Odd Fibonacci Mean Labeling of Some Special Graphs

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Abstract: Let G be a graph with p vertices and q edges and an injective function f:V(G)→{1,3,5,13,21,

55,89….} where each f_i is a Odd Fibonacci number and the induced edge labeling f: E(G)→ N are defined by

$$f(uv) = \begin{cases} \dfrac{f(u) + f(v)}{2}, & \text{if } f(u) + f(v) \text{ is even} \\ \dfrac{f(u) + f(v) + 1}{2}, & \text{if } f(u) + f(v) \text{ is odd} \end{cases}$$

and all these edge labeling are distinct is called a odd Fibonacci Mean Labeling, A graph which admits a Odd Fibonacci Mean Labeling is called a odd Fibonacci mean graph.

Keywords: Odd Fibonacci Mean Labeling, Wheel graph, Helm graph, Gear graph, Cycle graph, Banana tree, Tadpole graph, Fire Cracker graph, Pan graph, Star graph, Ladder graph, Cone graph, Haar graph, Dutch Windmill graph.

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I. Introduction

In this paper we consider a graph G by finite connected, undirected and simple graph with p vertices and q edges. For various graph, Theoretic notations and terminology we follow D. B. West [8].

The concept of mean labeling was introduced by Somasundaram and Ponraj [4].

In this paper we introduced a new concept of odd Fibonacci mean labeling we have derived different graph families are satisfying the conditions of Odd Fibonacci Mean Labeling.

Definition1.1

The Fibonacci numbers can be defined by the linear recurrence $F_n = F_{n-1} + F_{n-2}$; $n\geq3$. This generates the infinite sequence of integers beginning 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233………..

Definition 1.2

A graph G with p vertices and q edges is a mean graph if there is an injective function of from the vertices of G to {0, 1, 2….q} such that when each edge uv is labeled with

$\frac{f(u) + f(v)}{2}$ if f(u) + f(v) is even $\frac{f(u) + f(v) + 1}{2}$ is odd then the resulting edge labels are distinct. The

admits a mean labeling is called mean graph.

Definition 1.3

The Wheel graph W_n , n≥3 is join of the graphs C_n & K_1 $W_n = C_n + K_1$. Here vertices corresponding to C_n are called rim vertices & C_n is called rim of C_n are called rim vertices and C_n is called rim of W_n while the vertex corresponds to K_1 is called apex vertex.

Definition 1.4

A Helm graph H_n, n≥3 is the graph obtained from the wheel W_n by adding a pendant edge at each vertex on the wheel's rim.

Definition 1.5

A Gear graph is obtained from the wheel by adding a vertex between every pair of adjacent vertices of the cycle. The gear graph G_n has 2n+1 vertices and 3n edges.

Definition 1.6

Cycle is a closed walk in which all the vertices and edges are distinct except u= v. A non-trivial closed path is a cycle and a cycle is also called a circuit.

Definition 1.7

A Banana tree, as defined is a graph obtained by connecting one leaf of each of copies of a k-star graph with a single root vertex that is distinct from all the stars.

Definition 1.8

The (m, n)-tadpole graph is a graph is a special type of graph consisting of a graph on m (at least 3) vertices and a path graph on n. vertices connected with a bridge.

Definition 1.9

An Firecracker is a graph obtained by the concatenation of stars by linking one leaf.

Definition 1.10

The pan graph is the graph obtained by joining a cycle graph to a singleton graph with a bridge. The pan graph is therefore isomorphic with the tadpole graph. Pre computed properties of pan graphs are available.

Definition 1.11

A star graph with n vertices is a tree with one vertex having degree n-1 and other n-1 vertices having degree 1. A star graph with n + 1 vertices $K_{1,n}$.

Definition 1.12

The ladder graph L_n is a planar undirected graph with 2n vertices and n+2(n-1) edges (or) add the edges between corresponding vertices of two path of equal length.

Definition 1.13

An-gonal-cone graph also called the-point suspension of, is defined by the graph and is an empty graph. The case corresponds to the usual cone graph to the double cone etc. The-cone graph is isomorphic to the complete tripartite graph.

Definition 1.14

A haar graph is a bipartite regular vertex-transitive graph indexed by a positive integer and obtained by a simple binary encoding of cyclically adjacent vertices. Haar graph may be connected or disconnected. In general the graph union of copies of the cycle graph has haar index.

Definition 1.15

The Dutch windmill graph also called a friendship graph is the graph obtained by taking copies of the cycle graph with a vertex in common and therefore corresponds to the usual windmill graph.

