Unidominating Function of A Cycle

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Abstract*: Unidominating function was introduced by V.Anantha Lakshmi and B.Maheshwari for path, In this paper we present the unidominating function on cycle and determine the unidomination number of cycle. Further we find the number of unidominating function of minimum weight for a cycle graph.*

Keywords: *Rooted Product graph, Unidominating functions, Unidomination number.*

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I. INTRODUCTION

Domination theory is an extremely rich area in graph theory with large varities of domination numbers defined by many researchers. Hedeteniem introduced the concept of dominating function with variations as k-domination, sign domination, minus domination. Researchers introducing the concept of unidominating function for path with restriction of sum of the function values of the neighborhood vertices exactly equal to one when f(u)=0. They found the unidomination number of path P_n is $\gamma_u(P_n) = \left\lceil\frac{n}{3}\right\rceil$. In this paper we present the unidominating function for a cycle and determine the unidomination number. Further we find the number of unidominating function of minimum weight for a cycle.

Definition 1.1: Let G(V,E) be a graph. A function $f: V \to \{0,1\}$ is said to be a unidominating function

If $\sum_{u\in N[v]} f(u) \geq 1$ and $f(v) = 1$

$\sum_{u\in N[v]} f(u) = 1$ and $f(v) = 0$

$f(V) = \sum_{u\in v} f(u)$ is called the weight of the function f and is denoted by $\gamma_u(G)$.

Definition 1.2 : The unidomination number of a graph G (V,E) is

$\gamma_u(G)$ = min $\{f(V)/f \ \ is\ a\ uni\ dominating\ function\ f\ on\ G\}$

II. UNIDOMINATION NUMBER OF A CYCLE

In this section we find the unidomination number of a cycle and also the number of unidominating functions of minimum weight for a cycle. Further the results obtained are illustrated.

Theorem 2.1: The unidomination number of a cycle is

$$\gamma_u(C_n) = \begin{cases} \left\lceil\frac{n}{3}\right\rceil \ for\ n \equiv 0,1(mod3) \\ \left\lceil\frac{n}{3}\right\rceil + 1 \ \ for\ n \equiv 2(mod3) \end{cases}$$

Proof: Let C_n be a cycle with vertex set V = $\{v_1, v_2, v_3 \dots \dots \dots \dots . v_n\}$. The degree of each vertex of a cycle graph is two. If $f(v_i) = 1$ then $\sum_{u \in N[v_i]} f(u) = f(v_{i-1}) + f(v_i) + f(v_{i+1}) \geq 1$ satisfied unidominating as $f(v_i) = 1$. If $(v_i) = 1$, for some vertex v in C_n then $\sum_{u \in N[v_i]} f(u) = f(v_{i-1}) + f(v_i) + f(v_{i+1})$ to be equal to 1, it is necessary that exactly one of $f(v_{i-1})$ or $f(v_{i+1})$ =1, that means exactly weight of a vertex with function value 0 should have function value 1. Therefore we need to check unidominating condition only for two vertices if $f(v_i) = 0$

If f is a unidominating function of C_n, then we can see that amongst three adjacent vertices in C_n, atmost two vertices can have functional value 0 and at least one vertex must have function value one.

Therefore sum of the functional values of three adjacent vertices is greater than or equal to one.

That is $\sum_{i=1}^{3} f(v_i) \geq 1 \sum_{i=4}^{6} f(v_i) \geq 1 \dots \dots \dots \dots \dots \dots . . \sum_{i=n-2}^{n} f(v_i) \geq 1$

Therefore f(v) = $\sum_{i=1}^{3} f(v_i) + \sum_{i=4}^{6} f(v_i) + \cdots \dots \dots \dots \dots . + \sum_{i=n-2}^{n} f(v_i)$

$\geq$ 1+1+1+.................................1$\geq \frac{n}{3}$

Hence minimum { f(v) / f is a unidominating function } $\geq \frac{n}{3}$

That is $\gamma_u(c_n) \geq \frac{n}{3}$ --------------------------- (1)

To find the unidomination number the following three cases arises.

