M/M/1/WV Interdependent queueing model with controllable arrival rates, service rates with inspection, delayed repaired times and feedback

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Abstract

In this paper M/M/1/WV interdependent queueing model with controllable arrival rates, service rates with inspection delayed repair times and feedback is considered. For this markov model is developed to explore the performance analysis of a state interdependent working vacation queuing model with feedback. The server is subjected to breakdown randomly by providing services. When a breakdown occurs , the server is immediately sent to repair station where the repairmen takes a setup time before starting the repair. The failed server is inspected by the repairman and is there is minor problem the server is repaired and is sent back to the service station with probability q. if some major faults is diagnosed during inspection. The server requires second phase of the repair with probability p. The matrix geometric method is applied to obtained the system characteristics, queue size distributed and other performance indices for varying arrival rates when the arrival and services processes are interdependent. The analytical results are numerically illustrated and the effect of the nodal parameters on the system characteristics are studied and relevant conclusion is presented.

Keywords

Single server markovian queueing model, finite capacity working vacation, discouragement, delayed repair, controllable arrival rates, feedback.

I. INTRODUCTION

In the earlier work, Madhu Jain and Varsha Rani [3] have analysed the state dependent M/M/1/WV queueing system with inspection, controlled arrival rates & delayed repair are employed William J.Stewart [7] have studied the matrix geometric method for structured markov chain. M.Renisagaya Raj & B.Chandrasekar [6] has studied matrix geometric method for queueing model with subject to breakdown & N – policy vacation. Rani, G. & Srinivasan, A. [2] has studied M/M/1/k interdependent queueing model with controllable arrival rates & feedback. Rani, G. & Srinivasan, A. [1] have studied busy period analysis of M/M/1/∞ interdependent queueing model with controllable arrival rates and feedback. In this feedback is considered. In section II, the description of the queueing model is given stating the relevant postulates. In section III matrix geometric method is obtained. In section IV to provide some performance measures and in section V some special cases. In section VI the analytical results obtained by perform numerical experiments. In section VII we perform the sensitivity analysis.

II. DESCRIPTION OF MODELS

Consider single server finite capacity queuing system with controllable arrival rates and feedback customers arrive at the service station one by one according to a bivariate poisson stream with arrival rates λ_i-ε>0. Further system state probability are defined which are followed by chapman-kolmogorov equation. There is a single server providing service to all arriving customers either with feedback or without feedback service times are independent and identically distributed exponential random variable with service rate q (μ-ε). We consider an M/M/1WV unreliable server queuing model with interrupted working vacation and two phase repaired in case of server is failure. The first phase of the repair refers to the inspection of the server and is done by the repairman to remove minor faults after taking setup time as soon as the server breakdown occurs either with feedback or without feedback. The second phase is repaired is optional one and is rendered by the repairman when some faults are detected in the server. The failed server s inspected first and some minor faults are fixed up. It is sent to repair station for the second phase repaired with probability p= (1-q) ($0 \le p \le 1$) either with feedback or without feedback, in case some major faults is diagnosed during the inspection. Before starting is functioning the repairman takes a setup time. The repairman takes a setup time. The concept of discouragement, interdependent rates, controllable arrival rates, and state dependent arrival rates and state dependent arrival rates make the steady much closer to realistic situation after the completion of each service customer can either join at the end of queue with probability p or they can leave the system with probability q. The customer both newly arrived and those who opted for feedback are served in the order in which they join the tail of the original queue. It is assumed that there is no difference between regular arrival & feedback arrivals. The customers are served according to first server rule with the following assumptions are made. The arrival process $\{x_1(t)\}$ and service process $\{x_2(t)\}$ of the system are correlated and follow a bivariate poission distribution is given by

$$P(X_1 = x_1, X_2 = x_2, t) = e^{-(\lambda_i+\mu_i-\varepsilon)t} \frac{\sum_{j=0}^{\min(x_1,x_2)} (\varepsilon t)^j \left[(\lambda_i-\varepsilon)t\right]^{(x_1-j)} \left[(\mu_i-\varepsilon)t\right]^{(x_2-j)}}{j!(x_1-j)!(x_2-j)!} \qquad \lambda_i > 0, \mu_i > 0$$

