

Comparison of Various Methods for Solving Linearly Programmed Transportation Problem

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Abstract: The transportation problem is a particular case of linear programming problems in operations research. It is used to determine the minimum cost for transporting a homogenous commodity from 'm' sources to 'n' destinations. Transportation models are extensively used in industries to simulate real-life problems and hence reduce the overall cost by minimizing transportation costs. In this paper, three methods of obtaining an initial basic feasible solution (IBFS) viz. the Least-cost method, the North-west corner method, and Vogel's approximation method, are tested and compared.

Keywords – Transportation problem, Initial Basic Feasible Solution (IBFS), Allocation, Supply, Demand

I. Introduction

In operations research, mathematical programming models consisting of linear programming, non-linear programming, stochastic programming, integer programming, and dynamic programming are used for optimization.

The transportation problem is one of the sub-classes of linear programming problems in which the objective is to transport various quantities of a single homogeneous commodity, that are initially stored at various origins to different destinations in such a way that the total transportation cost is the minimum. Because of its unique structure, the usual simplex method is not suitable for solving transportation problems. These problems require a particular way of solution. The origin of a transportation problem is the location from which shipments are dispatched. The destination of a transportation problem is the location to which deliveries are transported. The unit transportation cost is the cost of transporting one unit of the consignment from an origin to a destination.

In the most general form, a transportation problem has several origins, having a certain amount of a commodity, and several destinations, requiring a certain amount of that commodity. The transportation solution needs to determine the minimum transportation costs without violating the availability and requirement constraints. The decision variables x_{ij} of a transportation problem indicate the amount to be transported from the i^{th} origin to the j^{th} destination. Two subscripts are necessary to describe these decision variables. A transportation problem can be formulated as a linear programming problem using decision variables with two indexes.

Let s_i is the number of supply units required at source i ($i=1, 2, 3... m$), d_j is the number of demand units required at destination j ($j=1, 2, 3...n$) and c_{ij} represent the unit transportation cost for transporting the units from sources i to destination j .

The objective function

$$\text{Minimize } Z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} \cdot x_{ij}$$

Subjected to

$$\sum_{j=1}^n x_{ij} = s_i \text{ for } i=1, 2, 3...m$$

$$\sum_{i=1}^m x_{ij} = d_j \text{ for } j=1, 2, 3...n$$

And $x_{ij} \geq 0$ for all i to j .

A transportation problem is said to be balanced if the total supply from all the sources is equal to the total demand in all the destinations.

$$\sum_{i=1}^m s_i = \sum_{j=1}^n d_j$$

Otherwise, it is said to be unbalanced.

To balance an unbalanced problem, a dummy origin or destination with zero cost is introduced. If the total supply is more than the total demand, we introduce an additional column, which will indicate the surplus supply with transportation cost zero. Likewise, if the overall demand is more than the total supply, an extra row is introduced in the Table, which represents unsatisfied demand with transportation cost zero.[1], [2], [3]

II. Finding Initial Basic Feasible Solution

In general, any basic feasible solution of a transportation problem with 'm' origins (such as factories) and 'n' destinations (such as warehouses) should have ' $m + n - 1$ ' non-zero basic variables.

A transportation problem is said to be a degenerate transport problem if it has a basic feasible solution with several non-zero basic variables less than $m + n - 1$.

According to Must fit, "A degenerate basic feasible solution in a transportation problem exists if and only if some partial sum of availabilities (row) is equal to a partial sum of requirements (column)."[1]

Steps involved in finding an initial basic feasible solution

- i. Balancing the given problem, i.e., checking whether the sum of availability of constraints is equal to the sum of requirement constraints. If they are equal, proceed to step II. If not, introduce a dummy row or column (with cost coefficients equal to zero) as required.
- ii. Any of the following three methods can be appointed to obtain the initial basic feasible solution.
 - a) Least cost method or Matrix-minima method (or MM).
 - b) North-west corner method (or NWCM).
 - c) Vogel's Approximation Method (or VAM).

A. Least-cost or Matrix Minima Method

- I. Make the maximum possible allocations to the cell containing the minimum cost. If there are multiple cells with the same least cost value, then select the cell by which maximum demand can be exhausted.
- II. Make maximum possible allocations to the second-lowest-cost cell depending upon the remaining demand/supply for the row/column containing that cell.
- III. Repeat the above step until all the requirements are exhausted.
- IV. Now, the objective function can be used to determine the initial basic feasible solution.

B. North-west Corner Method

- I. Make maximum allocation to the upper left cornered cell (also known as the North-west corner cell) depending upon the demand and supply corresponding to that cell.
- II. Move to the next cell of the first row depending upon the remaining supply for that row and the demand requirement for the next column. Go on until the row total is exhausted.
- III. Move to the next row, allocate the cell below the cell of the preceding row in which the last allocation was made, and follow Steps I and II.
- IV. Repeat steps I to III until all requirements are exhausted.
- V. Utilize the objective function to obtain the initial basic feasible solution.

