Some Results on Accurate Split Domination And Accurate Non-Split Domination of Some Special Graphs

T. Aswini#1, Dr. k. Ameenal Bibi#2, M. Hoorulayeen#3

1 and 3 M.Phil. scholars, PG and Research Department of Mathematics,
D.K.M College for Women (Autonomous).
Thiruvalluvar University
2 Associate Professor of Mathematics, D.K.M College for Women (Autonomous),
Thiruvalluvar University,
No.57, DKM college road, Sainathapuram, RV Nagar, Vellore, Tamil Nadu 632001, India

Abstract — *Let G = (V, E) be a simple, finite, connected and undirected graph. A non-empty subset D⊆V is called a dominating set if every vertex in V-D adjacent to at least one vertex in D. The minimum cardinality taken over all the minimal dominating sets of G is called the domination number of G, denoted by γ(G). A dominating set D of V is an accurate dominating set if V-D has no dominating set of |D|. The accurate domination number $\gamma_a(G)$ is the minimum cardinality of a minimal accurate dominating set of G. An accurate dominating set D of V is an accurate split dominating set if <V-D> is disconnected. The accurate split domination number $\gamma_{as}(G)$ is the minimum cardinality of a minimal accurate split dominating set of G. An accurate dominating set D of V is an accurate non-split dominating set if <V-D> is connected. The accurate non-split domination number $\gamma_{ans}(G)$ is the minimum cardinality of a minimal accurate non-split dominating set of G. In this paper, we obtained some results on $\gamma_a(G)$ and $\gamma_{ans}(G)$ of some special graphs.*

Keywords — *Dominating set, Accurate dominating set, Accurate split dominating set, accurate non-split dominating set, Accurate split domination number and Accurate non-split domination number.*

Mathematics subject classification: 05C69

I. INTRODUCTION

Let G = (V, E) be a simple, finite, connected and undirected graph. Let n and m be the order and size of the graph G. Any undefined term in this paper can be found in [1, 2]. In this paper, we obtained accurate split domination numbers and accurate non-split domination numbers of some special graphs. Graphs are used as models for both natural and human made structures. The transformation of graphs are often formalized and represented by graph rewrite systems. In 1958, Claude Berge in his book on Graph theory defined the concept of domination number of a graph under the name "coefficient of external stability." In 1962 O. Ore in his book on "Graph Theory" used the terminology **"dominating set"** and "domination number." In 1977, Cockayne and Hedetniemi in their survey article used γ(G) to denote the domination number of a graph and since then, it has become the accepted the notation.

Dominations in graphs have a wide range of real life applications. Theory of dominations is widely used in facility locations tasks, telecommunications, radar stations, coding theory, transhipment and assignment techniques.

II. BASIC DEFINITIONS

2.1 Accurate dominating set

A non-empty subset $D \subseteq V$ in a graph G = (V, E) is a dominating set of G if every vertex in V-D is adjacent to some vertex in D. The **domination number γ(G)** of G is the minimum cardinality of a minimal dominating set of G.

2.2 Accurate dominating set

A dominating set $D \subseteq V$ of a graph G = (V, E) is an accurate dominating set, if V-D has no dominating set of cardinality |D|. The **accurate domination number γ_a(G)** of G is the minimum cardinality of a minimal accurate dominating set of G.

2.3 Accurate split dominating set

An accurate dominating set $D \subseteq V$ is an accurate split dominating set, if the induced subgraph <V-D> is disconnected and it has no dominating set of cardinality |D|. The **accurate split domination number γ_{as}(G)** of G is the minimum cardinality of a minimal accurate split dominating set of G.

2.4 Accurate non-split dominating set

A dominating set $D \subseteq V$ is an accurate non-split dominating set, if the induced subgraph <V-D> is connected and it has no dominating set of cardinality |D|. The **accurate non-split domination number γ_{ans}(G)** of G is the minimum cardinality of a minimal accurate non-split dominating set of G.

III. DEFINITIONS OF SOME NAMED GRAPHS

3.1 Stacked book graph

It is a graph with 12 vertices and 16 edges.

3.2 Unit distance graph

It is a graph with 10 vertices and 17 edges.

