On Pre Generalized $\mu *$-Closed Set In Topological Spaces

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Abstract: In this article, we introduce a new class of sets called pre generalized μ *-closed sets in topological spaces (briefly pg μ *-closed set). Also we discuss some of their properties and investigate the relations between the associated topology.

AMS Subject Classification: 54A05

Key Words: pg μ *star-closed set, sg-closed sets, gb-closed sets, rgb-closed sets and gp$*$ -closed sets

I. INTRODUCTION

Levine [6] introduced the concept of semi-open sets and semi-continuity in topological spaces in 1963. In 1970 Levine [5] introduced the notion of generalized closed (briefly g-closed) set. Sundaram and Shiek John [10] introduced the concepts of weakly-closed set and Maki et al[7] introduced the notion of generalized pre-closed sets and investigated some of their basic properties. In 1995 Dontchev introduced the concept of generalized semi pre-open sets in topological spaces. Gnanambal [4] introduced the concept of gpr-closed sets and studied its properties in topological spaces in 1997. In this paper we define and study the properties of pre generalized μ * sets (pg μ *-closed) in a topological space, which is properly placed between pre-closed sets and generalized pre-closed sets.

II. Preliminaries

In this section, we referred some of the closed set definitions which was already defined by various authors.

Definition 2.1. [15] Let a subset A of a topological space (X, τ), is called a pre-open set if A ⊆ int(cl(A)).

Definition 2.2. [11] Let a subset A of a topological space (X, τ), is called a semi-open set if A ⊆ cl(int(A)).

Definition 2.3. [16] Let a subset A of a topological space (X, τ), is called a α -open set if A ⊆ int(cl(int(A))).

Definition 2.4. [4] Let a subset A of a topological space (X, τ), is called a b-open set if A ⊆ cl(int(A)) ∪ int(cl(A)).

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Definition 2.5. [10] Let a subset A of a topological space (X, τ), is called a generalized closed set (briefly g-closed) if cl(A) ⊆ U whenever A ⊆ U and U is open in X.

Definition 2.6. [12] Let a subset A of a topological space (X, τ), is called a generalized α closed set (briefly $g\alpha$ - closed) if αcl(A) ⊆ U whenever A ⊆ U and U is α open in X.

Definition 2.7. [2] Let a subset A of a topological space (X, τ), is called a generalized b- closed set (briefly gb- closed) if bcl(A) ⊆ U whenever A ⊆ U and U is open in X

Definition 2.8. [13] Let a subset A of a topological space (X, τ), is called a generalized α *-closed set (briefly g α *-closed) if αcl(A) ⊆ intU whenever A ⊆ U and U is α open in X.

Definition 2.9. [18] Let a subset A of a topological space (X, τ), is called a pre-generalized closed set (briefly pg- closed) if pcl(A) ⊆ U whenever A ⊆ U and U is pre-open in X.

Definition 2.10. [6] Let a subset A of a topological space (X, τ), is called a semi generalized closed set (briefly sg- closed) if scl(A) ⊆ U whenever A ⊆ U and U is semi open in X.

Definition 2.11. [19] Let a subset A of a topological space (X, τ), is called a generalized αb- closed set (briefly $g\alpha$b- closed) if bcl(A) ⊆ U whenever A ⊆ U and U is α open in X.

Definition 2.12. [14] Let a subset A of a topological space (X, τ), is called a regular generalized b- closed set (briefly rgb- closed) if bcl(A) ⊆ U whenever A ⊆ U and U is regular open in X.

III. Pre Generalized μ *-Closed Sets

In this section, we introduce pre generalized μ * -closed set and investigate some of their properties.

Definition 3.1. Let a subset A of a topological space (X, τ), is called a pre generalized μ *- closed set (briefly pgμ *- closed) if scl(A) ⊆ U whenever A ⊆ U and U is pre open in X.

Theorem 3.2. Every closed set is pgμ *-closed.

Proof. Let A be any closed set in X such that A ⊆ U, where U is pre open. Since scl(A) ⊆ cl(A) = A. There fore scl(A) ⊆ U. Hence A is pgμ *-closed set in X.

The converse of above theorem need not be true as seen from the following example. _

Example 3.3. Let X = {a, b, c} with τ = {X, ∅, {a, b}}. The set {a, c} is pg μ *-closed set but not a closed set.

Theorem 3.4. Every sg-closed set is pgμ *-closed.

Proof. Let A be any sg-closed set in X such that U be any pre open set containing A. Since A is sg closed, scl(A) = A. There fore scl(A) ⊆ U. Hence A is pg μ *-closed set.

The converse of above theorem need not be true as seen from the following example. _

Example 3.5. Let X = {a, b, c} with τ = {X, ∅ , {a}, {a, b}, }. The set {a, c} is pg μ *-closed set but not a sg-closed set.

Theorem 3.6. Every gα-closed set is pg μ *-closed set.

Proof. Let A be any gα-closed set in X and U be any pre open set containing A. Since A is gα closed, scl(A) $\subseteq\alpha$cl(A) $\subseteq$ U. There fore scl(A) $\subseteq$ U. Hence A is pg μ *-closed set.

The converse of above theorem need not be true as seen from the following example. _

Example 3.7. Let X = {a, b, c} with τ = {X, $\emptyset$, {a}, {c}, {a, c}}. The set {a} {c} is pg μ *-closed set but not a gα-closed set.

Theorem 3.8. Every gα*-closed set is pg μ *-closed set.

Proof. Let A be any gα*-closed set in X and U be any pre open set containing A. Since A is gα* closed, scl(A) $\subseteq\alpha$cl(A) $\subseteq$ U. There fore scl(A) $\subseteq$U. Hence A is pg μ *-closed set.

The converse of above theorem need not be true as seen from the following example. _

Example 3.9. Let X = {a, b, c} with τ = {X, $\emptyset$, {b}, {a, b}}. The set {c} is pg μ *-closed set but not a gα*-closed set.

Theorem 3.10. Every g-closed set is pg μ *-closed set.

Proof. Let A be any g-closed set in X and U be any open set containing A. Since every open set is pre open set, scl(A) $\subseteq$ cl(A) $\subseteq$ U. There fore scl(A) $\subseteq$ U. Hence A is pg μ *-closed set.

The converse of above theorem need not be true as seen from the following example. _

Example 3.11. Let X = {a, b, c} with τ = {X, $\emptyset$, {a}, {a, b}}. The set {a,b} is pg μ *-closed set but not a g-closed set.

Theorem 3.12. Every gαb-closed set is pg μ *-closed set.

Proof. Let A be any gαb-closed set in X and U be any α-open set containing A. Since every α-open set is pre open set, scl(A) $\subseteq$ cl(A) $\subseteq$ U. There fore scl(A) $\subseteq$ U. Hence A is pg μ *-closed set.

The converse of above theorem need not be true as seen from the following example. _

Example 3.13. Let X = {a, b, c} with τ = {X, $\emptyset$, {a}, {b}, {a, b}, {b, c}}. The set {a} is pg μ *-closed set but not a gαb-closed set.

Theorem 3.14. Every rgb-closed set is pg μ *-closed set.

Proof. Let A be any rgb-closed set in X and U be any regular open set containing A. Since every regular open sets are α-open sets and every α -sets are pre open set, scl(A) $\subseteq$ U and U is pre open. Hence A is pg μ *-closed set.

The converse of above theorem need not be true as seen from the following example. _

Example 3.15. Let X = {a, b, c} with τ = {X, ∅, {a}, {b}, { b}, {b, c}}.
The set {a} is pgμ *-closed set but not a rgb-closed set.

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