C. Vogel's Approximation Method[4]

- I. Find row penalties, i.e., the difference between the first minimum and second minimum in each row. If the minimum values occur twice, then the row penalty is zero.
- II. Find column penalties, i.e., the difference between the first minimum and second minimum in each column. If the minimum values occur twice, then the column penalty is zero.
- III. Find the minimum amongst the row penalties and column penalties, and identify whether it occurs in a row or in a column (break tie randomly). If the maximum penalty is in a row, go to step IV; otherwise, go to step VII.
- IV. Identify the cell for allocation that has the least cost in that row.
- V. Find the minimum of the supply and the demand values with respect to the selected cell.
- VI. Allocate this minimum value to that cell and subtract this minimum from the supply and demand values with respect to the selected cell and go to step X.
- VII. Identify the cell for allocation that has the least cost in that column.
- VIII. Find the minimum of the supply and the demand values with respect to the selected cell.
- IX. Allocate this minimum value to the selected cell and subtract this minimum from the supply and demand values with respect to the selected cell.
- X. Check whether precisely one of the rows and the columns corresponding to the selected cell has zero supply/zero demand, respectively. If yes, go to step XI; otherwise, go to step XII.
- XI. Delete the row/column, which has the zero supply/demand, and revise the corresponding row/column penalties. Then go to step XIII.
- XII. Delete both the row and the column with respect to the selected cell. Then revise the row and column penalties.
- XIII. Check whether exactly one row/column is left out. If yes, go to step XIV; otherwise, go to step III.
- XIV. Match the supply/demand of the left-out row/column with the remaining demands/supplies of the undeleted columns/rows.
- XV. Obtain an initial basic feasible solution using the objective function.

III. Illustrative Example.

Tadah Tractors ships its tractors from its three manufacturing units situated in Mumbai (M), Chennai (C) and Kochi (K) to its two distributors in Hyderabad (H) and Bengaluru (B). The monthly supply and demand of the tractors, along with their unit transportation cost (INR) is summarized in the transportation model in table 1.[5]

	Hyderabad	Bengaluru	Supply
Mumbai	16500	10600	48
Chennai	12200	12600	35
Kochi	10300	9240	22
Demand	48	57	

Table 1: Transportation Model for Example (Tadah Tractors)

First, we find whether the model is balanced or not. For that, we calculate the sum of all the supplies and the sum of all the demands.

$$\sum s_i = 48 + 35 + 22 = 102$$

$$\sum d_j = 48 + 57 = 102$$

Since, $\sum s_i = \sum d_j$, the model is balanced and we can proceed to finding the initial basic feasible solution.

iii. Least Cost or Matrix Minima Method

- a. Cell (2,3) has the minimum cost (=9240), the maximum number of tractors that can be shipped through (2,3) is $X_{ij} = \min.(22,57) = 22$. This leads to satisfying row 3; thus, we arbitrarily cross out row 3 and adjust the demand.

	Hyderabad	Bengaluru	Supply
Mumbai	16500	10600	48
Chennai	12200	12600	35
Kochi	10300	9240	22
Demand	48	57	35

Table 2: MM –Allocating Cell (2,3)

- b. Next, cell (1,2) has the least cost (=10600) and here $X_{ij} = \min.(48,35) = 35$. This satisfies column 2; thus, we arbitrarily cross it out and adjust the supply.

	Hyderabad	Bengaluru	Supply
Mumbai	16500	10600	35
Chennai	12200	12600	35
Kochi	10300	9240	22
Demand	48	35	0

Table 3: MM –Allocating Cell (1,2)

- c. Now, the least cost (=12200) is in cell (2,1) with $X_{ij} = \min.(35,48) = 35$. Thus, we allocate 35 tractors to cell (2,1) which satisfies row 2.

	Hyderabad	Bengaluru	Supply
Mumbai	16500	10600	35
Chennai	12200	12600	35
Kochi	10300	9240	22
Demand	48	13	0

Table 4: MM –Allocating Cell (2,1)

- d. Next, the cell (1,1) has the minimum cost (=16500). The row and column are satisfied since the demand and supply across this cell are equal. This results in the exhaustion of all the supplies and demands.

	Hyderabad		Bengaluru		Supply
Mumbai	16500	13	10600	35	48 0
Chennai	12200	35	12600		0
Kochi	10300		9240	22	0
Demand	48 0		0		

Table 5: MM – Allocating Cell (1,1)

- e. The initial basic feasible solution will be obtained as

$$\begin{aligned} \text{Minimize } Z &= \sum_{i=1}^m \sum_{j=1}^n C_{ij} \cdot x_{ij} \\ &= (9240 \times 22) + (10600 \times 35) + (12200 \times 35) + (16500 \times 13) \\ &= \text{INR } 1215780/- \end{aligned}$$

	Hyderabad		Bengaluru		Supply
Mumbai	16500	13	10600	35	0
Chennai	12200	35	12600		0
Kochi	10300		9240	22	0
Demand	0		0		

Table 6: Least Cost Method – IBFS

iv. North-west Corner Method

- a. We start from cell (1,1) having cost (=16500). Here $x_{ij} = \min.(48,48) = 48$. This satisfies both the row and the column corresponding to the cell.