3.3 Bull graph

The bull graph is a planar undirected graph with 5 vertices and 5 edges, in the form of a triangle with two disjoint pendants.

3.4 Herschel graph

The Herschel graph is a bipartite undirected graph with 11 vertices and 18 edges.

3.5 Pappus graph

The pappus graph is a bipartite 3-regular undirected graph with 18 vertices and 27 edges.

3.6 Grotzsch graph

The Grotzsch graph is a triangle –free graph with 11 vertices and 20 edges.

3.7 Franklin graph

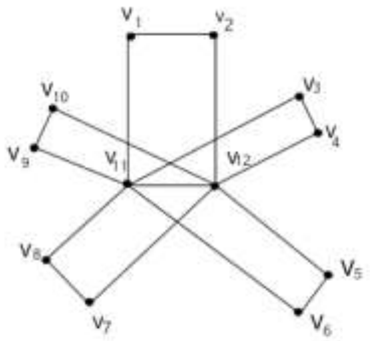
The Franklin graph is a 3-regular graph with 12 vertices and 18 edges. It was discovered by Philip Franklin.

3.8 Hexahedral graph

It is a graph with 6 vertices and 10 edges.

IV. RESULTS ON ACCURATE SPLIT AND NON-SPLIT DOMINATION NUMBERS OF SOME SPECIAL GRAPHS

4.1 Stacked book graph



From the figure,
$V = \{v_1, v_2, \ldots v_{12}\}$
$\gamma(G) = 2 \longrightarrow D = \{v_{11}, v_{12}\}$
$\gamma_a(G) = 2 \longrightarrow D_a = \{v_{11}, v_{12}\}$
$\gamma_{as}(G) = 2 \longrightarrow D_{as} = \{v_{11}, v_{12}\}$

Result

If G is a stacked book graph then $\gamma_a(G) = \gamma_{as}(G)$.

Proof

Let G = (V, E) be a stocked book graph with 12 vertices and 16 edges.

Let $V = \{v_1, v_2, \ldots v_{12}\}$.

The minimum dominating set is $D = \{v_{11}, v_{12}\}$

The induced sub graph <V-D> = $\{v_1, \ldots v_{10}\}$

The minimum dominating set of <V-D> is $\{v_1, v_3, v_5, v_7, v_9\}$ and the cardinality is $|D_1| = 5$

∴ <V-D> has no dominating set of cardinality |D|.

$\gamma_a(G) = 2$

Hence, $D = \{v_{11}, v_{12}\}$ is the accurate dominating set of G.

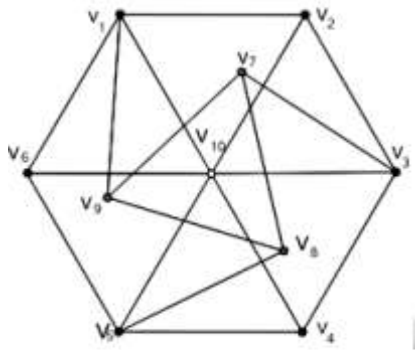
From the graph G, it is found that the induced sub graph <V-D> is disconnected.

∴ $D = \{v_{11}, v_{12}\}$ is the accurate split dominating set of G.

$\gamma_{as}(G) = 2.$

Hence, $\boldsymbol{\gamma_a(G) = \gamma_{as}(G)}$.

4.2 Unit distance graph



From the figure,

V= $\{v_1, v_2, \ldots v_{12}\}$

$\gamma(G) = 2 \longrightarrow D = \{v_7, v_{10}\}$

$\gamma_a(G) = 2 \longrightarrow D_a = \{v_7, v_{10}\}$

$\gamma_{ans}(G) = 2 \longrightarrow D_{ans} = \{v_7, v_{10}\}$

Result

If G is the unit distance graph then there exists an accurate non-split dominating set of G.

Proof

Let G = (V, E) be the unit distance graph with 10 vertices and 17 edges.

Let V = $\{v_1, v_2, \ldots v_{10}\}$.

The minimum dominating set is D = $\{v_{10}, v_7\}$

$|D| = 2$

Let <V-D> = $\{v_1, \ldots v_9\}$

The induced sub graph <V-D> has the minimum dominating set $D_1 = \{v_1, v_3, v_5\}$.

