Challenges in Teaching Linear Algebra for Undergraduate Engineering Students

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Abstract *: Linear algebra plays a vital role in various fields of engineering and its applications. Linear algebra is an essential subject for engineering students. This focuses not only on computation but also on reasoning and theory. Teaching Linear algebra has various challenges like time management to cover the syllabus, devoting class time for homework questions and in-depth problem solving. In paper discusses some of the particular challenges in teaching linear algebra and its methods and techniques used to overcome the challenges. Also, this paper survey significant pedagogical innovations employed in teaching linear algebra.*

Keywords — *Teaching Issues, Linear Algebra, Teaching Strategies, Technology Tools*

I. INTRODUCTION

The topics and techniques of linear algebra are especially important for under graduate students. Engineering and computer science majors have an inherent need for understanding the practical applications of linear algebra in their discipline. Many researches revel that linear algebra is an important. Further, mathematical reasoning skill will be developed by studying linear algebra in depth. The linear algebra at the sophomore level course, and it is a required for engineering, mathematics, and computer science majors. The challenge in teaching linear algebra is balancing the allotted time for theory and applications.

Sepideh Stewart et. Al. [1] presented issues in teaching and learning of linear algebra around the world. This paper discussed how to promote conceptual understanding of Linear Algebra, problems and issues, with a particular focus on mathematical problems that are productive for learning. The major key issues are [1]:

- How applications of Linear Algebra be used as motivation for studying?
- What are the advantages of proving results in Linear Algebra in different ways?
- In what ways can a linear algebra course be adapted to meet the needs of students from other disciplines, such as engineering, physics, and computer science?
- How can challenge problems be used in teaching Linear Algebra?
- In what way should technology be used in teaching Linear Algebra?
- What is the role of visualization in learning Linear Algebra?
- In what order (pictures, symbols, definitions, and theorems) should we teach Linear Algebra concepts?
- How can we educate students to appreciate the importance of deep understanding of Linear Algebra concepts?

In literature, researcher stated major issues faced by students in learning linear algebra. Staib [2] stated that the applications of linear algebra to real time problems are not immediate. Students felt that the Linear algebra is difficult subject. The reason of this is probably linear algebra has many notions and should have much concentration in understanding and solving.

II. THE FACTORS AFFECTING LINEAR ALGEBRA

2.1 Formalism

The theoretical approach of Mathematics is Formalism. In the twentieth century, modern formalist, mathematics is a play made by interferences. There is only way to evaluate the mathematics is to arrive proof [3]. The difficulty of Students' to understand the linear algebra is interpreting the underlying concepts of the formalism within linear algebra. Dorier [4] studies state that 'meta level activities' is that linear algebra students have to be used their previous knowledge and have to be related with new formal concepts in linear algebra. This point inferred that it is vital that students learning linear algebra should have previous knowledge or some idea about the concepts and axiomatic structures of linear algebra is one of the most important.

2.2 Teaching Strategies

Sinan AYDIN [5] described some strategies to develop the teaching method of linear algebra there are various alternatives, including using of projects, group activities, portfolios and technology. Three principles of teaching linear algebra is [6] the Concreteness Principle, the Necessity Principle and the Generalisability Principle. The Concreteness Principle is students to abstract a mathematical structure from a given model. Students must see a need to learn for what they are intended to be taught is Necessity Principle state. The last is Generalisability When instruction is concerned with a 'concrete' model should satisfy the Concreteness Principle, the instructional activities within this model should allow encouraging the students to the concepts".

2.3 Basic Curriculum for Linear Algebra

Linear Algebra Study Group recommended basic Core Syllabus and prerequisite for Linear Algebra course [7]. The following core syllabus are recommended for 26 – 28 days.

2.3.1 Matrix Addition and Multiplication

This topic includes the operations of matrix like addition multiplication transposition and scalar multiplication. In addition, it has the properties of algebraic such as associativity of matrix multiplication.

2.3.1.1 Systems of Linear Equations

- Gaussian elimination/elementary matrices.
- Echelon and reduced echelon form.
- Existence/uniqueness of solutions.
- Matrix inverses.
- Row reduction interpreted as an LU -factorization.

2.3.1.2 Determinants

- Determinants have the following topics in the linear algebra curriculum.
- Cofactor expansion
- Determinants and row operations
- det AB = det A det B
- Cramer's Rule (to show the sensitivity of solutions to Ax = b).

The determinants are useful while solving 2 by 2 and 3 by 3 general linear systems. Using the resulting expressions, the elementary factors of determinants can be encountered.

2.3.1.3 Properties of R^n

Vector Spaces is an important topic in Linear Algebra. The following topics of Vector Spaces are inevitable in the course curriculum.
Linear combinations: linear dependence and independence.