For i= 0 & 1;the repair rates are ($\mu_v - \varepsilon$) and ($\mu_b - \varepsilon$) where us arrival rates are λ_0 , λ_1 ,λ_2 , λ_3 , λ_4 respectively where

x_1,x_2=0,1,2,3 and ε,0<ε min(λ_i, μ_i) for i=0,1

- The single server Markovian system is considered with the provision of working vacation if there is no customer in the system. During the busy period (working vacation), the customers arrive in the system in Poisson fashion with rate $\lambda 1$ ($\lambda 0$) and the server provides service according to exponential distribution with mean rate 1/q(μ_b-ε) (1/μ_v-ε).
- If the system is empty, the server starts a successive working vacation after the termination of first vacation. The duration of vacation is assumed to follow an exponential distribution with a mean 1/qδ.
- The server is prone to random breakdown during normal busy period. The life time of the server is exponentially distributed withmean1/qθ.

- After breakdown, the server is instantly sent at a repair station where the repairman takes a setup time before starting the first phase repair, i.e., inspection.
- The setup time of the repairman follows exponential distribution with mean $1/q\psi$. During the setup of the repairman, the customers arrive in the system in Poisson fashion with rate $\lambda 2$.
- The failed server can be restored in two phases of repair. The first phase of the repair refers to inspection facility of the server. During inspection, the customers arrive in the system in Poisson fashion with rate $\lambda 3$.After the inspection the server is sent back to the service station with probability q; the time of inspection is exponentially distributed with mean $1/q\eta e$.
- If some major fault is diagnosed during inspection, the server is sent for repair with probability $p(1-q)$.The time of repair is exponentially distributed with mean$1/\eta$. During repair, the customers arrive in the system in Poisson fashion with rate $\lambda 4$.
- If long waiting time is anticipated, the customers may balk from the queue with rate β instead of waiting in the queue.

For the sake of mathematical of mathematical formulation, the following steady state probability is defined:

$P_{0,n}$ the steady state probability that there are n customers in the system when the server is on working vacation either with feedback or without feedback

$P_{1,n}$ the steady state probability that there are n customers in the system when server is busy in providing service either with feedback or without feedback

$P_{2,n}$ the steady state probability that there are n customers in the system when the server is broken down and repairman is in set up state either with feedback or without feedback

$P_{3,n}$ the steady state probability that there are n customer in the system when the server is being inspected either with feedback or without feedback

$P_{4,n}$ the steady state probability that there are n customer in the system when the server is under repair due to some major fault either with feedback or without feedback

The transition rate diagram depicting the in-flow and rates for the system states is shown in figure 1. The steady state equation governing the models are constructed by balancing the flows for different system states as follows.

State 0 The server is on working vacation.

At state (0,n),the in –flow takes place from (0,n+1) with probability $p_{0,n+1}$ due to a service and from (0,n-1)with probability $p_{0,n-1}$ due to an arrival. Thus, we have

$$\{(\lambda_0-\varepsilon)\beta+q(\mu_v-\varepsilon)+\delta\}p_{0,1}=(\lambda_0-\varepsilon)\beta p_{0,n-1}+q(\mu_v-\varepsilon)p_{0,n+1}\ ;\ n>1 \quad (1)$$

State 1 The server busy in providing service

At state (1,n), the in-flow occurs from states $p_{0,n}$ from (1,n-1) with probability $p_{1,n-1}$,from(1,n+1)with probability $p_{1,n+1}$,from (3,n)with probability $P_{3,n}$ and from(4,n) with probability $p_{4,n}$.thus ,by balancing the in -flow with out- flow, we get

$$\{(\lambda_1-\varepsilon)\beta+q(\mu_b-\varepsilon)+\theta\}p_{1,n}=\delta p_{0,n}+((\lambda_1-\varepsilon)\beta p_{1,n-1}+q(\mu_b-\varepsilon)\ p_{1,n+1}+q\eta_e p_{3,n}+\eta p_{4,n} \quad (2)$$

State 2 The server is broken down and repairman is under setup.

At state (2,n), the incoming flows happen from states (2,n-1) and (1,n) with respective probabilities , so that

$$\{(\lambda_2-\varepsilon)\beta+q(\mu_b-\varepsilon)\}p_{2,n} =(\lambda_2-\varepsilon)p_{2,n-1}+\theta p_{1,n} \tag{3}$$

State 3 The repairman is inspecting the broken down server.