Case (I): Let $\boldsymbol{n \equiv 0(mod3)}$

Define a function f: V → {0,1} by

$$f(v_i) = \begin{cases} 1 & if\ i \equiv 0(mod3) \\ 0 & if\ i \equiv 1,2(mod3) \end{cases}$$

Every vertex in C_n is of degree 2. Now we check the unidominating function at every vertex when f(v_i) = 0

Subcase (1) : Let i $\equiv 1(mod3)$ then f(v_i) = 0

When i$\neq$ 1, $\sum_{u \in N[V_i]} f(u) = f(v_{i-1}) + f(v_i) + f(v_{i+1}) = 0 + 0 + 1 = 1$

When i=1 , $\sum_{u \in N[V_1]} f(u) = f(v_n) + f(v_1) + f(v_2) = 0 + 1 + 0 = 1$

Subcase (2): Let i $\equiv 2(mod3)$ then f(v_i) = 0

When i$\neq$ 1 $\sum_{u \in N[V_i]} f(u) = f(v_{i-1}) + f(v_i) + f(v_{i+1}) = 0 + 0 + 1 = 1$

Since $\sum_{u \in N[v_i]} f(u) \geq 1$ for $f(v_i) = 1$

$\sum_{u \in N[v_i]} f(u) \geq 1$ for $f(v_i) = 0$

It follows that f is a unidominating function

Now f(V) = $\sum_{u \in V} f(u) = 0 + 0 + 1 + \cdots \dots \dots \dots \dots \dots \dots . . +0 + 0 + 1 = [\frac{n}{3}]$

$\therefore \gamma_u(c_n) \leq \left|\frac{n}{3}\right|$-------------------- (2)

From (1) and (2) for n$\equiv 0(mod3)$,$\gamma_u(c_n) = \left\lceil\frac{n}{3}\right\rceil$

Case (II) : Let n $\equiv$ $\mathbf{1}(\boldsymbol{mod3})$

Define a function $f: V \rightarrow \{0,1\}$ by

$$f(v_i) = \begin{cases} 1 & if\ i \equiv 0(mod3)\ and\ i = n-1 \\ 0 & if\ i \equiv 1,2(mod3) \end{cases}$$

So this function follows the pattern satisfies 001,001,001,……….001,0011. Which is identical with case(I) for $f(v_1), f(v_2) \dots\dots\dots\dots\dots\dots\dots f(v_{n-1})$. We need to check the unidomination condition at v_n with $f(v_n)$ =1.

$$\sum_{u\in N[V_n]} f(u) = f(v_{n-1}) + f(v_n) + f(v_{n+1}) = 1 + 1 + 0 = 2 \geq 1$$

It follows that ‘ f ‘ is a unidominating function with weight ,

f(V) $= \sum_{u\in V} f(u) = 0 + 0 + 1 + \cdots \dots\dots\dots\dots\dots\dots\dots.. + 0 + 0 + 1 + 1 = (\frac{n-1}{3}) + 1 = \frac{n+2}{3} = \left\lceil\frac{n}{3}\right\rceil$

From (1) and (3) for n$\equiv 1(mod3)$, $\gamma_u(c_n) = \left\lceil\frac{n}{3}\right\rceil$------------------ (3)

Case(III) : Let n $\equiv$ $\mathbf{2}(\boldsymbol{mod3})$

Define a function $f: V \rightarrow \{0,1\}$ by

$$f(V_i) = \begin{cases} 0\ if\ i \equiv 1,2(mod3) and\ i \neq n-1, n \\ 1\quad if\ i \equiv 0(mod3) and\ i = n-1, n \end{cases}$$

This function is identical to the function f defined in case I $f(v_1), f(v_2) \dots\dots\dots\dots\dots\dots\dots f(v_{n-2})$, except atv_{n-1}, v_n $with\quad f(v_{n-1}) = 1\ and\ f(v_n) = 1$

To check the unidomination condition i at v_n and v_{n-1}

$$\sum_{u\in N[V_{n-1}]} f(u) = f(v_{n-2}) + f(v_{n-1}) + f(v_n) = 1 + 1 + 1 = 3 \geq 1$$

$$\sum_{u\in N[V_n]} f(u) = f(v_{n-1}) + f(v_n) + f(v_1) = 1 + 1 + 0 = 2 \geq 1$$

It follows that f is an unidominating function

Now f(V) = $\sum_{u\in V} f(u) = 0 + 0 + 1 + 0 + 0 + 1 + \cdots \dots\dots\dots\dots.. + 0 + 0 + 1 + 1 + 1 = \left(\frac{n-2}{3}\right) + 1$

$$= \frac{n+1}{3} = \left\lceil\frac{n}{3}\right\rceil + 1$$

By the definition of unidomination number, we get

$\gamma_u(c_n) \leq \left\lceil\frac{n}{3}\right\rceil + 1$ ------------------------------(4)