$|D_1| = 3$

D = $\{v_7, v_{10}\}$ is the accurate dominating set of G

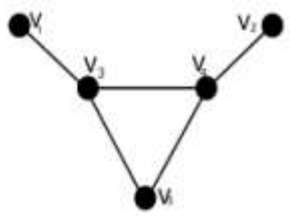
Hence, $\gamma_a(G) = 2$.

Moreover, the induced sub graph <V-D> is connected.

Hence, D = $\{v_7, v_{10}\}$ is an accurate non-split dominating set of G.

$\gamma_{ans}(G) = 2$.

4.3 Bull graph



From the figure,

V= $\{v_1, v_2, \ldots v_5\}$

$\gamma(G) = 2 \longrightarrow D = \{v_3, v_4\}$

$\gamma_a(G) = 2 \longrightarrow D_a = \{v_3, v_4\}$

$\gamma_{as}(G) = 3 \longrightarrow D_{as} = \{v_1, v_2, v_5\}$

Result

If G is the Bull graph then the graph G has an accurate split dominating set. For any bull graph, $\gamma_{as}(G) = n-k$ where k is the number of end vertices of G.

Proof

Let G = (V, E) be the Bull graph with 5 vertices and 5 edges.

Let V= $\{v_1,v_2,v_3,v_4,v_5\}$.

The minimum Dominating set D = $\{v_3,v_4\}$

$|D| = 2$

Hence, $\gamma(G) = 2$

Let the induced sub graph <V-D> = $\{v_1,v_2,v_5\}$ and it forms a minimum dominating set

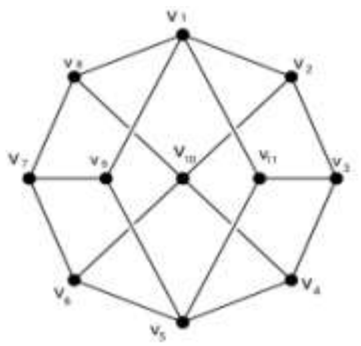
$D_1 = \{v_1,v_2,v_5\}$ and $|D_1| = 3$.

Then, D = $\{v_3,v_4,\}$ is the accurate dominating set of G and $\gamma_a(G) = 2$.

Moreover, the induced subgraph <V-D> is disconnected and it has isolated vertices.

Hence, $\boldsymbol{\gamma_{as}(G) = n\text{-}k}$

4.4 Herschel graph



From the figure,

V= $\{v_1,v_2, \ldots v_{11}\}$

$\gamma(G) = 3 \longrightarrow D = \{v_9, v_{10},v_{11}\}$

$\gamma_a(G) = 3 \longrightarrow D_a = \{v_9, v_{10},v_{11}\}$

$\gamma_{ans}(G) = 3 \longrightarrow D_{ans} = \{v_9, v_{10},v_{11}\}$

Result

If G is the Herschel graph then G has the accurate non-split dominating set of G.

Proof

Let G = (V, E) be the Herschel graph with 11 vertices and 15 edges.

Let V = $\{v_1,v_2,\ldots\ldots v_{11}\}$.

The minimum dominating set D = $\{v_9, v_{10},v_{11}\}$ and <V-D> = $\{v_1, v_2, v_3,v_4, v_5, v_6, v_7, v_8\}$.

Then, $D_1 = \{v_1, v_3, v_5, v_7\}$ and has no dominating set of cardinality $|D| = 3$

Hence, D = $\{ v_9, v_{10}, v_{11}\}$ is the accurate dominating set of G.

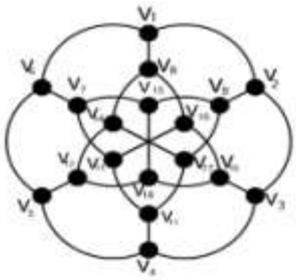
$\gamma_a(G) = 3$

Moreover, <V-D> is connected.

Thus, D = $\{ v_9, v_{10}, v_{11}\}$ is the accurate non-split dominating set of G.