At state (3,n) the incoming flows occur from (3,n-1) and (2,n) with respective probabilities, now

$$\{(\lambda_3-\varepsilon)\beta+q(\mu_b-\varepsilon)+\eta_e\}p_{3,n} =(\lambda_3-\varepsilon)\beta p_{3,n-1}+p_{2,n} \tag{4}$$

State 4 The server is under repair. in case some major fault is diagnosed during inspection.

At state (4,n), the incoming flows happen from(4,n-1) and from (3,n) with respective probabilities. Thus , by balancing the in and out flows , we get

$$\{(\lambda_4-\varepsilon)\beta+\eta+q(\mu_b-\varepsilon)+\eta_e\}p_{4,n} =(\lambda_4-\varepsilon)\beta p_{4,n-1}+p\eta_e p_{3,n} \tag{5}$$

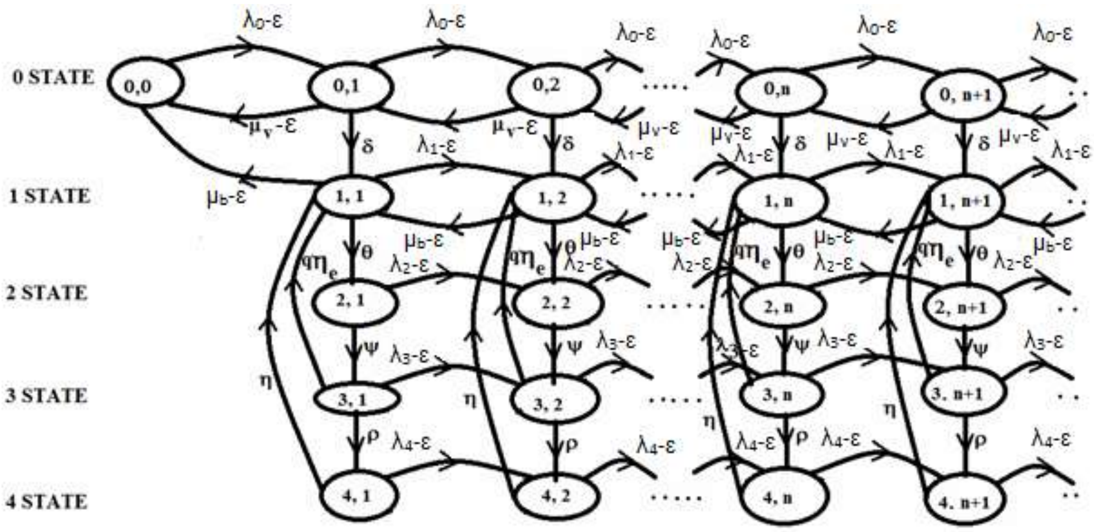

Figure: 1 Transition rate diagram

III. MATRIX GEOMETRIC SOLUTION

A closed form solution for the QBD process presented in section 2. In order to obtain an efficient and numerically state solution, we employ matrix geometric method to obtain the probabilities for markov chain. As there are repetitive block sub matrices in transition rate matrix, we can easily employ matrix geometric method to evaluate the stationary

Probability vector $P_n= \{p_{0,n,}\ p_{1,n}\ p_{2,n}\ p_{3,n}\ p_{4,n}\}$.

The transition rate matrix Q of the markov chain corresponding to the coefficients of equation (1)-(5) has the block tri diagonal from given by:

$$
Q = \begin{bmatrix}
B_0 & C_0 & & & & & & & & \\
A_0 & B_0 & C_1 & & & & & & & \\
 & A_1 & B_1 & C_1 & & & & & & \\
 & & A_1 & B_1 & C_1 & & & & & \\
 & & & & - & - & - & & & \\
 & & & & & - & - & - & & \\
 & & & & & & A_1 & B_1 & C_1 & \\
 & & & & & & & A_1 & B_1 & C_1 \\
 & & & & & & & & - & - & -
\end{bmatrix}
$$

The rate matrix Q of this stationary system is similar to QBD process. The sub matrices of the matrix Q are listed as below:

