Solving the Solution to the River Flow Problem which caused due to Dumping of Plastic wastes into the Rivers by using Nonlinear Method of Characteristics

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Abstract: *We have discussed in this paper how to solve river flow problems by applying the method of characteristics including conservation of plastics. We studied a nonlinear method of characteristics to model the flow dynamics in all simulations presented here. Actually, characteristicsmethod or technique is another way to solve partial differential equations of nonlinear problem, an example of a conservation law .We introduced river flow problem as a nonlinear hyperbolic equation and deal with its solution. We consider the number of plastics particles and their floating density in a river channel by using the relationship between flow rate, density and velocity.*

Keywords: *River flow problem, Flow rate, Method of characteristics, River velocity, Plastics density.*

I. INTRODUCTION

Gaspard Monge (1746-1818) developed the method of characteristics (MOC) and it was first practically used by JuniusMassau (1889, 1990) to find the solution of partial differential equations for graphical integration of the unsteady open-channel flow equations. The method of characteristics is considered one of the best practical tools for studying a variety of unsteady flow problems (Abbott, 1966). It was excessively used for numerical solution of unsteady open-channel flow equations along with the origin of modern digital computer (Lai, 1965; Amein, 1966; Liggett and Woolhiser, 1967). It is also used to solve the continuity equation (i.e., conservation law) with initial conditions. A basic significance of partial differential equations is that of conservation laws. In general terms, conservation laws are nonlinear in nature. Therefore, we apply the method of characteristics to examine a one dimensional scalar conservation law. This method of characteristics is applicable to solve initial value problems for general first order of partial differential equations. River flow patterning is a method of characteristic system which enables the solution to flow side by side in a channel. The approach is based on the assumption that pressure of plastics and velocity briefly propagate with the speed of river flow within the channel system. In the x–t domain plane, these paths are called characteristic curves. The method of characteristics (MOC) approach is based on the consideration that all plastic wastes move on these paths, the partial differential equations are reduce to ordinary differential equations that explained the propagation along these characteristic curves . These curves define the plastic wastes distribution in both the upstream anddownstream directions of rivers which enables to calculate the unknown variables at points where these characteristic curves intersect each other. The aim of this method of characteristics (MOC) is to change coordinates from (x,t) to a new coordinates system $(x\ ,s)$ in which the partial differential equation converted to ordinary differential equation along certain curves in the x,t-plane along the $x=0$. This approach can be obtained by forming the characteristics for the equations over the region of known initial conditions and proceeding along these lines to determine the solutions for new regions.

We suggests that $q(x,t)=\rho(x,t)v(x,t)$ which is a correlation of velocity, density, and flow rate , and a simple way to prove this is to examine the number of plastics that carry away by river at $x=x<0, x>0$ in a very small time Δt, i.e., between t_0 and $t_0+\Delta t$. [Richard Haberman (1998)].

We found dumping of plastic waste throughout the marine environment, yet its global abundance and weight density of plastics have not enough data particularly from remote regions. The speed of river flow is directly depending on its density of plastic which enable us to predict the rate of river flow. However, river flow problem is a problem which changes the natural flow pattern due to various reasons. One of such factors is that dumping of plastic wastes on a river is commonly known which we cannot prevent it immediately rather we find a solution by developing mathematical model. We select a particular region of river channel where dumping of plastics blockage the river flow at interval between $x=a$ and $x=b$ which is bounded in such a way that when the velocity is at maximum, the number of plastics decreases for it is flushed away passing the region at implies $x=b$ density decreases. In contrast, river flow slows down when the plastic density increases.

We consider three variables which are velocity, density, and flow rate (flux) such that the relation between them is given as

i.e., $q=\rho v(\rho)$……………………………………………………………………………….(1)

where $q=$ flow rate, $\rho=$ density and $v=$ velocity

then, the river flow equation of conservation of plastics is given as $\frac{\partial\rho}{\partial t}+\frac{\partial q}{\partial\rho}\frac{\partial\rho}{\partial x}=0$ ……..…..(2)

must be solved using the initial condition $t=0$.

$$\rho\,(x,0)=\begin{cases}\rho_{\max}\ if\ \ x<0\\ 0\ if\ \ x>0\end{cases}$$ ………………………………………………………...……….(3)

We consider an infinite length of river drainage system where the velocity and density are confirmable; can we predict the river flow direction easily? Let us consider the two most essential variables density, $\rho(x,t)$ and velocity, $v(x,t)$ which encounter in this river flow problem solution. We assume $\rho(x,t)$ as plastic density, which is the number of plastics at time t at position x and $v(x,t)$ as river velocity at position x and time t. We defined the term flow rate of a river as a measure of the volume of water along with the dumping plastic that moves in a river drainage with a certain amount of time along density , ρ (i.e., $\rho = \frac{m}{V}$,mass per unit volume).Thus,

$$q(x,t) = \rho(x,t)v(x,t) \quad \text{..............................(4)}$$

Where $q(x,t)$ is the number of plastics per hour passing position x at time t .