II. Main Results

Definition 2.1

Let G be a graph with p vertices and q edges. An injective function $f : V(G) \rightarrow \{1,3,5,13,21, 55..\,.\}$

Where each f_i is a odd Fibonacci number and the induced edge labeling $f^* : E(G) \rightarrow N$ defined by

$$f^*(e = uv) = \begin{cases} \dfrac{f(u) + f(v)}{2} & \text{if } f(u) + f(v) \text{ is even} \\[2ex] \dfrac{f(u) + f(v) + 1}{2} & \text{if } f(u) + f(v) \text{ is odd} \end{cases}$$

and all the edge labeling are distinct.

Theorem 2.2:

The Wheel graph W_n admits odd Fibonacci mean labeling.

Proof:

Let $G = W_n$ be a wheel graph.

Here $|V| = n$, $|E| = n-1$.

Let the function $f:V(G) \rightarrow \{1,3,5,13,21,55\ldots..\}$ where each f_i is a odd Fibonacci number.

$f(u_1) = 2i-1; i=1$

$f(u_2) = 2i-1; i=2$

$f(u_3) = 2i-1; i=3$

$f(u_4) = 3i+1; i=4$

$f(u_5) = 4i+1; i=5$

$f(u_6) = 9i+1; i=6$ and so on.

Such that the induced edge labeling using the condition of

$$f^*(e = uv) = \begin{cases} \dfrac{f(u) + f(v)}{2}, & \text{if } f(u) + f(v) \text{ is even} \\ \dfrac{f(u) + f(v) + 1}{2}, & \text{if } f(u) + f(v) \text{ is odd} \end{cases}$$

Here all the edges labeling are distinct.

Hence we proved a Wheel graph W_n admits odd Fibonacci mean labeling.

Theorem 2.3:

A Helm graph H_n admits odd Fibonacci mean labeling.

Proof:

Let $G = H_n$ be a helm graph.

Here $|V| = n$, $|E| = n-1$.

Let the function $f:V(G)\rightarrow\{1,3,5,13,21,55,89,233,377\ldots.\}$ where each f_i is a odd Fibonacci number.

Here the vertices are assigned by the labeling.

$f(u_1) = 2i-1; i = 1$

$f(u_2) = 2i-1; i=2$

$f(u_3) = 2i-1; i=3$

$f(u_4) = 3i+1; i=4$

$f(u_5) = 4i+1; i=5$

$f(u_6) = 9i+1; i=6$

$f(u_7) = 12i+5; i=7$

$f(u_8) = 29i+1; i=8$

$f(u_9) = 42i-1; i=9$ and so on.

Such that the induced edge labeling using the condition of

$$f^*(e = uv) = \begin{cases} \dfrac{f(u) + f(v)}{2}, & \text{if } f(u) + f(v) \text{ is even} \\ \dfrac{f(u) + f(v)+1}{2}, & \text{if } f(u) + f(v) \text{ is odd} \end{cases}$$

Hence all the edges labeling are distinct.

Hence proved a Helm graph H_n admits odd Fibonacci mean labeling.

Theorem 2.4:

A Gear graph G_n admits odd Fibonacci mean labeling.

Proof:

Let $G = G_n$ be a gear graph.

Here $|V| = n$, $|E| = n-1$.

Let the function $f:V(G) \rightarrow \{1,3,5,13,21,55,89\ldots\ldots\}$ where each f_i is a odd Fibonacci number.

$f(u_1) = 2i-1; i = 1$

$f(u_2) = 2i-1; i=2$

$f(u_3) = 2i-1; i=3$

$f(u_4) = 3i+1; i=4$

$f(u_5) = 4i+1; i=5$

$f(u_6) = 9i+1; i=6$

$f(u_7) = 12i+5; i=7$ and so on.

Such that the induced edge labeling using the condition of

$$\frac{f(u) + f(v)}{2} \quad \text{if } f(u) + f(v) \text{ is even}$$

$$\frac{f(u) + f(v) + 1}{2} \quad \text{if } f(u) + f(v) \text{ is odd}$$

then all the edge labeling are distinct.

Thus we proved a Gear graph G_n admits odd Fibonacci mean labeling.

Theorem 2.5:

The Cycle C_n, n≥3 and n is odd then admits odd Fibonacci mean labeling.

Proof:

Let G = C_n be a cycle graph.

Here |V| = n, |E| = n-1.

Let the function f :V(G)→{1, 3, 5, 13, 21, 55…}where each f_i is a odd Fibonacci mean labeling and the assignment of vertex labeling are.

$f(u_1) = 2i-1; i=1$

$f(u_2) = 2i-1; i=2$

$f(u_3) = 2i-1; i=3$

$f(u_4) = 3i+1; i=4$

$f(u_5) = 4i+1; i=5$

$f(u_6) = 9i+1; i=6$ and so on.

Here all the edge labeling are distinct.

Hence proved a Cycle C_n admits odd Fibonacci mean labeling.