$B_0 = [-(\lambda_0-\varepsilon)]$, $C_0 = [(\lambda_0-\varepsilon)]$,

$$
A_0 = \begin{bmatrix} q(\mu_v-\varepsilon) \\ q(\mu_b-\varepsilon) \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$
A_1 = \begin{bmatrix}
q(\mu_v-\varepsilon) & 0 & 0 & 0 & 0 \\
0 & q(\mu_b-\varepsilon) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
B_1 = \begin{bmatrix}
-((\lambda_0-\varepsilon)\beta+q\mu_v+\delta) & \delta & 0 & 0 & 0 \\
0 & -((\lambda_1-\varepsilon)\beta+q\mu_v+\theta) & 0 & 0 & 0 \\
0 & 0 & -((\lambda_2-\varepsilon)\beta+\psi & \psi & 0 \\
q\eta_e & 0 & 0 & -((\lambda_3-\varepsilon)\beta+\eta_e q & p\eta_e \\
0 & \eta & 0 & 0 & -((\lambda_4-\varepsilon)\beta+q\eta
\end{bmatrix}
$$

$$
C_1 = \begin{bmatrix}
(\lambda_0-\varepsilon)\beta & 0 & 0 & 0 & 0 \\
 & (\lambda_1-\varepsilon)\beta & 0 & 0 & 0 \\
0 & 0 & (\lambda_2-\varepsilon)\beta & 0 & 0 \\
0 & 0 & 0 & (\lambda_3-\varepsilon)\beta & 0 \\
0 & 0 & 0 & 0 & (\lambda_4-\varepsilon)\beta
\end{bmatrix}
$$

Let P be the corresponding steady state probability vector of Q . By partitioning the vector as $P=\{P_{0,} P_1 P_2 ,\dots,P_n\}$ where $P_0=[P_{0,0}]$ is a non negative real number and $P_n=\{ P_{0,n}\ P_{1,n}\ P_{2,n},P_{3,n},P_{4,n}\}$; $n\geq 1$, is a row vector of dimension 5. The vector P satisfies PQ=0 and $P_e=1$ where e is a column vector of an appropriate dimension having each element as unit ‘1’. Apparently when the stability condition is satisfied, the sub vectors of P, corresponding to the different levels are given by $P_j = P_1 R^{i-1} = P_{j-1} R$, where $j \geq 1$. R is the minimal non – negative solution of the matrix of quadratic equation given by:

$$R^2A_1 + RB_1 + C_1 = 0 \tag{6}$$

The matrix R, $P_{0,}$ P_1 and P_n are obtained by solving

$$P_0B_0 + P_0RA_0 = 0 \tag{7}$$

$$P_0C_0 + P_1B_1 + P_2A_1 = 0 \tag{8}$$

$$P_{n-1}C_0 + P_nB_1 + P_{n-1}A_1 = 0 \tag{9}$$

Normalising condition $P_0 + P_1(1-R)^{-1}e = 1$ (10)

and boundary state equation $P_1(B_1 + RA_1) = 0$ (11)

IV. PERFORMANCE MEASURES

In order to analyse the characteristics of the single unreliable working vacation queue, it is necessary to provide some performance measures. we facilitate some formulae in terms of steady state probabilities in order to enhance the applicability of our analytical results. The performance indices such as expected number of customers during working vacation, busy period, setup, inspection and repair either with feedback or without feedback are formulated as follows:

- expected number of customers either with feedback or without feedback when the server is on working vacation, is:

$$E(V) = \sum_{n=0}^{\infty} nP_{0,n} \tag{12}$$

- expected number of customers either with feedback or without feedback when the server is busy in providing service is:

$$E(B) = \sum_{n=1}^{\infty} nP_{1,n} \tag{13}$$

- expected number of customers either with feedback or without feedback when the server is broken-down and the repairman is under setup state, is:

$$E(S) = \sum_{n=1}^{\infty} nP_{2,n} \tag{14}$$

- expected number of customers either with feedback or without feedback when the server is under inspection is obtained by using

$$E(R_j) = \sum_{n=1}^{\infty} nP_{3.n} \tag{15}$$

- expected number of customers either with feedback or without feedback when the server is under repair, is

$$E(R) = \sum_{n=1}^{\infty} nP_{4,n} \tag{16}$$

- expected number of customers in the system is obtained using

$$E(N)=E(V)+E(B)+E(S)+E(Rj)+E(R) \qquad (17)$$

- The throughput can be obtained as:

$$TP = \mu_v \sum_{n=0}^{\infty} P_{0,n} + \mu_b \sum_{n=1}^{\infty} P_{1,n} \qquad (18)$$