	Hyderabad		Bengaluru		Supply
Mumbai	16500	48	10600		48 0
Chennai	12200		12600		35
Kochi	10300		9240		22
Demand	48 0		57		

Table 7: NWCM - Allocating Cell (1,1)

- b. Since row 1 and column 1 have been eliminated, we now move to cell (2,2), which is at the north-west corner position. The cell having cost (=12600) has $x_{ij} = \min.(35,57) = 35$. Thus, we allocate 35 tractors to the cell and balance the remaining demand.

	Hyderabad		Bengaluru		Supply
Mumbai	16500	48	10600		0
Chennai	12200		12600	35	35 0
Kochi	10300		9240		22
Demand	0		57 22		

Table 8: NWCM - Allocating Cell (2,2)

- c. Now, only cell (3,2) is left. We allocate the remaining supply to the corresponding demand to satisfy them.

	Hyderabad		Bengaluru		Supply
Mumbai	16500	48	10600		0
Chennai	12200		12600	35	0
Kochi	10300		9240	22	22 0
Demand	0		22 0		

Table 9: NWCM - Allocating Cell (3,2)

- d. The initial basic feasible solution will be obtained as

$$\text{Minimize } Z = \sum_{i=1}^m \sum_{j=1}^n C_{ij} \cdot x_{ij}$$

$$= (16500 \times 48) + (12600 \times 35) + (9240 \times 22)$$

$$= \text{INR } 1436280/-$$

	Hyderabad		Bengaluru		Supply
Mumbai	16500	48	10600		0
Chennai	12200		12600	35	0
Kochi	10300		9240	22	0
Demand	0		0		

Table 10: North-west Corner Method – IBFS

v. **Vogel's Approximation Method**

- a. First, we allot penalties to the rows and columns. Now the maximum penalty (=5900) is corresponding to row 1. We allocate cell (1,2) having cost (=10600) with $x_{ij} = \min.(48,57) = 47$ supplies, satisfying row 1 and adjusting the demand for column 2.

	Hyderabad	Bengaluru		Supply	Penalties
Mumbai	16500	10600	48	48 0	5900
Chennai	12200	12600		35	400
Kochi	10300	9240		22	1060
Demand	48	57 9			
Penalties	1900	1360			

Table 11: VAM - Allocating Cell (1,2)

- b. We continue to select maximum penalties and allocating the supplies for the corresponding demands, as illustrated in the models below.

	Hyderabad	Bengaluru		Supply	Penalties
Mumbai	16500	10600	48	0	5900
Chennai	12200	12600		35	400
Kochi	10300	9240	9	22 13	1060
Demand	48	9 0			
Penalties	1900	1360			
	1900	3360			

Table 12: VAM - Allocating Cell (3,2)

	Hyderabad		Bengaluru		Supply	Penalties
Mumbai	16500		10600	48	0	5900
Chennai	12200	35	12600		35 0	400
Kochi	10300		9240	9	13	1060
Demand	48 13		0			
Penalties	1900		1360			
	1900		3360			

Table 13: VAM - Allocating Cell (2,1)

	Hyderabad		Bengaluru		Supply	Penalties
Mumbai	16500		10600	48	0	5900
Chennai	12200	35	12600		0	400
Kochi	10300	13	9240	9	13 0	1060
Demand	13 0		0			
Penalties	1900		1360			
	1900		3360			

Table 14: VAM - Allocating Cell (1,3)

- c. The initial basic feasible solution will be obtained as

$$\begin{aligned} \text{Minimize } Z &= \sum_{i=1}^m \sum_{j=1}^n C_{ij} \cdot X_{ij} \\ &= (10600 \times 48) + (9240 \times 9) + (12200 \times 35) + (10300 \times 13) \\ &= \text{INR } 1152860/- \end{aligned}$$

	Hyderabad		Bengaluru		Supply	Penalties
Mumbai	16500		10600	48	0	5900
Chennai	12200	35	12600		0	400
Kochi	10300	13	9240	9	0	1060
Demand	0		0			
Penalties	1900		1360			
	1900		3360			

Table 15 Vogel's Approximation Method – IBFS

IV. Results

It can be seen that the North-west Corner Method is used when demand number 1 (column 1) and supply number 1 (row 1) is to be completed first, followed by the similar allocations depending upon the remaining rows and columns. NWCM generates a quicker solution as computations take a shorter time, but it yields a bad solution as the basic solution is very far from the optimal solution.

On the other hand, the Least-cost Method and Vogel's Approximation Method are used to obtain shorter paths for the initial basic feasible solution. These methods, although requiring longer computations, yield solutions that are closer to the optimal solution.

As can be observed, the IBFS for Vogel's Approximate Method (=1152860) is less than those obtained from North-west Corner Method (=1436280) and Least-cost Method (=1215780).

V. Conclusion

It can be concluded that if the decision-maker requires a quicker initial basic feasible solution, then the North-west Corner Method should be appointed. However, if a solution that is closer to the optimal solution is needed, then either the Least-cost Method or the Vogel's Approximate Method should be used.

Also, it should be noted that while applying VAM care should be taken as this method deals with a more significant number of calculations (corrections), increasing the possibility of error.

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