Theorem 2.6

A Banana tree admits odd Fibonacci mean labeling.

Proof:

Let G be a banana tree

Here |V| = n, |E| = n-1.

Let the function f:V(G)→{1,3,5,13,21,55,89,233,377……} where each f_i is a odd Fibonacci number.

$f(u_1) = 2i-1; i=1$

$f(u_2) = 2i-1; i=2$

$f(u_3) = 2i-1; i=3$

$f(u_4) = 2i-1; i=4$

$f(u_5) = 4i+1; i=5$

$f(u_6) = 9i+1; i=6$

$f(u_7) = 12i+5; i=7$

$f(u_8) = 29i+1; i=8$

$f(u_9) = 42i\text{-}1;i=9$ and so on.

Such that the induced edge labeling using the condition of

$$f^*(e = uv) = \begin{cases} \dfrac{f(u) + f(v)}{2} & \text{if } f(u) + f(v) \text{ is even} \\ \dfrac{f(u) + f(v) + 1}{2} & \text{if } f(u) + f(v) \text{ is odd} \end{cases}$$

Here all the edge labeling are distinct.

Thus we proved a Banana tree admits an odd Fibonacci mean labeling.

Theorem 2.7:

The (m, n)-tadpole graph admits odd Fibonacci mean labeling.

Proof:

Let G be a (m, n)-tadpole graph.

Here $|V| = n$, $|E| = n\text{-}1$.

Let the function $f : V(G) \rightarrow \{1,3,5,13,21,55\ldots.\}$ where each f_i is a odd Fibonacci number.

Here the vertices are assigned by the labeling.

$f(u_1) = 2i\text{-}1;i=1$

$f(u_2) = 2i\text{-}1;i=2$

$f(u_3) = 2i\text{-}1;i=3$

$f(u_4) = 3i\text{+}1;i=4$

$f(u_5) = 4i\text{+}1;i=5$

$f(u_6) = 9i\text{+}1;i=6$ and so on.

To find the edge labeling we use the condition of

$$f^*(e = uv) = \begin{cases} \dfrac{f(u) + f(v)}{2} & \text{if } f(u) + f(v) \text{ is even} \\ \dfrac{f(u) + f(v) + 1}{2} & \text{if } f(u) + f(v) \text{ is odd} \end{cases}$$

Then all the edge labeling are distinct.

Hence we proved the theorem.

Theorem 2.8:

An Fire Cracker graph admits odd Fibonacci mean labeling.

Proof:

Let G be a fire cracker graph.

Here |V| = n, |E| = n-1.

Let the function f :V(G)→{1,3,5,13,21,55,89,233….} where each f_i is a odd Fibonacci number and the assignment of vertex labeling are.

$f(u_1) = 2i-1; i=1$

$f(u_2) = 2i-1; i=2$

$f(u_3) = 2i-1; i=3$

$f(u_4) = 3i+1; i=4$

$f(u_5) = 4i+1; i=5$

$f(u_6) = 9i+1; i=6$

$f(u_7) = 12i+5; i=7$

$f(u_8) = 29i+1; i=8$ and so on.

Then all the edge labeling are distinct.

Thus we proved the theorem.

Theorem 2.9:

The Pan graph admits an odd Fibonacci mean labeling.

Proof:

Let G be a pan graph.

Here |V| = n, |E| = n -1.

Let the function f :V(G) → {1, 3, 5, 13, 21, 55…} Where each f_i is a odd Fibonacci number.

Here the vertices assigned by the labeling.

$f(u_1) = 2i-1; i=1$

$f(u_2) = 2i-1; i=2$

$f(u_3) = 2i-1; i=3$

$f(u_4) = 3i+1; i=4$

$f(u_5) = 4i+1; i=5$

$f(u_6) = 9i+1; i=6$ and so on.

To find the edge labeling we use the condition of

$$f^*(e = uv) = \begin{cases} \dfrac{f(u) + f(v)}{2} & \text{if } f(u) + f(v) \text{ is even} \\ \dfrac{f(u) + f(v) + 1}{2} & \text{if } f(u) + f(v) \text{ is odd} \end{cases}$$

Hence all the edge labeling are distinct.

Thus we proved the Pan graph admits odd Fibonacci mean labeling.

Theorem 2.10:

A Star graph S_n admits odd Fibonacci mean labeling.

Proof:

Let G = S_n be a star graph.

Here |V| = n-1, |E| = n-1.

Let the function f :V(G)→{1,3,5,13,21,55,89….} where each f_i is a odd Fibonacci number and the assignment of vertex labeling are.