The expected delay time is given by:

$$E(D) = \frac{E(N)}{TP} \qquad (19)$$

Now we provide the cost function to minimize the expected total cost incurred per unit time by considering the following cost element

C_1 holding cost per customer per unit time

C_2 per unit time cost of the server during the working vacation

C_3 per unit time cost of the server when the server is in busy state

C_4 per unit time cost of the server during the broken down state when the repairman is under setup

C_5 per unit time cost of the server during the broken down state when the repairman is under inspection

C_6 per unit time cost of the server when the server is broken down and under repair

The expected total cost per unit time is:

$$E(TC)= C_1E(N)+ C_2E(V)+ C_3E(B)+ C_4E(S)+ C_5E(Ri)+ C_6E(R) \qquad (20)$$

V. SPECIAL CASES

1) This model includes the models studied earlier as particular cases. For example, when c=1,the model reduces to M/M/1/k model with interdependent inter arrival and service times, controlled arrival rates & feedback. When $\lambda_0=\lambda_1=\lambda$ and $\varepsilon=0$ & p=q=1, this model reduces to the M/M/c/k model discussed by Gross& Harris (1974) when p=q=1,this model reduces to M/M/c/k model with interdependent queueing model with controllable arrival rates discussed by Thiagarajan & Srinivasan (2007)
2) M/M/1/WV queueing system with delayed repair facility & inspection. Setting $\varepsilon=0,\beta=1$ $\lambda_0= \lambda_1 =\lambda_2 = \lambda_3= \lambda_4= \lambda$ our results can be used for the M/M/1/WV queueing system with delayed repair facility & inspection.

VI. NUMERICAL ILLUSTRATION

For verifying the computational tractability of the analytical results obtained for the model under consideration, we encode a computer programme in MATLAB software and perform numerical experiments.

For illustration purpose, probability vectors and various performance measures are obtained by fixing the default system parameters as

λ_0=0.9, λ_1=0.8, λ_2=0.3, λ_3=0.2, λ_4=0.1, μ_v =2 μ_b =4 δ=0.2 ,ε=0.02, β=0.2, ψ=0.5, θ=0.7, η_e=0.6, η=0.4, p=1/,

q=1/2

For these parameters we get sub matrices of the generator matrix as follows:

B_0= -[0.8800] , C_0= [0.8800 0 0 0 0]

A_0 = [0.99 1.99 0 0 0]

$$A_1 = \begin{bmatrix} 0.99 & 0 & 0 & 0 & 0 \\ 0 & 1.99 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_1 = \begin{bmatrix} -(1.3760) & 0.2 & 0 & 0 & 0 \\ 0 & -(1.8560) & 0.7 & 0 & 0 \\ 0 & 0 & -(0.5560) & 0.5 & 0 \\ 0.3 & 0 & 0 & -(0.3360) & 0.3 \\ 0 & 0.4 & 0 & 0 & -(0.2160) \end{bmatrix}$$

$$C_1 = \begin{bmatrix} 1.0800 & 0 & 0 & 0 & 0 \\ 0 & 0.9800 & 0 & 0 & 0 \\ 0 & 0 & 0.4800 & 0 & 0. \\ 0 & 0 & 0 & 0.3800 & 0 \\ 0 & 0 & 0 & 0 & 0.2800 \end{bmatrix}$$

The minimal non-negative rate matrix R is

$$R = \begin{bmatrix} 1.6719 & 0.4102 & 0.5164 & 0.7685 & 1.0674 \\ 1.6540 & 4.0492 & 5.0980 & 7.5862 & 10.5364 \\ 0.7088 & 1.0494 & 2.1845 & 3.2508 & 4.5150 \\ 0.6239 & 0.9239 & 1.6131 & 2.8618 & 3.9747 \\ 0.2888 & 0.7069 & 0.8900 & 1.3244 & 3.1368 \end{bmatrix}$$

The probability vectors are obtained as

$p_0 = [0.3777]$

$p_1 = [0.3450, 0.1190, 0.041, 0.014]$

$p_2 = [0.0225, 0.000008, 0.00000025, 0.000000000007, 1.61\text{x}10^{-14}]$

$P_3=[0.0005, 3.34\text{x}10^{-14}, 6.920\text{x}10^{-20}, 1.2433\text{x}10^{-27}, 1.40875\text{x}10^{-36}]$