From (1) and (4) we get for n$\equiv$ 2 (mod3) , $\gamma_u(c_n) = \left\lceil\frac{n}{3}\right\rceil + 1$

From case (I), (II) and (III) we write

$$\gamma_u(C_n) = \begin{cases} \left\lceil \frac{n}{3} \right\rceil \ for\ n \equiv 0,1(mod3) \\ \left\lceil \frac{n}{3} \right\rceil + 1 \ \ for\ n \equiv 2(mod3) \end{cases}$$

Theorem 2.2 : The number of unidominating functions with minimum weight for the cycle c_n is

$$\gamma_u(c_n) = \begin{cases} 1 & for\ n \equiv 0,1\ (mod3) \\ 1 + \dfrac{\left\lceil \frac{n}{3} \right\rceil (\left\lceil \frac{n}{3} \right\rceil - 1)}{2} & for\ \mathrm{n}\ \equiv 2(mod3) \end{cases}$$

Proof : Let c_n be a cycle with vertex set V = $\{v_1, v_2, v_3 \ldots \ldots \ldots \ldots . v_n\}$. To find the number of unidominating functions with minimum weight the following three cases arises

Case (I) : Let $n \equiv 0(mod3)$

The function f defined in case (I) of theorem is given by the function values of f are 001 001 --------001. Taking a = 001. Then the functional value of f are in the pattern of aaa…..aa. The functional value of f can also be arranged in different ways (i) 010 010……….010 (ii) 100 100 …….100. Consider b=010 then f can be written as bbb……b also c= 100 then f can be written as ccc…….c but as f is defined on a cycle graph , $\theta = \frac{2\pi}{n}$ anticlockwise rotation of f= aaa…….a will be bbb……..b and $\theta = \frac{2\pi}{n}$ anticlockwise rotation of f= bbbb…….b will be cccc…….c . We state that f=bbb…….b and ccc….….c are isomorphic to f= aaaa…..a.

Hence there is only one unidominating function of c_n with minimum weight $\left\lceil \frac{n}{3} \right\rceil$

Case (II) : Let n $\equiv 1(mod3)$

As explained in case (2) of theorem [2.1] is given by the functional value of f are 001 001……..0011. Taking a =001. Then the functional value of f can be written as the pattern of f = a a a …….a 1. This ' 1' can be placed anywhere at the $\left\lceil \frac{n}{3} \right\rceil$ positions between a's, the possible patterns are (i) a 1 a a………….a (ii) a a 1 a………….a (iii) a a a 1 a …………a etc. As these a's are arrange on a cycle , taking the starting point at the 'a ' next to position '1' we get every sequence (i) , (ii), (iii)….. are isomorphic copy to a a a ……..a 1.

Therefore there is unique unidominating function of c_n with minimum weight $\left\lceil \frac{n}{3} \right\rceil$.

Case (III) : Let n $\equiv 2(mod3)$ let n=3k+2

As explained in case (III) of theorem [2.1]is given by the functional value of f are

001 001 ………..001 11 . Taking a = 001. Then the functional value of f can we written as the pattern of f = a a a ………a 1 1 with minimum weight $\left\lceil \frac{n}{3} \right\rceil + 1$. The last two '11's can be placed anywhere between 'a' which is again isomorphic to the pattern a a a ………a 1 1due to cycle nature as discussed in case (ii) . Next consider b = 0011 then the pattern is a a a ………a b b is another pattern with the same minimum weight. These two b's and (k-2)a's can be arranged in cyclic permutation of repeated things in

$\frac{k!}{(k-2)!2!} = \frac{k(k-1)}{2}$ ways. So there are $\frac{k(k-1)}{2}$ non-isomorphic patterns of minimum weight $\left\lceil \frac{n}{3} \right\rceil + 1$

Therefore the total unidominating function of minimum weight $\left\lceil \frac{n}{3} \right\rceil + 1$ are

$1+\frac{k(k-1)}{2}$ where $k = \left\lceil \frac{n}{3} \right\rceil$

Hence the number of unidominating functions with minimum weight for the cycle c_n is

$$\gamma_u(c_n) = \begin{cases} 1 & for\ n \equiv 0,1\ (mod 3) \\ 1 + \frac{\left\lceil \frac{n}{3} \right\rceil (\left\lceil \frac{n}{3} \right\rceil - 1)}{2} & for\ n \equiv 2 (mod 3) \end{cases}$$

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