$f(u_1) = 2i-1; i=1$

$f(u_2) = 2i-1; i=2$

$f(u_3) = 2i-1; i=3$

$f(u_4) = 3i+1; i=4$

$f(u_5) = 4i+1; i=5$

$f(u_6) = 9i+1; i=6$

$f(u_7) = 12i+5; i=7$ and so on.

Then all the edge labeling are distinct.

Hence proved a Star graph S_n admits odd Fibonacci mean labeling.

Theorem 2.11:

A Ladder graph L_n admits odd Fibonacci mean labeling.

Proof:

Let G = L_n be a ladder graph.

Here $|V| = n$, $|E| = n-1$.

Let the function f :V(G)→{1,3,5,13,21,55,89,233….} where each f_i is a odd Fibonacci number.

$f(u_1) = 2i-1; i=1$

$f(u_2) = 2i-1; i=2$

$f(u_3) = 2i-1; i=3$

$f(u_4) = 3i+1; i=4$

$f(u_5) = 4i+1; i=5$

$f(u_6) = 9i+1; i=6$

$f(u_7) = 12i+5; i=7$

$f(u_8) = 29i+1; i=8$ and so on.

Such that the induced edge labeling using the condition of

$$f^*(e = uv) = \begin{cases} \dfrac{f(u) + f(v)}{2} & \text{if } f(u) + f(v) \text{ is even} \\ \dfrac{f(u) + f(v) + 1}{2} & \text{if } f(u) + f(v) \text{ is odd} \end{cases}$$

Here all the edge labeling are distinct.

Hence proved a Ladder graph L_n admits odd Fibonacci mean labeling.

Theorem 2.12

The Cone graph admits odd Fibonacci mean labeling.

Proof:

Let G be a cone graph.

Here $|V| = n$, $|E| = n-1$.

Let the function f:V(G)→{1,3,5,13,21,55…..} Where each f_i is a odd Fibonacci number.

$f(u_1) = 2i-1; i=1$

$f(u_2) = 2i-1; i=2$

$f(u_3) = 2i-1; i=3$

$f(u_4) = 3i+1; i=4$

$f(u_5) = 4i+1; i=5$

$f(u_6) = 9i+1; i=6$ and so on.

Such that the induced edge labeling using the condition of

$\frac{f(u) + f(v)}{2}$ if f(u) + f(v) is even

$\frac{f(u) + f(v) + 1}{2}$ if f(u) + f(v) is odd

Then all the edge labeling are distinct.

Thus we proved the theorem.

Theorem 2.13

A Haar graph H(n) admits odd Fibonaaci mean labeling.

Proof:

Let G = H(n) be a haar graph.

Here |V| = n, |E| = n-1.

Let the function f:V(G)→{1,3,5,13,21,55,89,233…} Where each f_i is a odd Fibonacci number and the assignment of vertex labeling are

$f(u_1) = 2i-1; i=1$

$f(u_2) = 2i-1; i=2$

$f(u_3) = 2i-1; i=3$

$f(u_4) = 3i+1; i=4$

$f(u_5) = 4i+1; i=5$

$f(u_6) = 9i+1; i=6$

$f(u_7) = 12i+5; i=7$

$f(u_8) = 29i+1; i=8$ and so on.

Then all the edge labeling are distinct.

Hence we proved a Haar graph admits odd Fibonacci mean labeling.

Theorem 2.14

The Dutch Windmill graph admits odd Fibonacci mean labeling.

Proof:

Let G be a Dutch windmill graph.

Here |V| = n, |E| = n-1.

Let the function f:V(G)→{1,3,5,13,21,55,89…} Where each f_i is a odd Fibonacci number.

$f(u_1) = 2i-1; i=1$

$f(u_2) = 2i-1; i=2$

$f(u_3) = 2i-1; i=3$

$f(u_4) = 3i+1; i=4$

$f(u_5) = 4i+1; i=5$

$f(u_6) = 9i+1; i=6$

$f(u_7) = 12i+5; i=7$ and so on.

Such that the induced edge labeling using the condition of

$$f^*(e = uv) = \begin{cases} \frac{f(u)+f(v)}{2} & \text{if } f(u)+f(v) \text{ is even} \\ \frac{f(u)+f(v)+1}{2} & \text{if } f(u)+f(v) \text{ is odd} \end{cases}$$

Here all the edge labeling are distinct.

Hence proved the Dutch Windmill graph admits odd Fibonacci mean labeling.

III. Conclusion

In this article we have investigated Odd Fibonacci Mean Labeling of Wheel Graph, Helm Graph, Gear Graph, Cycle Graph, Banana tree, Tadpole Graph, Fire Cracker Graph, Pan Graph, Star Graph, Ladder Graph, Cone Graph, Haar Graph, Dutch Windmill Graph. Analogous work can be carried out for other families and in the context of different types of graph labeling techniques.

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