$\therefore \gamma_{ans}(G) = 3$

4.5 Pappus graph



From the figure,

V= $\{v_1, v_2, \ldots v_{18}\}$

$\gamma_a(G) = 6 \longrightarrow D_a = \{v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\}$

$\gamma_{as}(G) = 6 \longrightarrow D_{as} = \{v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\}$

Result

If G is the pappus graph then G has the accurate split dominating set and $\gamma_a(G) = \gamma_{as}(G) = 6$.

Proof

Let G = (V, E) be the pappus graph with 18 vertices and 27 edges.

Let V = $\{v_1, v_2, \ldots\ldots\ldots v_{18}\}$.

The minimum dominating set D = $\{v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\}$.

Let <V-D> = $\{v_1, v_2, v_3, v_4, v_5, v_6, v_{13}, v_{14}, v_{15}, v_{16}, v_{17}, v_{18}\}$

The minimum dominating set $D_1 = \{v_1, v_4, v_{13}, v_{14}, v_{15}\}$

$|D_1| = 5$.

Hence, D = $\{v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\}$ is the accurate dominating set of G.

$\therefore \gamma_a(G) = 6$

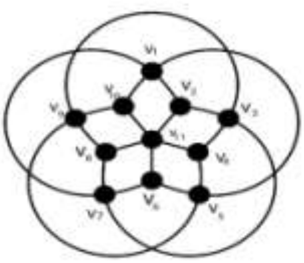
Moreover, <V-D> is disconnected.

Thus, D = $\{v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\}$ is the accurate split dominating set of G

$\therefore \gamma_{as}(G) = 6$

Hence, $\boldsymbol{\gamma_a(G) = \gamma_{as}(G) = 6}$

4.6 Grotzsch graph



From the figure,

V= $\{v_1, v_2, \ldots v_{11}\}$

$\gamma(G) = 3 \longrightarrow D = \{v_1, v_3, v_{11}\}$

$\gamma_{as}(G) = 3 \longrightarrow D_{as} = \{v_1, v_3, v_{11}\}$

Result

If G is the Grotzsch graph then G has the accurate split dominating set.

Proof

Let G = (V, E) be the Grotzsch graph with 11 vertices and 20 edges and the vertex set

V = $\{v_1, v_2, \ldots v_{11}\}$.

The minimum dominating set is D = $\{v_1, v_3, v_{11}\}$ and $\gamma(G) = 3$.

Consider, the induced sub graph <V-D> = $\{v_{2,} v_{4,} v_{5,} v_6, v_7, v_8, v_9, v_{10}\}$.

The minimum dominating set is $D_1 = \{v_{2,} v_{4,} v_{7,} v_9\}$.

$|D_1| = 4$.

Hence, <V-D> has no dominating set of cardinality $|D| = 3$.

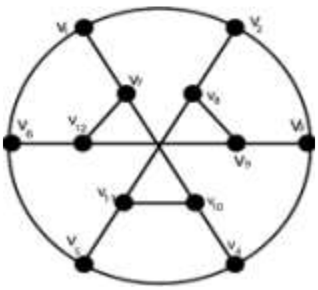
Thus, $D = \{v_{1,} v_{3,} v_{11}\}$ is the accurate dominating set of G.

Moreover, the induced sub graph <V-D> is disconnected.

Hence, $D = \{v_{1,} v_{3,}v_{11}\}$ is the accurate split dominating set of G.

$\therefore\ \gamma_{as}(G) = 3$

4.7 Franklin Graph



From the figure,

$V= \{v_1,v_{2,} \ldots..v_{12}\}$

$\gamma_a(G) = 4 \longrightarrow D_a = \{v_{1,} v_{4,} v_{7,} v_8\}$

$\gamma_{ans}(G) = 4 \longrightarrow D_{ans} = \{v_{1,} v_{4,} v_{7,} v_8\}$

Result

If G is the Franklin graph then $\gamma_a(G) = \gamma_{ans}(G) = 4$.

Proof

Let G = (V, E) be the Franklin graph with 12 vertices and 18 edges.

Let $V = \{v_1,v_2,\ldots\ldots\ldots..v_{12}\}$.

The minimum dominating set $D = \{v_{1,} v_{4,} v_{7,} v_8\}$ and $\gamma(G) = 4$.