Assuming the initial plastic density in a river is $\rho(x,0)$ at position x and time 0 , and the river velocity field remains constant for all time t . The mobility of each plastic in a river flow is obtained with the derivative of position x w.r.t. t which proves the following first order differential equation:

$$\frac{dx}{dt} = v(x,t) \text{ with initial condition } x(0) = x_0 \quad \text{..(5)}$$

Apparently, we can determine the position of plastics flow in river drainage at a particular time. It is necessary to know the river velocity and the initial plastic density to be able to predict the plastic density at a future time likely to happen when plastic consumption increases. If we know the velocity of a river flow, it would be easily calculate the plastic density at a later time. Suppose, we consider an interval on any particular river drainage pattern between $x = a$ and $x = b$ as follows:

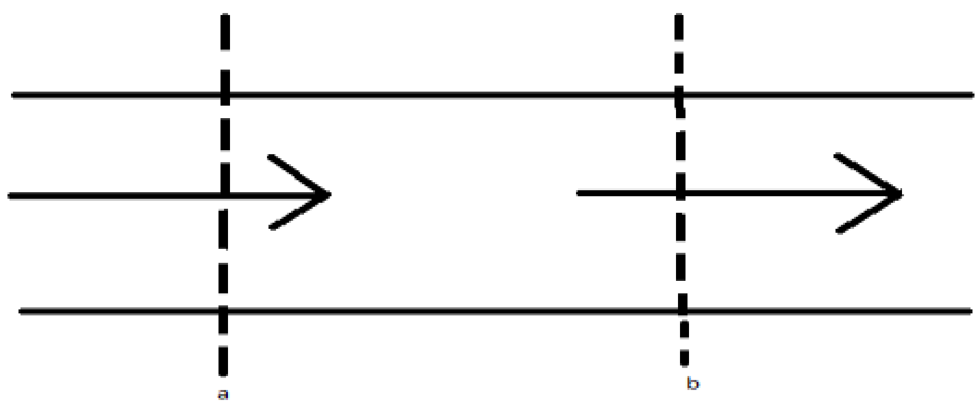

Figure 1: Flow chart of plastic wastes in a river segments.

The plastics density integrated in a river flow problem between the intervals $[a,b]$ such that,

$$N_p = \int_{x=a}^{x=b} \rho(x,t)dx \quad \text{,...(6)}$$

where N_p is denoted as the number of plastics dumping in a river.

Disposal of plastic wastes in a particular area of river drainage between the interval $x = a$ and $x = b$, the number of plastics could still be change in time, even if there is no addition of dumping or losing it due to the process of decomposition. Dumped a plastic wastes at $x = a$, the number of plastics increases and as plastics flow pass at $x = b$, the number of plastics decreases, therefore, the river flow rate $q(a,t)$ and $q(b,t)$is not same for all the time.

The rate of change of the number of plastics with respect to time, $\frac{dN_p}{dt}$ is equal to the number of plastics per unit time intruding the interval $[a,b]$at $x = a$ minus the number of plastics per unit time departing the interval $[a,b]$at $x = b$, where the plastics wastes always flow to the right direction along with the river flow direction , as explained in the equation given below since the rate of change of the number of plastics per unit time is the river flow at position $x = a$ minus position $x = b$ both at time t:

$$\frac{dN_p}{dt} = q(a,t) - q(b,t) \quad \text{.. (7)}$$

By taking the derivative of equation of (6) with respect to time, we get

$$\frac{dN_p}{dt} = \frac{d}{dt}\int_{x=a}^{x=b} \rho(x,t)dx \quad \text{...(8)}$$

From eqn.(7) and eqn.(8), we get,

$$\frac{d}{dt}\int_{x=a}^{x=b} \rho(x,t)dx = q(a,t) - q(b,t) \quad \text{..(9)}$$

R.H.S of eqn. (9) can be written by taking the partial derivative w.r.to x and also take the integral from $x = b$ to $x = a$ as follows

$$\frac{d}{dt}\int_{x=a}^{x=b} \rho(x,t)dx = \int_{x=b}^{x=a} \frac{\partial q(x,t)}{\partial x}dx \quad \text{..(10)}$$

Using an integral property, we get,

$$\frac{d}{dt}\int_{x=a}^{x=b}\rho(x,t)dx = -\int_{x=a}^{x=b}\frac{\partial q(x,t)}{\partial x}dx \quad \text{..(11)}$$

$$\frac{d}{dt}\int_{x=a}^{x=b}\rho(x,t)dx = \int_{x=a}^{x=b}-\frac{\partial q(x,t)}{\partial x}dx \quad \text{..(12)}$$

We convert derivative into partial derivative by moving the derivative inside the integral, thus

$$\int_{x=a}^{x=b}\frac{\partial}{\partial t}\rho(x,t)dx = \int_{x=a}^{x=b}-\frac{\partial q(x,t)}{\partial x}dx \quad \text{..(13)}$$

$$\int_{x=a}^{x=b}\left[\frac{\partial}{\partial t}\rho(x,t)+\frac{\partial q(x,t)}{\partial x}\right]dx = 0$$

$$\frac{\partial}{\partial t}\rho(x,t)+\frac{\partial q(x,t)}{\partial x} = 0$$

$$\frac{d\rho(x,t)}{dt}+\frac{\partial q(x,t)}{\partial x} = 0$$

An expression excluding the variables of the function gives

$$\frac{d\rho}{dt}+\frac{\partial q}{\partial x} = 0 \quad \text{..(14)}$$

This equation is known as the conservation of plastics.