2.3.1.4 Bases of R^n

- Subspaces of R^n: spanning set, basis, dimension, row space and column space (range of A as a mapping), null space.
- Matrices as linear transformations.
- Rank: row rank = column rank, products, connections with invertible submatrices.
- Systems of equations revisited: solution theory, rank + nullity = number of columns.
- Inner product: length and orthogonality, orthogonal/orthonormal sets and bases, orthogonal matrices.

2.3.1.5 EigenValues & EigenVectors

Numerous geometrics examples may be used to teach Eigenvalues and Eigenvectors. For wide range of application Eigenvalues and Eigenvectors are essential.

- The equation $Ax = \lambda x$.
- The characteristic polynomial
- Identification of its coefficients (trace, determinant),
- Algebraic multiplicity of eigenvalues.
- Eigenspaces, geometric multiplicity.
- Similarity: distinct eigenvalues and diagonalization (with emphasis on AP = PD).
- Symmetric matrices: orthogonal diagonalization, quadratic forms.

2.3.1.6 Orthogonality

- Orthogonal projection onto a subspace;
- Gram-Schmidt orthogonalization and interpretation as a QR factorization;
- The least square solutions of inconsistent linear systems, with applications to data-fitting.

III. Technology in Teaching Linear Algebra

Linear algebra teachers and mathematics education researches are agree that using technology is necessary at teaching and learning linear algebra. There are several different roles that technology can play instruction, from eliminating computational drudgery in applications, to providing environments for actively exploring the properties of mathematical concepts and structures. The technology preferred by teachers to teach linear algebra are:

- MATLAB
- Maple
- Mathemtica
- Mathwright
- Cabri
- Linalg
- Geogebra

Some of teachers prefer computer projects to be done outside of class. Some of teachers use computer demos and examples to enrich lectures and to make a helpful visualization. Others rarely lecture at all with a significant proportion of class time spent interacting with the computer. Computer programs provide students a means of instantly and effortlessly performing in linear algebra, and thus free them to concentrate on what the computations mean, and when and why to perform them.

Many teachers use software in this context. The focus is not necessarily on linear algebra applications. Rather, students are intended to answer questions about what happens when certain computations are performed, without having to think too much about the mechanics of carrying out the operations.

For example, students might experiment with the effect of triangular matrices as multipliers, without actually performing all the matrix multiplications by hand. There are two classic approaches about using computers in linear algebra course; first, some instructors think that doing some of the matrix multiplications by hand provides insight about why results appear as they do. Second, some instructors think that the ability to rapidly investigate a large number of examples makes a contribution to understanding. uses computer projects in linear algebra.

The purpose of each of these projects is to introduce students to a new subject in linear algebra through a hands-on approach. They are intended to provide motivation for new definitions, show the need for the new theorems, make conjectures, and realize the usefulness of the new theorems by applying them to solve various problems. They can be used with any linear algebra software. Each student is allowed to choose an applied problem in the student’s area of interest. The written project can be combined with an oral presentation. There are many good sources of problems, since many of the current linear algebra textbooks emphasize applications to a wide choice of different fields.

IV. OTHER METHODS

One of the techniques the researchers explored in literature is Inquiry based learning (IBL). This technique can concentrate vigorously on verification composing. The researchers felt that utilizing just this strategy could forget about an excessive number of the computational abilities and applications. Another instructional method is Problem based learning (PBL). A PBL course can be revolved around an intricate issue which requires the strategies of the course to tackle. Understudies work in gatherings and become familiar with the substance as they need it to take care of the issue. The creators felt utilizing exclusively this methodology could lead the understudies to concentrate on "simply taking care of the issue" and maybe scam the hypothesis behind the application. Another methodology is utilizing innovation as a reason for showing the substance. Now, the creators feel that utilizing this strategy alone may likewise lean too vigorously toward application and away from hypothesis. In building up the altered talk with exercises design, the creators attempted to pick assignments and partition class time to mirror the requirements of the understudies. Right now, the changed talk with exercises group contains components of customary talk, IBL and PBL. The test for development rests in the ideal results for this specific straight variable based math course. The pre-designing majors need computational aptitudes just as the capacity to realize when to apply straight variable based math procedures. Fundamentally, straight variable based math ought to be a critical thinking instrument for them. No doubt, the hypothesis will be auxiliary for the pre-designing majors as they progress through the building educational program. The math majors need the computational abilities too; however, they will likewise need to comprehend the confirmations of the hypotheses to be capable take care of issues as their educational program advances. The creators accept that the two abilities are significant for every understudy of direct polynomial math. Be that as it may, the test of adjusting these requests frequently implies the requirements of the distinctive understudy populaces may not be completely met. The two creators have considerable experience with unadulterated science, and subsequently they frequently feel the course could be reinforced with more order explicit applications

CONCLUSIONS

Linear algebra is an important subject in engineering and its various applications. This paper reviewed the challenges in teaching and learning linear algebra in undergraduate level. Also, reviewed curriculum for teaching linear algebra and technologies to teach linear algebra course.

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