$P_4= [0.000380, 6.321\text{x}10^{-10}, 1.7408\text{x}10^{-13}, 4.2536\text{x}10^{-18}, 4.8193\text{x}10^{-22}]$

Using equation (12)-(19) and above probability vectors, various performance measures are obtained as

E(N)=10.04,E(V)=4.11,E(B)=4.14, E(S)=1.41, E(R_j)=0.34, E(R)=0.04,TP=2.48,E(D)=

VII. SENSITIVITY ANALYSIS

In order to examine the sensitivity of different parameters on the performance measures, we perform the numerical experiments. The results of numerical simulations are facilitated in tabular forms.

Table 1(a) and Table 1(b) display the effect of μ_b on different performance indices for heterogeneous and homogeneous arrival rates, respectively. It is seen that as μ_b increases, the indices E(V), E(B), E(S), E(R_j) and E(R) decreases because an efficient service rate results into enhancement of efficiency of the system and shortens the queue length eventually. An increased μ_b reduces the delay time of the customers and improves throughput which coincides with our expectation. The comparative study of Table 1(a) and 1(b) reveals that the queue length in case of homogeneous arrival rates.

Table 1 (a) Performance measure for heterogeneous arrival rates w.r.t. μ_b
(b)) Performance measure for homogeneous arrival rates w.r.t. μ_b

(a)

μ_b	E(V)	E(B)	E(S)	E(R_j)	E(R)	TP	E(D)
4	2.43	2.47	0.8	0.2	0.49	1.47	4.34
5	2.43	2.46	0.6	0.13	0.01	1.71	3.29
6	2.43	2.46	0.56	0.09	0	1.96	2.79
7	2.43	2.46	0.48	0.06	0	2.21	2.45
8	2.43	2.45	0.42	0.05	0	2.45	2.18

(b)

μ_b	E(V)	E(B)	E(S)	E(R_j)	E(R)	TP	E(D)
4	2.43	2.47	0.8	0.2	0.49	1.47	4.34
4.5	2.43	2.46	0.75	0.16	0.01	1.59	3.65
5	2.43	2.46	0.6	0.13	0.01	1.71	3.29
5.5	2.43	2.46	0.61	0.11	0.01	1.84	3.05
6	2.43	2.46	0.56	0.09	0	1.96	2.79

Table 2(a) and 2(b) μ_v increases ,the indices E(V),E(B) decreases but E(S),E(R_j),E(R) remains same and improves throughput.

Table 2 (a) Performance measure for heterogeneous arrival rates w.r.t.μ_v

(b)) Performance measure for homogeneous arrival rates w.r.t.μ_v

(a)

μ_v	E(V)	E(B)	E(S)	E(R_j)	E(R)	TP	E(D)
1	2.11	2.97	1.82	0.44	0.05	1.47	5.06
2	1.42	2.01	1.82	0.44	0.05	1.47	3.9
3	1.2	1.69	1.82	0.44	0.05	1.47	3.53
4	1.08	1.53	1.82	0.44	0.05	1.47	3.34
5	1.02	1.44	1.82	0.44	0.05	1.47	3.24

(b)

μ_v	E(V)	E(B)	E(S)	E(R_j)	E(R)	TP	E(D)
1	2.11	2.97	1.82	0.44	0.05	1.47	5.06
1.5	1.65	2.33	1.82	0.44	0.05	1.47	4.27
2	1.42	2.01	1.82	0.44	0.05	1.47	3.9
2.5	1.29	1.82	1.82	0.44	0.05	1.47	3.68
3	1.02	1.44	1.82	0.44	0.05	1.47	3.24

Table 3(a) and 3(b) explore the effect of inspection rate η_e on the performance measures for heterogeneous and homogeneous traffic . It is clear from the table that an improved repair facility causes an increased availability of the server during different states of the server i.e, E(V) remains same, E(B) and E(S) is increases ,E(R_j) and E(R) decreases, the through (TP) increases but E(D) decreases.