We know that $q = \rho v$, and hence we can rewrite as

$$\frac{\partial q}{\partial x} = \frac{\partial}{\partial x}q(\rho,v)$$

$$\frac{\partial q}{\partial x} = \frac{\partial q}{\partial \rho}\frac{\partial \rho}{\partial x}+\frac{\partial q}{\partial v}\frac{\partial v}{\partial x}$$

From eqn. (14) we get

$$\frac{\partial \rho}{\partial t}+\frac{\partial q}{\partial \rho}\frac{\partial \rho}{\partial x}+\frac{\partial q}{\partial v}\frac{\partial v}{\partial x} = 0 \quad \text{..(15)}$$

This equation eqn.(14) is also known as conservation of plastics, as this eqn. is similar to eqn.(13)

Suppose ,we consider $\nu = \nu(\rho)$ and its derivative w.r.to x which indicates ,

$$\frac{\partial \nu}{\partial x} = 0 \quad \text{.....(16)}$$

Putting the value of eqn.(15) in eqn.(14), we obtained the result which gives as ,

$$\frac{\partial \rho}{\partial t} + \frac{\partial q}{\partial \rho}\frac{\partial \rho}{\partial x} = 0 \quad \text{.....(17)}$$

Velocity-density relationship

Several sources interrupt the flow of water within a river like sediment load, amount of soil and rocks etc. One of such major sources of marine pollution is also the dumping of plastics into the rivers hold-up the flow of river. The rate of discharge of water in a river channel cannot flow naturally due to habitat nature of society on the environment. Plastics have become highly influence the consumers in the marketplace since their commercial development in the 1930s and 1940s. With increase of utilization, dumping of plastics wastes also increases which results into river flow problem. Based on all these observations density of plastics plays an important role in controlling of flow velocities. We can clarify this assumption that the velocity of a river flow only depends on the density of plastics along the river at any point. This is explained in the equation below based on the conservation of plastics.

$$\nu = \nu(\rho)$$

We have discussed that, river velocity is at maximum when there is little or no dumping of plastics wastes in a river. This shows that the density is at zero when is no plastic waste at all. Consequently, the velocity will be at maximum with the density at zero which represents below:

$$\nu(0) = \nu_{\max}$$

When direct dumping of plastics into the rivers is increasing within the selected region, it is evidently will slow down the river flow and as the plastics density keeps increasing, the velocity of the river will also continue to decrease. As a result, the rate of change which is the derivative of the velocity with respect to density is given below:

$$\frac{d\nu}{d\rho} \equiv \nu'(\rho) \leq 0$$

At a point when the density of plastics is in its maximum, the river flow will be at zero velocity or reach a condition that there is no movement at all.

$$\nu(\rho_{\max}) = 0$$

Thus, the river velocity versus the plastic density is steady decreasing.

II. Conclusion

The method of characteristics is a technique for solving and converting a partial differential equation to a family of ordinary differential equations. However, in general terms, the method of characteristics is valid for any hyperbolic partial differential equation. Several numerical methods have also become approval from the viewpoint of computer programming and computational efficiency. However, the method of characteristics give more special emphasis for the reason close relationship between physical and mathematical properties enable this method to comprehend a basic concept and tool more simple in analyzing the complex problem of unsteady flow. The method is commonly considered to be more precise than others (Liggett and Woolhiser, 1967; McDowell, 1976) and is often used for comparison of yardstick. It appears to be more effective than other methods in controlling of relatively rapidly changing flows. The method of characteristics (MOC) has been approach to find the potential river flow because it is explicit and thus easy to layout and solve. The method of characteristics discovers curves called characteristic curves along which the partial differential equation(PDE) becomes anordinary differential equations (ODE). Onceanordinary differential equationsis obtained, it can be easily solved along the characteristic curves and converted into a solution for the original PDE. Geometrically, the method of characteristics is often used to interpret the required difficulties of nonlinear case in which the differential equation should be tangent to the graph of the solution. The directions of the characteristic lines specify the flow of values through the solution. We can think of each characteristics line suggested a solution to ν along itself. This type of information is very essential and used when solving partial differential equations numerically as it point out which finite differencescheme is best suited for the problem. Characteristics are also a powerful tool for obtaining qualitative comprehension into a PDE. It is generally examined and provides a mathematical modeling for the solution, along which the solution can be integrated from some initial data given on a suitable dimension of hypersurface. This mathematical modeling solves the solution to the problem of plastic dumping in a river so as to prevent river flow problem in future times. This numerical approach of solution is introduced to reverse this growing dumping plastic in a river channel which caused huge environmental problem. In general, it is more difficult and complicated to develop numerical methods for nonlinear hyperbolic conservation laws than to develop numerical methods for parabolic and elliptic partial differential equations. Furthermore, it is of great significance for maintaining wave behavior that is required for fast, short and brief calculations in flow dynamics.

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