ_2, V_2) . If (g, Y) is a fuzzy soft V_2 -hyperideal (Λ_2 -hyperideal) over L_2 , then $(\varphi, \psi)^{-1}(g, Y)$ is a fuzzy soft V_1 -hyperideal (Λ_1 -hyperideal) over L_1 .

Proof:

Let $x \in \psi^{-1}(Y)$ and $a_1, a_2 \in L_1, c \in a_1 \Lambda_1 a_2$. Suppose that $\varphi(a_1) = b_1$ and $\varphi(a_2) = b_2$. Since (g, Y) is a fuzzy soft V_2 -hyperideal over L_2 , we have $\varphi^{-1}(g)_x(a_1) \cap \varphi^{-1}(g)_x(a_2) = g_{\psi(x)}(\varphi(a_1)) \cap g_{\psi(x)}(\varphi(a_2)) = g_{\psi(x)}(b_1) \cap g_{\psi(x)}(b_2) \leq g_{\psi(x)}(t)$, for all $t \in b_1 \Lambda_2 b_2 = \varphi(a_1 \Lambda_1 a_2)$. Hence, for all $c \in a_1 \Lambda_1 a_2, \varphi^{-1}(g)_x(a_1) \cap \varphi^{-1}(g)_x(a_2) \leq g_{\psi(x)}(\varphi(c)) = \varphi^{-1}(g)_x(c)$, that is,

$$\bigcap_{c \in a_1 \Lambda_1 a_2} \varphi^{-1}(g)_x(c) \geq \varphi^{-1}(g)_x(a_1) \cap \varphi^{-1}(g)_x(a_2)$$

Similarly, we obtain

$$\bigcap_{c \in a_1 V_1 a_2} \varphi^{-1}(g)_x(c) \geq \varphi^{-1}(g)_x(a_1) \cup \varphi^{-1}(g)_x(a_2)$$

Therefore, $(\varphi, \psi)^{-1}(g, Y)$ is a fuzzy soft V_1 -hyperideal over L_1 .

Similarly, we can prove that $(\varphi, \psi)^{-1}(g, Y)$ is a fuzzy soft Λ_1 -hyperideal over L_1 .

CONCLUSION

Hence, we have successfully introduced the fuzzy join hyperlattice. And we investigated some of their properties.

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