Table 3 (a) Performance measure for heterogeneous arrival rates w.r.t.η_e

(b) Performance measure for homogeneous arrival rates w.r.t. η_e

(a)

η_e	E(V)	E(B)	E(S)	E(R_j)	E(R)	TP	E(D)
0.2	1.42	1.34	1.21	0.87	0.1	0.82	6.02
0.4	1.42	1.39	1.26	0.91	0.11	0.84	6.01
0.6	1.42	1.44	1.31	0.32	0.04	0.86	5.26
0.8	1.42	1.49	1.35	0.33	0.04	0.88	5.26
1	1.42	1.54	1.4	0.34	0.04	1.16	4.08

(b)

η_e	E(V)	E(B)	E(S)	E(R_j)	E(R)	TP	E(D)
0.3	1.42	1.37	1.24	0.3	0.03	0.83	5.61
0.5	1.42	1.42	1.28	0.31	0.03	0.85	5.24
0.7	1.42	1.47	1.33	0.32	0.04	0.87	5.21
0.9	1.42	1.52	1.38	0.34	0.04	0.89	5.12
1.1	1.42	1.57	1.42	0.34	0.04	0.91	5.03

Table 4(a) and 4(b) explore the effect of inspection rate η on the performance measures for heterogeneous and homogeneous traffic . It is clear from the table that an improved repair facility causes an increased availability of the server during different states of the server i.e, E(V) remains same, E(B), E(S), E(R_j) is increases ,E(R) decreases, the through (TP) increases but E(D) decreases.

Table 4 (a) Performance measure for heterogeneous arrival rates w.r.t.η

(b) Performance measure for homogeneous arrival rates w.r.t. η

(a)

η	E(V)	E(B)	E(S)	E(R_j)	E(R)	TP	E(D)
0.2	1.47	1.34	1.22	0.3	0.04	0.82	5.32
0.4	1.47	1.44	1.31	0.32	0.04	0.86	5.32
0.6	1.47	1.54	1.4	0.34	0.03	0.9	5.31
0.8	1.47	1.64	1.49	0.36	0.03	0.94	5.3
1	1.47	1.74	1.58	0.39	0.03	0.98	5.28

(b)

η	E(V)	E(B)	E(S)	E(R_j)	E(R)	TP	E(D)
0.3	1.47	1.39	1.26	0.31	0.04	0.84	5.32
0.5	1.47	1.49	1.35	0.33	0.03	0.88	5.3
0.7	1.47	1.59	1.44	0.35	0.03	0.92	5.3
0.9	1.47	1.69	1.53	0.37	0.03	0.96	5.21
1.1	1.47	1.79	1.62	0.4	0.03	1	5.2

Tables 5(a) and 5(b) exibit the sensitivity of the performance measure, depicting the behaviour of the system, with respect to the breakdown rate θ. We observe that E(V) is same , E(B) and E(R) decreases ,E(S) ,E(R_j) , TP ,E(D) increases .

Table 5 (a) Performance measure for heterogeneous arrival rates w.r.t.θ

(b) Performance measure for homogeneous arrival rates w.r.t.θ

(a)

θ	E(V)	E(B)	E(S)	E(Rj)	E(R	TP	E(D)
0.2	1.47	1.53	0.39	0.09	0.01	0.89	3.92
0.4	1.47	1.41	0.73	0.18	0	0.84	4.51
0.6	1.47	1.3	1.01	0.25	0	0.8	5.03
0.8	1.47	1.21	1.26	0.31	0	0.77	5.51
1	1.47	1.14	1.47	0.36	0	0.74	6

(b)

θ	E(V)	E(B)	E(S)	E(R_j)	E(R)	TP	E(D)
0.3	1.47	1.41	0.54	0.13	0	0.84	4.22
0.5	1.47	1.35	0.87	0.21	0	0.82	4.75
0.7	1.47	1.26	1.14	0.28	0	0.7	5.25
0.9	1.47	1.17	1.37	0.33	0	0.75	5.78
1.1	1.47	1.1	1.57	0.38	0	0.72	6.27

CONCLUSIONS

The working vacation queueing model analysed in this investigation, when the concept of breakdown and phase repair is incorporated. our study is that after breakdown, the server is sent for the repair where it is inspected first and then transferred to the repair station if some major fault is diagnosed during inspection. The concept of interdependent rates together with impatient nature of the customers makes the study more versatile and much closer to many practical queueing scenarios. The cost function will prove helpful to the system designers and decision makers to take optimal decisions.

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