Consider, the induced sub graph <V-D> = $\{v_{2,} v_{3,} v_{5,} v_6,v_7,v_9,v_{10},v_{11},v_{12}\}$ and the minimum dominating set $D_1 = \{v_{3,} v_{6,} v_{10}\}$.

$|D_1| = 3$

Hence, $D = \{v_{1,} v_{4,} v_{7,} v_8\}$ is the accurate dominating set of G.

$\therefore\ \gamma_a(G) = 4$

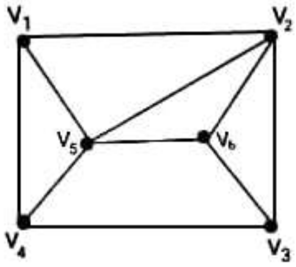
Moreover, the induced subgraph <V-D> is connected.

$\therefore D = \{v_{1,} v_{4,} v_{7,} v_8\}$ is the accurate non-split dominating set of G.

$\therefore\ \gamma_{ans}(G) = 4$

Hence, $\boldsymbol{\gamma_a(G) = \gamma_{ans}(G) = 4}$.

4.8 Hexahedral graph



From the figure,

$V = \{v_1, v_2, \ldots v_6\}$

$\gamma(G) = 2 \longrightarrow D = \{v_1, v_3\}$

$\gamma_{ans}(G) = 2 \longrightarrow D_{ans} = \{v_1, v_3\}$

Result

If G is the hexahedral graph then the graph G has the accurate non-split dominating set. Also, $\gamma(G) = \gamma_{as}(G) = 2$.

Proof

Let G = (V,E) be the Hexahedral graph with 6 vertices and 10 edges.

Let $V = \{v_1, v_2, \ldots\ldots, v_6\}$.

Then the minimum dominating set $D = \{v_1, v_3\}$.

$$\therefore \gamma(G) = 2$$

Then, the induced sub graph $\langle V\text{-}D\rangle = \{v_2, v_4, v_5, v_6\}$ and it has the minimum dominating set $D_1 = \{v_5\}$.

$$|D_1| = 1$$

$\langle V\text{-}D\rangle$ has no dominating set of cardinality $|D| = 2$

Hence, $D = \{v_1, v_3\}$ is the accurate dominating set of G.

Moreover, the induced subgraph $\langle V\text{-}D\rangle$ is connected.

$\therefore D = \{v_1, v_3\}$ is the accurate non-split dominating set of G and $\boldsymbol{\gamma_{ans}(G) = 2}$

Hence, the Hexahedral graph has an accurate non-split dominating set and

$$\boldsymbol{\gamma(G) = \gamma_{as}(G) = 2.}$$

V. CONCLUSIONS

In this paper, we obtained the accurate split domination and accurate non-split domination of some special graphs. The relationships between some domination parameters are also verified.

REFERENCES

[1] T.W. Hynes, S.T. Hedetriemi and P.J. Slater Fundamentals of Domination in Graphs, Marcel Dekkar, New York (1990).
[2] T.W. Hynes, S.T. Hedetriemi and P.J. Slater, Domination in Graph, Advanced Topic: Marcel Dekkar, New York (1998).
[3] O. Ore, Theory of Graphs, American math colloq (1962).
[4] V.R. Kulli, Advances in Domination Theory I & II Vishwa International publications, Gulbarga, Indian (2012, 2013).
[5] V.R. Kulli, College Graph Theory, Vishwa International publications, Gulbarga, Indian (2012).
[6] V.R. Kulli and B. Janakiram, The non-split domination number of a graph, Indian Journal of pure and applied mathematics (1996) 27(6), 537-542.
[7] V.R. Kulli and M.B. Kattimani, The accurate domination number of a graph, Technical Report 2000-2001 Gulbarga university, Gulbarga (2000).
[8] V.R. Kulli and B. Janakiram, The split domination number of a graph, Graph theory Notes of New York, New York Academy of science XXXII pp: 16-19.
[9] S. Arumugam and S. Ramachandran, Invitation of Graph Theory, Scitech publications, India (2010).
[10] Bordy J and Murthy V.S.R Graph Theory with Applications Mac Millan Press Landon (1976).